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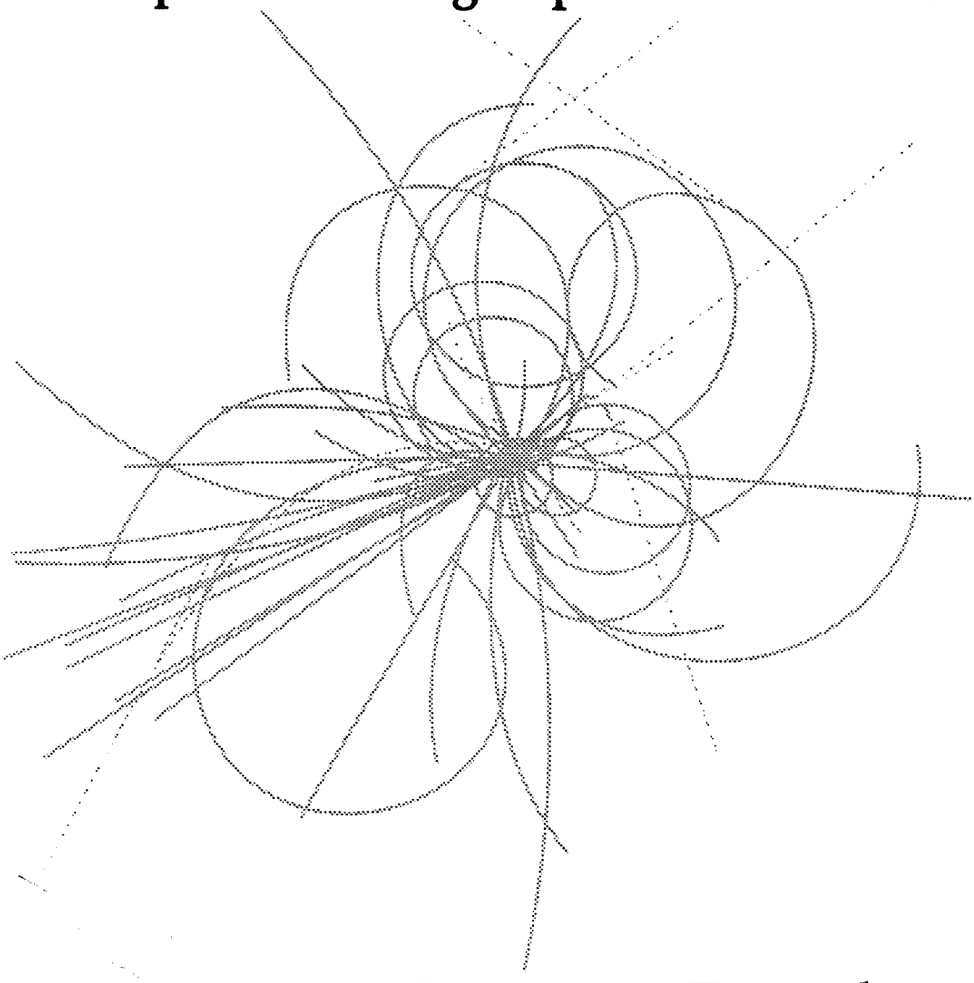

A Possible Screening Procedure for Random Multipole Field Errors Before Tracking Studies

S. Peggs, M. Furman and A. Chao

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A Possible Screening Procedure for Random Multipole Field Errors Before Tracking Studies

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1. Introduction

It is an elaborate process to evaluate the stability limit of particle motion in the presence of multipole field errors. Extensive analysis on the linear and nonlinear lattices and tracking studies are often required. It is therefore very useful to design "screening" procedures to tell if a particular set of multipole errors is unacceptable without extensive studies.

Although passing the screening does not necessarily mean good stability limits, failing the screening does mean the stability region is too small to be acceptable.

In this note we propose one possible screening procedure to be applied to the random multipole field errors in the SSC. Other screening procedures are not excluded from being applied simultaneously.

2. Analysis

Consider a dipole magnet that has a vertical magnetic field error given by

$$\frac{\Delta B}{B} = \sum_n b_n x^n \tag{1}$$

A particle passing through this dipole gets an orbital kick due to the error field

(2) $$\Delta x' = \vartheta_B \frac{\Delta B}{B}$$

where ϑ_B is the bending angle of the dipole magnet. In its subsequent motion, the particle oscillates with an amplitude

(3) $$\Delta A = \beta \vartheta_B \frac{\Delta B}{B}$$

Every time the particle comes around to the magnet, it gets a kick. These kicks are correlated according to the tune of the accelerator. Close to a nonlinear resonance, the kicks add up to a large orbital deviation. Away from the resonances, the orbital deviation remains of the order of that caused by a single kick. The resonance behavior is not what we address in this screening procedure. The orbital deviation caused by the particular dipole magnet is therefore given by eq.(2).

Now consider a particle executing a betatron oscillation

(4) $$x = A \sin\psi$$

for one revolution around the storage ring. We have ignored the beta-function variation in eq.(4). The amplitude A is to be evaluated at an average beta-function in the cells.

The particle passes through all dipoles in the ring, each having a random multipole field error. Assuming the multipole errors are uncorrelated from magnet to magnet, the orbital deviation of the particle accumulates statistically for one revolution, i.e.

(5) $$\Delta A_{rms} = \bar{\beta}\, \theta_B \sqrt{N_B} \left(\frac{\Delta B}{B}\right)_{rms}$$

with

(6) $$\left(\frac{\Delta B}{B}\right)^2_{rms} = \sum_n \langle b_n^2 \rangle A^{2n} \langle \sin^{2n}\psi \rangle$$

and N_B is the number of dipole magnets. In deriving eq.(6) we have assumed that the random multipole error coefficients b_n have zero average and that they are uncorrelated, that is, $\langle b_n \rangle = 0$ and $\langle b_n b_m \rangle = \langle b_n \rangle \delta_{nm}$

In eq.(5), we sum over all multipole orders n=2,3,... We assume that the linear (n=1) term can be cancelled in practice by engineering corrections on individual magnets, by shuffling and by the correction skew quadrupoles.

The average over $\sin^{2n}\psi$ in eq.(6) is

(7) $$C_n \equiv \langle \sin^{2n}\psi \rangle = \frac{1}{2\pi}\int_0^{2\pi} d\psi \, \sin^{2n}\psi = \frac{(2n)!}{2^{2n}\,(n!)^2}$$

Since away from resonances the orbit deviation does not increase further, eq.(5) gives the perturbation on the betatron oscillation amplitude due to nonlinearities as a function of the unperturbed amplitude A.

Two definitions of apertures can be made [1]. The first is when the amplitude variation due to nonlinearities is comparable to the unperturbed amplitude; then the motion is most likely unstable. The second definition is when the amplitude variation reaches, say, 10% of the unperturbed amplitude; then the motion becomes nonlinear although it is very likely stable. In other words,

(8)

stability aperture: $$\frac{\Delta A_{rms}}{A} = 100\%$$

linear aperture: $$\frac{\Delta A_{rms}}{A} = 10\%$$

The proposed screening procedure is, then, as follows: For a given set of rms multipole errors, compute

$$(9) \qquad F(A) = \frac{\Delta A_{rms}}{A} = \bar{\beta}\,\theta_B \sqrt{N_B}\,\frac{1}{A}\sqrt{\sum_{n=2} C_n \langle b_n^2 \rangle A^{2n}}$$

as a function of amplitude A. Stability aperture is given by the value of A when F(A) = 1. Linear aperture is given by the value of A when F(A)=0.1. When the obtained aperture(s) is too much smaller than that required for beam operation and manipulation, the proposed rms multipole errors are to be rejected without extensive analytical and tracking studies.

3. Examples and Results

It is necessary to point out that the procedure suggested is rather crude. In particular, no resonance effects are included. Also, no counterpart of the skew components of multipole errors is considered. Rather, the procedure serves as a screening technique and not as a substitute for aperture evaluation.

The rms values for the random multipole error coefficients b_n [2] are taken from the SSC database and are summarized in table (1). The bending angle θ_B and the effective value of the beta function, $\bar{\beta}$, are defined by

$$(10) \qquad \theta_B = \frac{2\pi}{N_B} \qquad\qquad \bar{\beta} = \frac{R}{\bar{\nu}}$$

where R is the radius of the ring and $\bar{\nu}$ is the part of the tune contributed by the normal cells. These values are summarized in table (2). (The number of dipoles for each design is taken from the Reference Design Study).

The four curves in fig.(1) represent the function F(A) evaluated for the four test lattice designs TLA1, TLA2, TLC1 and TLC2 [3]. The horizontal

lines at 0.1 and 1 define the linear and stable apertures, respectively, according to the criteria described above. The resulting values of the linear and stable aperture values are also summarized in table (2).

n	1	2	3	4	5	6
designs A1,A2	.73	2.7	.5	.7	.06	.085
designs C1,C2	1.0	1.0	.5	.4	.2	.5

Table 1

RMS values of the coefficients b_n for designs A and C. Units are $10^{-4}\ cm^{-n}$.

	N_B	θ_B (mrad)	$\bar{\beta}$ (m)	linear aperture (cm)	stable aperture (cm)
TLA1	3870	1.62	212	0.3	2.0
TLA2	3870	1.62	144	0.45	2.35
TLC1	990	6.35	285	0.3	1.4
TLC2	990	6.35	186	0.45	1.6

Table 2

Number of dipole magnets, bending angle per dipole, $\bar{\beta}$, and resulting values for linear and stable apertures for each lattice design.

3. References

[1]. D. Edwards, Aperture Task Force, Group B Interim Report, April 1985.

[2]. E. Fisk, J. Peterson, SSC Aperture Task Force, Group C, 1985.

[3]. E. Courant, D. Douglas, A. Garren and D. Johnson, SSC-19, 1985.

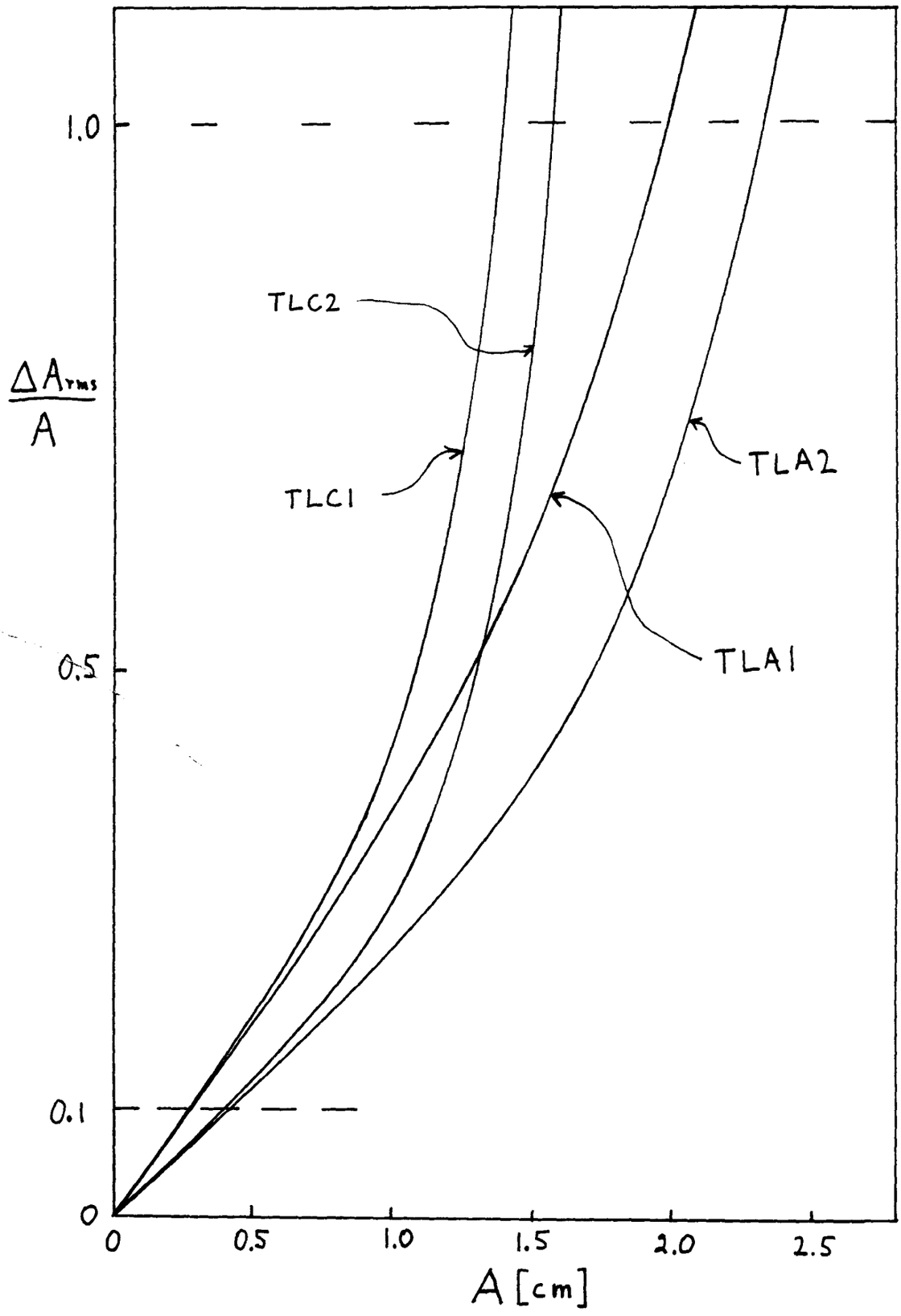
1.0
0.5
0.1
0
$\frac{\Delta A_{rms}}{A}$
TLC2
TLC1
TLA2
TLA1
0
0.5
1.0
1.5
2.0
2.5
A [cm]

Fig. 1