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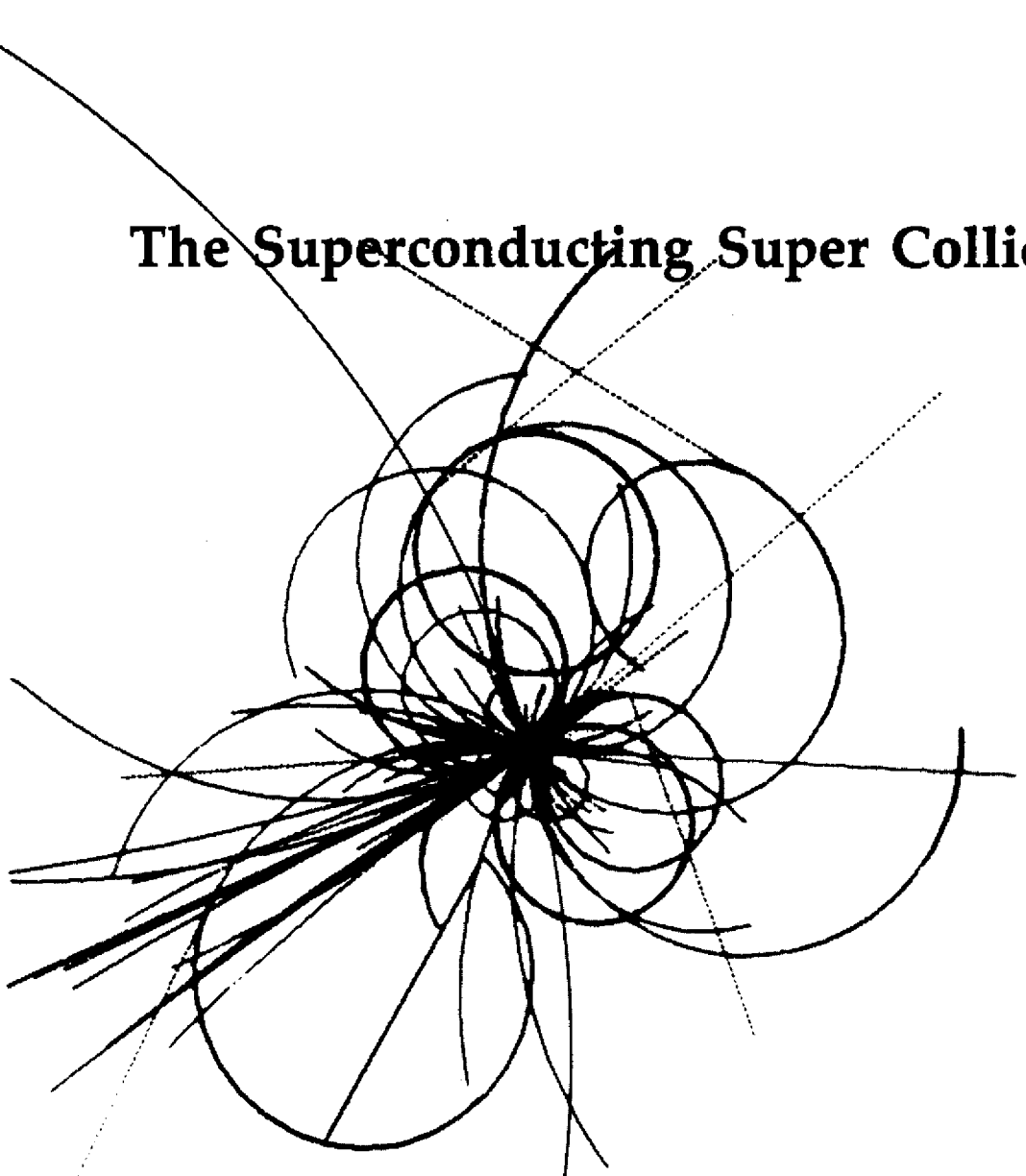

Study of the Aberrations of a Periodic Arc Using the Lie Algebraic Techniques

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ABSTRACT

The aberrations of a periodic achromat are analyzed using the map of a single periodic cell. An action-angle analysis of the Lie operators describing a cell is used to illustrate the differences amongst various phase advances per cell.

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1. Introduction

Since the introduction of second order matrix methods in accelerator physics, algebraic expressions for the matrix have been used in designing systems with little aberration.

Our purpose is to show how one can use a Lie algebraic representation to gain additional insight.

This paper will not be an exposé on Lie algebraic techniques. Indeed, some advanced tools used in our analysis require separate documentation. We expect that future papers, in collaboration with Alex Dragt and his group, will provide the general theory.

Here our goal is more modest. We intend to show how simple symmetries permit a classification of the aberrations in an arc of a particle accelerator. This work arose in connection with the design of the Superconducting Super Collider (SSC).

We will adopt the Einstein summation convention: Repeated indices are summed over.

2. *The matrix representation of a map*

Our objective is to study the motion of a charged particle around an ideal orbit. It is therefore useful to expand around this orbit. Let us denote by $\vec{Z}$ a complete set of canonically conjugate variables: [1,2]

$$\vec{Z} = (x,\ y,\ t,\ p_x,\ p_y,\ p_t) \tag{2.1}$$

The variables x and y measure the transverse displacements of a particle from the ideal orbit. The variable t measures the differential time of flight. It is closely related to the usual arc length used in matrix theory.

The variables p_x and p_y are to first order just

$$\frac{dx}{ds} \quad \text{and} \quad \frac{dy}{ds}$$

respectively, where the coordinate s is the arc length measured along the ideal orbit. Finally, p_t is the negative of the differential energy. This is a simple function of the more commonly used differential momentum δp.

In matrix theory, one describes the motion from an initial position s^{in} to a final one s^{fin} by a Taylor series. The Taylor series is said to be a representation of the map $\mathcal{M}$ from s^{in} to s^{fin}.

$$Z_i^{fin} = R_{ij} Z_j^{in} + T_{ijk} Z_j^{in} Z_k^{in} + W_{ijk\ell} Z_j^{in} Z_k^{in} Z_\ell^{in} + \ldots \tag{2.2a}$$

or symbolically,

$$\vec{Z}^{fin} = \mathcal{M}\, \vec{Z}^{in}. \tag{2.2b}$$

The matrices R, T and W contain all the relevant information concerning the particle, up to third order in Z^{in}. They are, of course, functions of s^{in} and s^{fin}.

Expressions for the aberration matrices T and W do exist. These expressions, in the case of T, have been used extensively to understand and design systems without any second order aberrations. [3]

However, the matrix representation for the map $\mathcal{M}$ is not ideal. First of all, it does not make use of the symplectic nature of the map $\mathcal{M}$. Secondly, as a direct consequence, it is hard to analyze the various symmetries of these matrices. For these reasons, we will use a different parametrization of $\mathcal{M}$.

3. The Lie algebraic representation of the map $\mathcal{M}$

In this section, we introduce the Lie algebraic representation of $\mathcal{M}$. We invite the reader to consult the appropriate references for a more complete exposition. [1, 2, 4, 5]

Consider two functions of the initial phase space $\vec{Z}^{in}$. We define the Poisson bracket in the usual manner:

$$[f(\vec{Z}^{in}), g(\vec{Z}^{in})] = \frac{\partial f}{\partial Z_i^{in}} J_{ij} \frac{\partial g}{\partial Z_j^{in}} \quad , \tag{3.1a}$$

$$J = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix} \tag{3.1b}$$

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad . \tag{3.1c}$$

We then define a Lie operator :f: associated to the function f with the help of the Poisson bracket:

$$:f: g = [f, g] \ . \tag{3.2}$$

Finally, we construct the Lie transformation M_f associated to f as follows:

$$M_f = \exp\ (:f:) \tag{3.3a}$$

$$\exp\ (:f:) = \sum_{n=0}^{\infty} \frac{:f:^n}{n!} \tag{3.3b}$$

$$:f:^0 = \text{identity} \ . \tag{3.3c}$$

We remark that M_f is a symplectic map.

It is therefore tempting to replace the matrix representation of equation (2.2a) by a Lie polynomial representation. This is the content of the factorization theorem. [5]

This theorem states that M can be factored as a product of symplectic transformations. Each one is of increasing order in the deviation around the ideal orbit. Mathematically, it goes as follows:

$$M = M_2 M_3 M_4 \cdots \tag{3.4a}$$

$$M_n = \exp(:f_n:) \tag{3.4b}$$

$$f_n(\vec{Z}^{in}) = \text{homogeneous polynomial of degree n in } \vec{Z}^{in} . \tag{3.4b}$$

The content of M_2 is equivalent to that of the matrix R_{ij}. M_2 is a linear symplectic map transforming $\vec{Z}^{in}$ in the following fashion:.

$$M_2 Z_i^{in} = R_{ij} Z_j^{in} . \tag{3.5}$$

In general, the transformation $M_n (n>2)$ will produce aberrations of order n-1 and higher. For example, all the conventional second order transport theory can be recovered by the study of f_2 and f_3. In fact, all the polynomial f_n can be computed in quadrature involving the Hamiltonian. These formulae are similar to the expression relating the aberrations matrices to the Hamiltonian and the linear matrix R. In this paper, we will only look at properties derived from simple symmetry arguments. This allows us to avoid the use of the actual expression for the matrices and the polynomials. However, for the sake of completeness we give the connection between f_3 and the Hamiltonian [4]:

$$f_3 = - \int_{s^{in}}^{s^{fin}} H_3 \, (R(s \leftarrow s^{in}) \, \vec{Z}^{in}; \, s) \, ds . \tag{3.6}$$

H_3 = Third order term of H .

The reader familiar with matrix theory, will recognize that the presence of R in (3.6) is linked to the Green's function of matrix theory. It is counter-productive to try to establish more connections, as doing so would

destroy the symplectic nature of the Lie representation. Despite this, it is often useful to connect a matrix to the Lie representation. For example, the T_{ijk} matrix is connected to f_3 by the formula.

$$f_3 = \frac{1}{3} J_{ia} T_{ajk} R^{-1}_{j\ell} R^{-1}_{km} Z^{in}_i Z^{in}_\ell Z^{in}_m \tag{3.7}$$

After this brief introduction, we introduce a decomposition of the Lie polynomial which provides an insight, as to the symmetries present in a system.

4. The resonance basis

In the standard Hamiltonian theory of accelerator physics, it is convenient to rewrite the Hamiltonian in terms of action-angle variables. We will follow a similar path with the map M . To set up a realistic problem, we assume that M is a map for which M_2(or R) is stable. In other words, the eigenvalues of R are on the unit circle. We also assume that the energy coordinate p_t is conserved.

In such cases, it is possible to make a linear symplectic similarity transformation A_2 on M_2 such that M_2 reduces to a map N_2 of the form:

$$N_2 = \exp(:-\mu_x J_x - \mu_y J_y - \frac{\alpha}{2} p_t^{in^2}:) \tag{4.1a}$$

$$J_x = \frac{1}{2}(X^{in^2} + p_x^{in^2}),\ J_y = \frac{1}{2}(Y^{in^2} + p_y^{in^2}) \tag{4.1b}$$

Using the properties listed in the appendix, we can derive the effect of A_2 on M .

$$N = A_2 M A_2^{-1} = A_2 \exp(:f_2:) A_2^{-1} A_2 \exp(:f_3:) A_2^{-1} \ldots$$

$$= \exp(:g_2:) \exp(:g_3:) \exp(:g_4:) \ldots \tag{4.2a}$$

$$g_n = A_2 f_n(\vec{Z}^{in}) = f_n(A_2 \vec{Z}^{in}) \tag{4.2b}$$

$$g_2 = -\mu_x J_x - \mu_y J_y - \frac{\alpha}{2} p_t^{in^2} . \tag{4.2c}$$

Of course, A_2 is selected such that g_2 takes the simple form of equation (4.2c). This can be done by looking at the eigenvectors of the transpose of the matrix R. [6]

It should be noted, that we assume that the temporal (or longitudinal) momentum is conserved. This implies that the variable t is absent from the f's or the g's.

As we already said, we have to introduce a new basis for the description of M. This is easily done using N. Consider the linear eigenvectors of $:g_2:$. There are four of them [6]:

$$h_\pm = \frac{1}{\sqrt{2}} (x^{in} \pm i\, p_x^{in}) = J_x^{1/2} e^{\mp i\phi_x} \tag{4.3a}$$

$$v_\pm = \frac{1}{\sqrt{2}} (y^{in} \pm i\, p_y^{in}) = J_y^{1/2}\, e_\pm^{i\phi_y} \tag{4.3c}$$

The linear functions $h_\pm$ and $v_\pm$, have the following properties:

$$:g_2: h_\pm = \mp i\mu_x h_\pm \tag{4.4a}$$

$$:g_2: v_\pm = \mp i\mu_y\, v_\pm . \tag{4.4b}$$

These functions, with the variable p_t, form a new basis for the polynomial g_n. In general, we can write g_n as follows:

$$g_n(\vec{Z}^{in}) = \sum_{a+j+k+\ell+m=n} A^m_{ajk\ell}\, h_+^a\ h_-^j\ v_+^k\ v_-^\ell\ p_t^{in^m} \tag{4.5a}$$

or

$$g_n(\vec{Z}^{in}) = \sum_{a+j+k+\ell+m=n} A^m_{ajk\ell} p_t^{in^m} \; J_x^{\frac{a+j}{2}} J_y^{\frac{k+\ell}{2}} \exp\{-i(a-j)\phi_x - i(k-\ell)\phi_y\} \qquad (4.5b)$$

To simplify the notation, we rewrite (4.5b) using a new notation:

$$g_n(\vec{Z}^{in}) = \sum_{a+j+k+\ell+m=n} A^m_{ajk\ell} \; |m>|\, ajk\ell> \quad . \qquad (4.6)$$

We now observe that the monomial $|m>|\, ajk\ell>$ is an eigenvector of degree $n(=a+j+k+\ell+m)$ of $:g_2:$.

$$:g_2: \; |m>|\, ajk\ell> = -i\,\{(a-j)\mu_x + (k-\ell)\mu_y\}\; |m>|\, ajk\ell> \qquad (4.7a)$$

It follows that this relation is true:

$$\exp(:bg_2:)\; |m>|\, ajk\ell> = \exp\{-ib((a-j)\mu_x - i(k-\ell)\mu_y)\}|m>|\, ajk\ell>. \qquad (4.7b)$$

The term "resonance basis" is suggested by the eigenvalue of $|m>|\, ajk\ell>$. We will see that "problems" arise precisely when the eigenvalue in (4.7a) is a multiple of $2\pi i$.

In the next section, we will raise the map M to an arbitrary power.

5. The power of a map and the second order achromat

The process of tracking N turns is equivalent to raising the map for a full turn of the machine to an integer power N. In a large machine, like the contemplated SSC, producing a full arc of a machine is also equivalent to raising a map to a power. For example, the distributed lattice of the SSC will have at a least of the order of 60 cells in one superperiod. What is the effect of raising the map for one cell of the 60th power?

This question can be readily answered by a "resonance basis" analysis. As a first example, we will consider a map M known to second order only.

$$M = \exp(:f_2:)\; \exp(:f_3:)\; \ldots \qquad (5.1a)$$

$$M = A_2^{-1} \; N \; A_2 \tag{5.1b}$$

Raising M to the Nth power, is equivalent to raising the map N to the same power. In fact, M^N is just:

$$M^N = A_2^{-1} \; N^N \; A_2 \tag{5.2}$$

Furthermore, N^N is given by:

$$N^N = \exp(:Ng_2:) \exp(:G_3:) \; \ldots \tag{5.3a}$$

$$G_3 = \sum_{n=0}^{N-1} \exp(:-ng_2:) g_3 \quad . \tag{5.3b}$$

This last expression for G_3 can be derived using the formulae in the appendix. To proceed further, we decompose G_3 using the "resonance" basis. Then G_3 becomes:

$$G_3 = \sum_{m+a+j+k+\ell=3} A^m_{ajk\ell} \sum_{n=0}^{N-1} \exp(in\lambda_{ajk\ell}) |m> |\, ajk\ell> \tag{5.4c}$$

$$\lambda_{ajk\ell} = (a-j)\mu_x + (k-\ell)\mu_y \; . \tag{5.4b}$$

Now, we turn to a specific example, that of the achromatic arc. Consider an arc made of N cells containing chromaticity correcting sextupoles. For clarity in this presentation, we ignore the presence of any other insertions (low beta regions etc...). For such an achromatic arc, G_3 is extremely simple:

$$G_3 = A^3_{0000} |3> |\, 0000> = A^3_{0000} \; p_t^{in^3} . \tag{5.5}$$

How can this be realized? It was discovered using matrix theory [3] that a special choice of phases for the cells insures the result of (5.5). We will rederive this result.

Let us choose the phase advance per cell in both planes to be

$$\frac{2\pi}{p} \; (= \mu_x = \mu_y).$$

We now look at all the possible values of $\lambda_{ajk\ell}$ with the restriction $m+a+j+k+\ell=3$. These are given in Table 1.

For all the terms in (5.4) for which d/p is not an integer, we can perform the sum using a geometric series. The result is:

$$G^m_{3,\ ajk\ell} = \frac{1 - \exp(-i2\pi\frac{Nd}{p})}{1 - \exp(\frac{-i2\pi d}{p})} A^m_{ajk\ell} \ . \tag{5.6}$$

$$\frac{d}{p} \neq 0, \pm 1, \ldots$$

For phase advances smaller than 120° (p=3), the denominator of (5.6) is always non-vanishing. Therefore we have the result:

$$\left.\begin{array}{l} p > 3 \\ d \neq 0 \\ \frac{N}{p} = \text{integer} \end{array}\right\} G^m_{3,\ ajk\ell} = 0 \quad . \tag{5.7}$$

There are five terms for which d=0.

$$\left.\begin{array}{l} |2>|1100> \\ |2>|0011> \end{array}\right\} \text{first order chormaticities} \tag{5.8a}$$

$$\left.\begin{array}{l} |2>|0110> \\ |2>|1001> \end{array}\right\} \nu_x - \nu_y \text{ driving terms} \tag{5.8b}$$

$$|3>|0000> \qquad p_t^3 \text{ term} \ . \tag{5.8c}$$

The chromaticities are locally corrected, hence the terms in (5.8a) are set to zero in every cell. The $\nu_x - \nu_y$ terms are absent if mid-plane symmetry is satisfied. As advertized, only (5.8c) remains.

In the next section, we look at the third order map.

6. The third order content of a map raised to a power

Let us consider the map for one cell up to the order of g_4. Our goal is to compute M^N. Again in terms of N we must have:

$$M^N = A_2^{-1} N^N A_2 = A_2^{-1} \exp(:Ng_2:) \exp(:G_3:) \exp(:G_4:) A_2 \tag{6.1}$$

$$= A_2^{-1} \exp(:A^3_{0000} p_t^{in^3}:) \exp(:G_4:) A_2$$

And, G_4 is given by:

$$G_4 = \sum_{n=0}^{N-1} \exp(:-ng_2:) \gamma_4 \tag{6.2}$$

This quantity γ_4 is computed using canonical perturbation theory [6]. It is given by the expression

$$\gamma_4 = g_4 + \frac{1}{2} [F_3, f_3] \tag{6.3a}$$

$$F_3 = \frac{1}{1 - \exp(:-g_2:)} \left\{ g_3 - A^3_{0000} \, p_t^{in^3} \right\} . \tag{6.3b}$$

We have to subtract the secular term in $p_t^{in\ 3}$ otherwise the denominator in (6.3b) would vanish.

As we did before, we proceed to analyze G_4 in the "resonance" basis:

$$G_4 = \sum_{m+a+j+k+\ell=4} \Gamma^m_{ajk\ell} \sum_{n=0}^{N-1} \exp(:-ng_2:) \, |m>| \, ajk\ell> \tag{6.4a}$$

$$\gamma_4 = \sum_{m+a+j+k+\ell=4} \Gamma^m_{ajk\ell} |m>| \, ajk\ell> \tag{6.4b}$$

Again, the effect of the map is easily evaluated in this eigenbasis:

$$G_4 = \sum_{m+a+j+k+\ell=4} \Gamma^m_{ajk\ell} \sum_{n=0}^{N-1} \exp(in\lambda_{ajk}\ell) \, |m>| \, ajk\ell>. \tag{6.5}$$

We now produce in Table 2 every possible value for m and d. Table 2 tells us that secular terms are produced for all values of p. These are the d = 0 terms. They are given in Table 3.

These terms are always present even if the product Nd/p is an integer. Symmetry alone cannot remove these terms. As before their value for one arc is N-times the individual cell value.

One notices that d/p can also take the values ±1. This adds an additional set of secular terms for the commonly used 90° lattices. Indeed, for a 90° phase advance d/p can be ±1. Table 4 lists the additional 90° secular terms. It should be noted that these terms are more detrimental to the dynamic aperture of a lattice than those of Table 3. They are all sum resonances.

It can also be proven that the dispersion is related to terms of the form:

$$|m>|\ ajk\ell> \quad \Rightarrow d = \pm 1. \qquad (6.6)$$
$$a+j+k+\ell=1$$

It should be noted that these terms cancel by group of "p" cells. The larger p is, or, the smaller the phase advance is, the sooner the cancellation occurs. This is a well known result. The dispersion of a 90° lattice tend to be smaller than the 60° lattice dispersion.

Another famous consequence of this analysis resides in the terms of the form:

$$|1>|\ ajk\ell> \ = > d = \pm 2 \qquad (6.7)$$

Suppose we combine two cells. For simplicity, let us assume that the map of one cell as the trivial form:

$$N = \exp(:g_2:)\ \exp(:a\,|1>|\ 2000> + b\ |1>|0200>:) \qquad (6.8)$$

We can compute N^2:

$$N^2 = \exp(:2g_2:)\ \exp(:G_3:)\ \exp(:G_4:) \qquad (6.9a)$$

$$G_3 = (1 + \exp(\frac{i4\pi}{p}))\ a|1>|2000> + (1 + \exp(\frac{-i4\pi}{p}))b|1>|0200> \qquad (6.9b)$$

$$G_4 = -iab\ \sin\frac{4\pi}{p}\ [|1>|2000>,\ |1>|\ 0200>] \qquad (6.9c)$$

$$= -4ab\ \sin\frac{4\pi}{p}\ |2>|\ 2000> = -4ab\ \sin\frac{4\pi}{p}\ p_t^{in^2} J_x$$

The polynomial G_4 for this simple model is modulated by sin $4\pi/p$. If the phase advance is 90°, this contribution to G_4 vanishes. Notice that (6.9c) is a chromaticity term. Chromatic terms of a 90° lattice are usually better.

7. Conclusion

The purpose of this paper was to show how one can use the concept of a map to understand the various symmetries of an ideal lattice. We stress here the word ideal. Indeed, the presence of errors will break this symmetry. On the contemplated SSC, our analysis, confirmed by tracking, had to be modified to take into account magnet errors.

This resonance analysis can also be used to understand the behavior of a full ring with or without errors.

Appendix

Some Useful Formulae

If A is a Lie algebraic map:

$$A\, f(\vec{Z}^{in}) = f(\, A\, \vec{Z}^{in}) \tag{A1}$$

$$A{:}f{:}\, A^{-1} = :\, Af{:}$$

$$A\, \exp(:f:)\, A^{-1} = \exp(:\, A\, f:)$$

$$\exp(:f:)^{-1} = \exp(:-f:)$$

If f_2 is quadratic in $\vec{Z}^{in}$, then $\exp(:f_2:)$ is a linear map:

$$\vec{Z}_i^{fin} = \exp(:f_2:)\, Z_i^{in} = R_{ij} Z_j^{in} \tag{A2}$$

R_{ij} is a matrix.

In a ring with mid-plane symmetry, if $\exp(:f_2:)$ corresponds to a full turn of the machine (or a cell), then we have:

$$f_2 = f_{2x} + f_{2y} \tag{A3}$$

$$f_{2x} = \frac{-\mu_x}{2} (\gamma_x X^{in^2} + 2\alpha_x\, X^{in} p_x^{in} + \beta_x p_x^{in^2})$$

$$f_{2y} = \frac{-\mu_y}{2} (\gamma_y Y^{in^2} + 2\, \alpha_y\, Y^{in} p_y^{in} + \beta_y p_y^{in^2})$$

If the following are true:

$$\mathcal{M}_f = \exp(:f_2:)\ \exp(:f_3:)\ \exp(:f_4:) \tag{A4}$$

$$\mathcal{M}_g = \exp(:g_2:)\ \exp(:g_3:)\ \exp(:g_4:)$$

$$\mathcal{M}_f\ \mathcal{M}_g = \mathcal{M}_h = \exp(:h_2:)\ \exp(:h_3:)\ \exp(:h_4:)$$

then,

$$h_3 = g_3 + \exp(:-g_2:)\ f_3$$

$$h_4 = \frac{1}{2}\,[\exp(:-g_2:)\ f_3,\ g_3] + \exp(:-g_2:)\ f_4 + g_4\ .$$

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Table 1

Possible values of m and d for a third order polynomial

	$m = 0$	$m = 1$	$m = 2$	$m = 3$
$d = \frac{\lambda p}{2\pi}$	$\pm 1, \pm 3$	$0, \pm 2$	± 1	0

Table 2

Possible values of m and d for a fourth order polynomial

	m = 0	m = 1	m = 2	m = 3	m = 4
$d = \frac{\lambda D}{2\pi}$	0, ±2, ±4	±1, ±3	0, ±2	±1	0

Table 3

Table of fourth order resonance terms for a second order achromatic arc

$m = 0,\ d = 0$	$\|0>\|1111>$	tune shifts with amplitude
	$\|0>\|2200>$	
	$\|0>\|0022>$	
	$\|0>\|2002>$	$2\,\nu_x - 2\,\nu_y$ coupling terms
	$\|0>\|0220>$	
$m = 2,\ d = 0$	$\|2>\|1100>$	second order chromaticities
	$\|2>\|0011>$	
	$\|2>\|1001>$	$\nu_x - \nu_y$ coupling terms.
	$\|2>\|0110>$	(Absent if mid-plane symmetry is satisfied)
$m = 4,\ d = 0$	$\|4>\|0000>$	p_t^4 time of flight term.

Table 4

Table of additional resonances found in a 90° phase advance arc

m = 0, d = 4	$\|0>\|\ 4000>$	$4\nu_x$ terms
	$\|0>\|\ 0400>$	
	$\|0>\|\ 0040>$	$4\nu_y$ terms
	$\|0>\|\ 0004>$	
	$\|0>\|\ 2020>$	$2\nu_x + 2\nu_y$ terms.
	$\|0>\|\ 0202>$	