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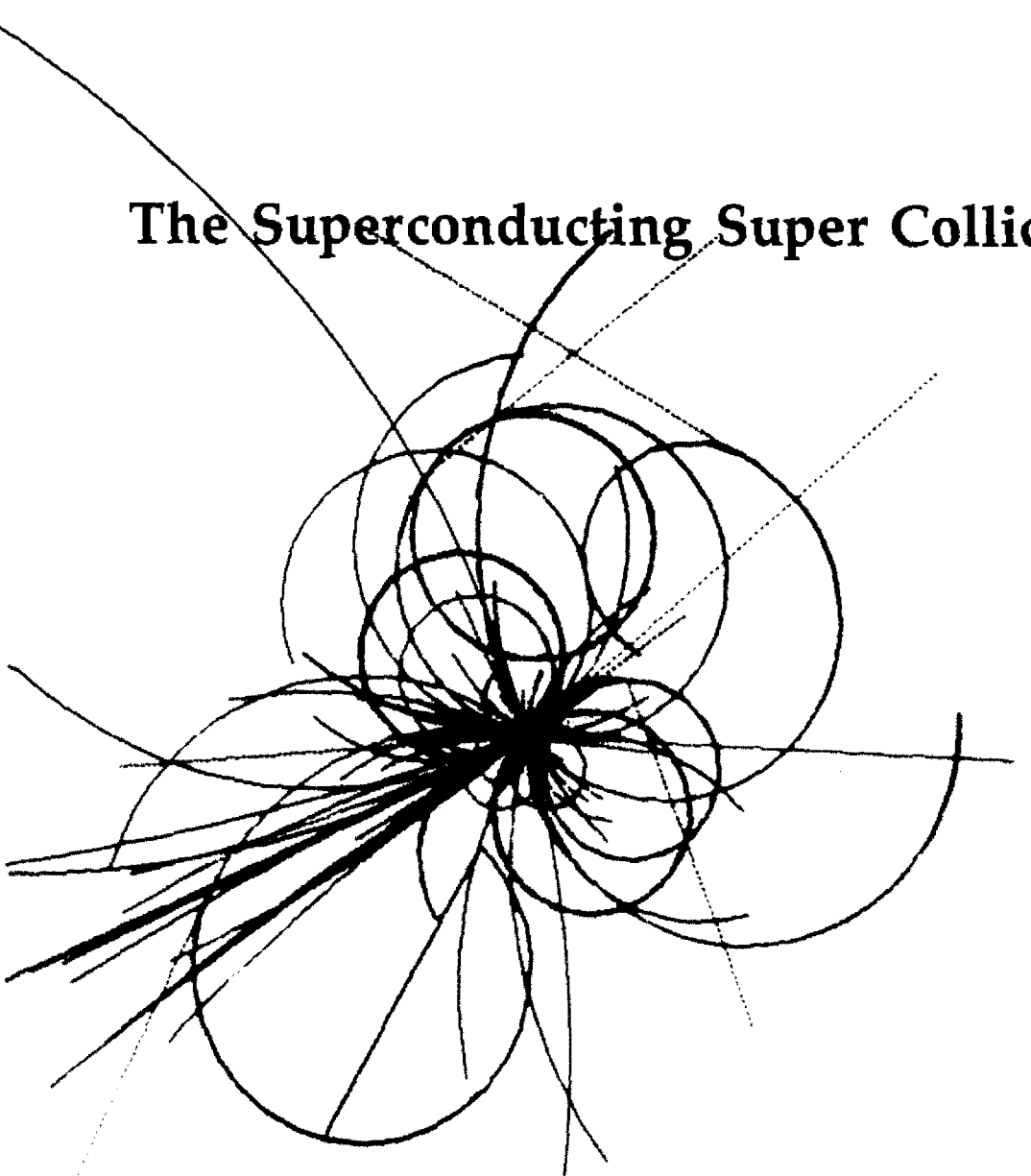
The Superconducting Super Collider



Three Dimensional Ring Configuration

SSC Central Design Group

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ABSTRACT

A wide variety of three dimensional ring configurations are possible for the SSC, ranging from a horizontal plane ring to one that follows the terrain. This report discusses an efficient method of folding the ring along its major axis in a way that eliminates coupling effects over most of the ring.

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Introduction

The ideal configuration for a synchrotron is a ring in a horizontal plane, which minimizes circumference, simplifies surveying, and avoids hydraulic and cryogenic problems. Furthermore, the beam dynamics is simplified — dispersion only exists in the horizontal plane and the linear motion consists of decoupled horizontal and vertical oscillations. Because of these simplifications, the machine is relatively easy to understand, measure, and correct.

Nonetheless, the geographical extent of the SSC assures that for many potential sites, the cost savings in configurations that conform more closely to the local terrain will lead to consideration of non-planar ring geometries.

In addition to numerous beamlines, there exists at least one non-planar ring, the Fermilab Main Ring, which has vertical bypasses over the interaction regions of the Tevatron, following a scheme invented by Collins[1] and discussed further by Ohnuma.[2,3] This scheme avoids betatron coupling by keeping the horizontal direction invariant. If applied to a very large ring such as the SSC, however, the circumference would be greater than that of the method to be discussed below.

Tilted Plane Ring. The simplest alternative to the horizontal plane is a tilted one. This configuration has been selected for LEP, in order to keep the ring in a layer of good rock. In this case the beam dynamics are the same as that of the horizontal ring; however, some of the practical difficulties mentioned above exist. It is believed that these undesirable consequences will be manageable if the SSC ring plane is tilted by not more than one half degree from the horizontal.

Folded Ring. The next possibility, and the subject of this paper, is to fold the ring along a diameter, so that it lies in two planes inclined at a small angle to each other. Such an arrangement might be attractive for valley sites — the long axis of the ring would be placed along the valley bottom, and the two sides that contain the IR's would tilt upwards to higher ground. It is believed that the effects of this type of distortion can be tolerated if the relative inclination of the two planes is not more than one degree, which would permit

IR elevations of up to about 100 meters.

A combination of tilting and folding might also be envisioned, provided that the maximum angle to the horizontal does not exceed one-half degree. For example, the ring could be tilted along the long NS axis while the EW sides are folded upwards, or the West side could lie in a true horizontal plane while the East side tilted upwards (or downwards) by up to 0.5°. Also the tilt axis is arbitrary.

We now discuss the geometrical and orbital consequences of folding the ring along a diameter.

Geometry of a Folded Ring

The lattice considered is that of the CDR,[4] which is described in more detail in an SSC note.[5] A schematic diagram of the ring folded in a way to raise the two clusters of straight sections is shown in Fig. 1. To be sure, the ring is not really folded, since the orbit cannot have discontinuous changes of direction. Two hinge sections are needed to produce a relative inclination of the two planes continuously. In the scheme proposed, each hinge consists of six regular 60° cells, or a multiple thereof.* Both the main and the hinge arcs lie in planes and have common tangent lines at their intersection points. The hinges contain six cells in order to have unit transfer matrices; hence they are first and second order achromats.[6] †

A similar method is used in the Stanford Linear Collider,[7] where the arcs consist of a sequence of achromatic circular arcs, with rolls about the beam direction at the intersections. For the scheme proposed here, however, only the small hinge arcs are achromats.

If the ring has its long axis in the N-S direction, it is divided into four sections produced by the intersections AA', BB', CC', and DD' of a horizontal plane with the four tilted planes that contain the four arcs of the ring (see Fig. 1). The long arcs lie in planes that are tilted upwards about AA' and BB' by an angle ω, while the small hinge arcs are tilted

* Six 60° cells were chosen because the lattice cells have this phase advance, but one could also use four 90° cells, etc..

† To make the hinge be a second order achromat, the two sextupole families within it must be tuned to produce zero chromaticity locally. However this may not be necessary for the SSC.

downwards about CC' and DD' by an angle ω'. All of the dipoles and quadrupoles of each arc are oriented normally with respect to their local planes.

The angles ω, ω' are related to each other and to the bend angles of the arcs in such a way that the long and short arcs have a common tangent line at their points of intersection, and so that their relative rotation, of magnitude $\Omega = \sqrt{\omega^2 + \omega'^2}$, is about the beam axis. This condition links the rotation angles ω, ω' and the bend angles Φ, Φ' of the large and small arcs with θ, the projection of the angle Θ on the horizontal plane (see Fig. 2). Here Φ, Φ' are equal to the bend half-angles of the long and short arcs respectively. For the planar case, $\Phi + \Phi' = 90°$.

The complete ring consists of arcs with constant radius of curvature r. The long arcs are centered at O and the short ones at O'. From Fig. 2 one may verify the following relations:

$$\begin{aligned} \tan\Phi &= \cot\theta \sec\omega \\ \tan\Phi' &= \tan\theta \sec\omega' \\ \sin\omega' &= \sin\omega \sin\Phi / \sin\Phi'. \end{aligned} \tag{1}$$

The first of Eqs. (1) is obtained from the triangles NPQ, NPR, and OPQ, noting that the angles OQN, QNP, QNR, and PRN are right angles. The second is the corresponding equation obtained from the triangles of the small arc and the relation $\theta + \theta' = 90°$. The third equation is obtained by comparing h'/h as derived from the triangles NPQ, $N'P'Q'$ on the one hand, with that ratio derived from PQR, $P'Q'R'$ on the other. From these equations it can be shown that

$$\sin\Phi' = \sqrt{\sin^2\theta + \cos^2\theta \sin^2\Phi \sin^2\omega}. \tag{2}$$

Eliminating θ , we obtain

$$\begin{aligned} \cos\omega &= \cos\Phi' / \sin\Phi \\ \cos\omega' &= \cos\Phi / \sin\Phi'. \end{aligned} \tag{3}$$

The SSC lattice has 96 cells per quadrant. If the hinge has six cells, then Φ, Φ' are the bend angles of 93 and 3 cells respectively, so that their ratio ρ must be the same as that

of the cell numbers:

$$\rho = \Phi/\Phi' = 93/3 = 31. \tag{4}$$

Equations (3) and (4) define a one-dimensional family of solutions. Tables I and II display solutions for the six and twelve cell ($\rho = 90/6 = 15$) hinges respectivily, as well as the rotation angle Ω about the beam axis, the angular increase factor $f = (\Phi + \Phi')/90°$, and the heights h and h' of the arc centers, which are given by

$$\begin{aligned} h &= r(1 - \cos\Phi)\sin\omega \\ h' &= -r(1 - \cos\Phi')\sin\omega', \end{aligned} \tag{5}$$

where $r = 11734.2$ m is the arc radius of the SSC.

Table I

Parameters for various tilt angles ω for six cell hinges with arc ratio $\rho = 31$, (angles in degrees).

ω	$\Phi - 87°$	ω'	Φ'	Ω	$10^3(f-1)$	h(m)	h'(m)
0.25	0.1983	5.095	2.8128	5.101	0.1236	48.7	−1.26
0.50	0.2308	10.227	2.8139	10.239	0.4972	97.6	−2.51
0.75	0.2860	15.436	2.8157	15.454	1.1294	146.3	−3.77
1.00	0.3650	20.768	2.8182	20.792	2.0361	195.4	−5.03
1.25	0.4702	26.278	2.8216	26.307	3.2421	244.7	−6.30

The increase of total bending angle can be obtained either by raising the magnetic field, or by lengthening the dipoles in the cells by the factor f. If the latter procedure is followed, the quadrupole and drift lengths are unaltered. For $\omega = 0.5°$ and a six cell hinge, each half-cell increases in length by 8.22 cm, and the circumference by 35.5 m.

Table II

Parameters for various tilt angles ω for twelve cell hinges with arc ratio $\rho = 15$, (angles in degrees).

ω	$\Phi - 84°$	ω'	Φ'	Ω	$10^3(f-1)$	h(m)	h'(m)
0.25	0.3802	2.5390	5.6253	2.5512	0.0616	46.2	−2.50
0.50	0.3958	5.0821	5.6264	5.1067	0.2466	92.4	−5.01
0.75	0.4219	7.6336	5.6281	7.6704	0.5560	138.7	−7.51
1.00	0.4586	10.1978	5.6306	10.2467	0.9914	185.0	−10.02
1.25	0.5062	12.7793	5.6337	12.8403	1.5554	231.5	−12.54

Two further relations can be derived, which are especially useful if there are more than two hinges. Then each hinge should be able to produce a different relative tilt angle 2ω between an adjacent pair of great circles on a single sphere, but all of the hinges should have the same bend angle $2\Phi'$, so that each can be made of an identical $M = 1$ achromat with the same number of cells. It is possible to obtain relations giving the unknown quantities ω' and Φ in terms of the chosen ω value and the fixed value of Φ':

$$\begin{aligned} \sin\omega' &= \tan\omega \cot\Phi' \\ \sin\Phi &= \sec\omega \cos\Phi'. \end{aligned} \tag{6}$$

The angle from the vertex of two great circles to the beginning of their hinge is $90° - \Phi$.

Orbital Effects in a Folded Ring

The effect of tilting the four sectors of the ring is to introduce rotations Ω and $-\Omega$ about the beam direction at the beginning and end of each hinge sector. Since each hinge is an achromat, these rotations cancel each other so far as their effect on the large arcs is concerned. However, they produce mismatches of the dispersion and betatron oscillations within the hinge cells. The analysis is simplified if the entrance and exit of the six-cell hinge are both midway between an upstream QF and a downstream QD. This is also the most symmetrical choice, since the centers of the north and south arcs are at such points.

Dispersion. When the beam enters the tilted six-cell hinge it sees a rotation of the transverse coordinate system of magnitude Ω. Therefore the purely horizontal dispersion at the hinge entrance is projected on the new horizontal and vertical planes. These projections oscillate about the new matched dispersion vector. The maxima occur in the third and sixth QF's with values

$$\begin{aligned} \hat{D}_x &= \hat{D}_0(1-\cos\Omega)\cos 15^\circ \\ \hat{D}_y &= -\hat{D}_0 \sin\Omega \cos 15^\circ, \end{aligned} \tag{7}$$

where $\hat{D}_0$ is the maximum value of the dispersion in the regular cells. For a tilt angle $\omega = 0.5^\circ$, we obtain $\hat{D}_x = 1.0154\hat{D}_0$ and $\hat{D}_y = -0.1717\hat{D}_0$.

Transverse Oscillations. Similarly, two initial betatron oscillations, one horizontal and the other vertical, are rotated so that each has components in both of the new horizontal and vertical directions. Since the emittances are equal, we find that the horizontal displacement in a QF at phase Δ from the hinge entrance is

$$x = \sqrt{\hat{\beta}\epsilon}\ \{\ \cos\Omega\cos(\psi_x+\Delta) + \sin\Omega\ [\cos(\psi_y+\Delta) + 2\alpha_0\cos\psi_y\sin\Delta]\ \}, \tag{8}$$

where $\hat{\beta}$ is the matched value of β_x in a QF and α_0 is the value of α_x at the mid half-cell point, and ψ_x and ψ_y are the phases of the horizontal and vertical oscillations at the entrance. In Eq.(8), the terms proportional to $\cos\Omega$ and $\sin\Omega$ come respectively from the horizontal and vertical oscillations upstream of the hinge. The horizontal maxima occur in

the second and fifth QF's, where $\Delta = 105°$ and $285°$. The first term has a maximum when $\psi_y + \Delta = 0$, and the second when $\cot\psi_y = -\cot\Delta - 2\alpha_0$. For 60° cells, $\alpha_{0x} = 1.1682$, whence $\psi_y = -21.0°$. Consequently the maximum horizontal displacement in the hinge cells is

$$\hat{x} = \sqrt{\hat{\beta}\epsilon}\,(\cos\Omega + 0.6772\sin\Omega). \tag{9}$$

The peak vertical displacements, $\hat{y}$, which occur in certain QD's, have magnitudes equal to those of the horizontal peak displacements. For a tilt angle $\omega = 0.5°$, we obtain $\hat{x} = \hat{y} = 1.1058\sqrt{\hat{\beta}\epsilon}$.

Generalization

As was mentioned earlier, the two large arcs of our example have a common center, so they are great circles on a sphere. It would in principle be possible to cause the design orbit to be some arbitrary closed sequence of great circles on a sphere, connected by achromatic hinges (which leave the sphere) at their vertices. The choice of great circles would be made by placing the imaginary sphere on a topographic and geological map, and choosing them to lie within a chosen depth from the surface, or within certain strata. The theory developed above could be extended to give the geometry and orbital effects in this case. Iterations would be necessary to insure that all of the arcs consist of integral numbers of cells.

Acknowledgments

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FIGURE CAPTIONS

1. SSC ring folded along lines parallel to the IR clusters.
2. Geometry of the folded ring. Two long arcs are in planes tilted upwards by ω, and two short arcs are in planes tilted downwards by ω'. The arcs have common tangent lines at their intersections, *e.g.*, at Q.

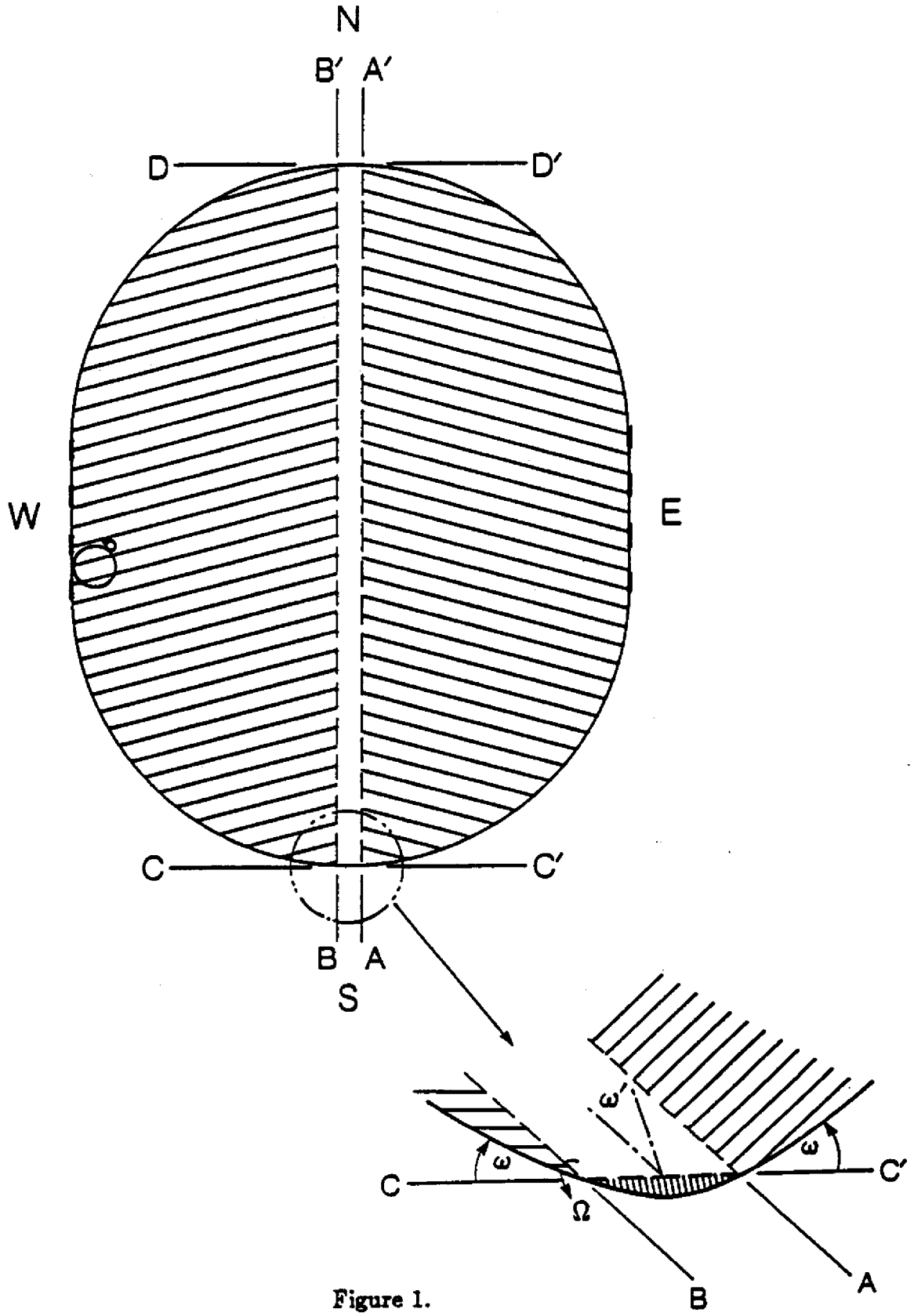

Figure 1.

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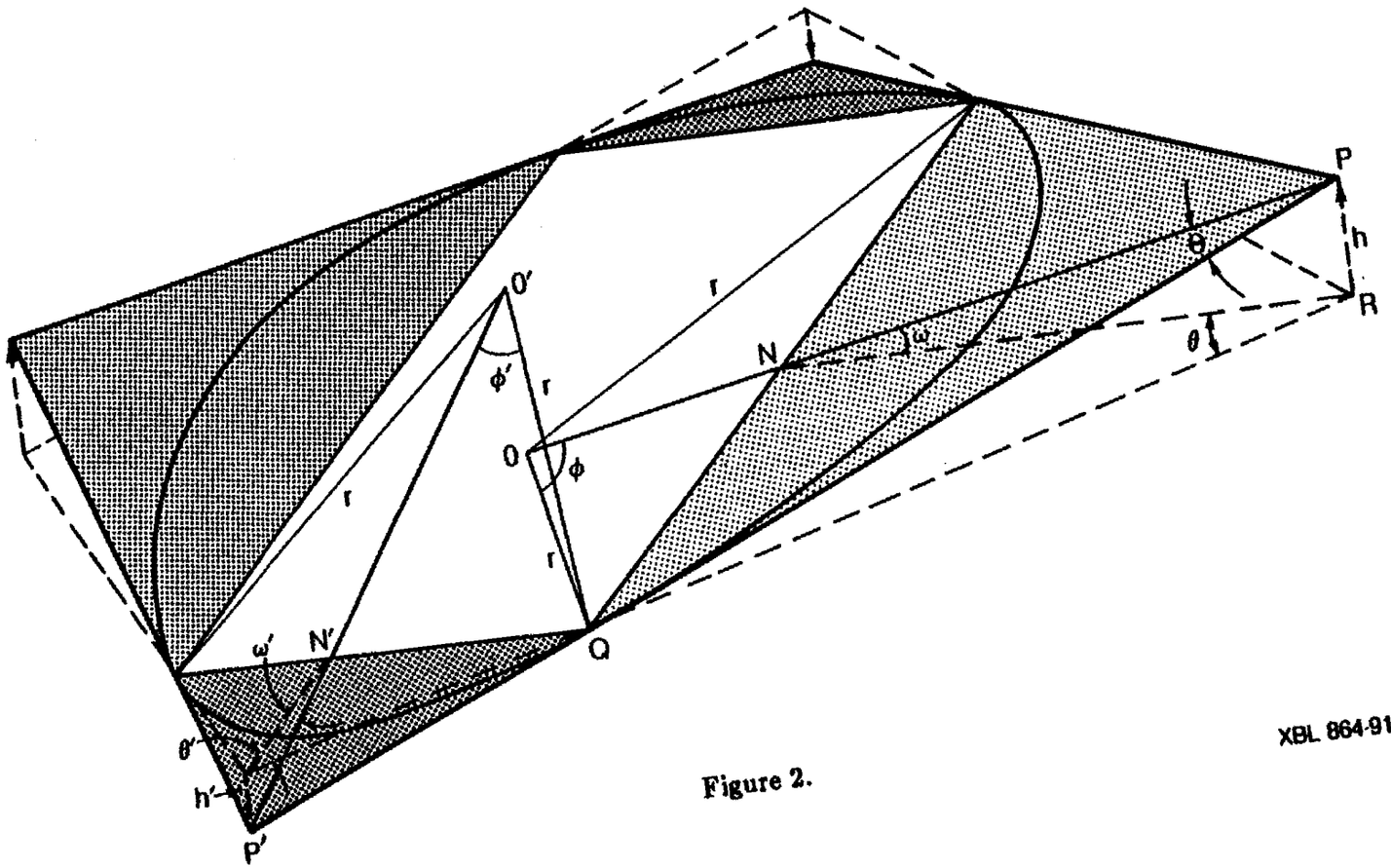

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Figure 2.