The Superconducting Super Collider

General Formulas for the Adiabatic Propagation Velocity of the Normal Zone

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Abstract

The influence on the propagation velocity of the normal zone of four phenomena was investigated:
1) The temperature dependence of the specific heat and the thermal conductivity, 2) The current-sharing zone, 
3) The electromagnetic diffusion of current through a possible super-stabilizer, and 4) The thermal diffusion through a possible insulator. At the beginning, these influences were studied independently of each other. In all cases, after creating a model of the particular phenomenon, the equations of thermal and electromagnetic behavior were solved analytically and the expressions for the propagation velocity obtained. These expressions were then put into nondimensional forms, which have allowed us to define four correction factors depending only on one or two nondimensional parameters.

Subsequent studies were made to investigate how to combine these correction factors in order to obtain general formulas for the velocity, taking into account a part or all of these influences. A review is presented here of these formulas, which are of interest for the superconducting windings, where the heat transfer to helium can be neglected on the time scale of the quench process.

Introduction

Hypothesis of adiabaticity

The study of the propagation of the normal zone along the superconducting windings has already been the object of many publications. However, all these publications have one point in common: they are all concerned with conductors immersed in helium baths, and their authors direct their efforts primarily toward modelization, calculation, and measurement of thermal exchanges between the conductors and the helium.

In this study we are more specifically interested in indirectly-cooled magnets, such as the layer of conductors representative of the ALEPH solenoid presented in Figure 1.

In this configuration, the annular ring and especially the layer of insulation (between the annular ring and the layer of conductors) introduce important thermal resistance. In more precise terms, a characteristic time $T_4$ of thermal diffusion across the width $L_4$ of the layer insulation is given by:

$$T_4 = \frac{L_4^2}{D_4}$$

(1)

where $D_4$ is the thermal diffusivity of the layer insulation ($m^2s^{-1}$). Typically:

$D_4 = 10^{-6} m^2s^{-1}$  \hspace{1cm}  $L_4 = 10^{-3} m$

Hence: $T_4 = 1 s$

In addition, a characteristic time $T_p$ of the propagation phenomenon is given by:

$$T_p = \frac{D_1}{v^2}$$

(2)

where $D_1$ is the equivalent longitudinal thermal diffusivity of the elementary winding pattern ($m^2s^{-1}$) and $v$ is the longitudinal propagation velocity ($ms^{-1}$). Typically:

$D_1 = 0.1 m^2s^{-1}$  \hspace{1cm}  $v = 10 ms^{-1}$

Hence: $T_p = 10^{-3} s$

It appears that $T_p$ is small in comparison to $T_4$. During $T_p$, the thermal diffusion will be limited to a small width of the layer insulation. What follows in particular is that at the scale of propagation, the heat flux transferred to the annular ring and to the helium is negligible.
Consequently, we see that the layer of insulated conductors—or, as the case may be, the elementary winding pattern—is thermally decoupled from the exterior and thus shows an ideally adiabatic behavior.

Adiabatic propagation velocity, with but one exception, has only been regarded, in terms of theory, as a limiting case of the propagation velocity when immersed in a helium bath, with the coefficient of exchange with helium tending towards zero.\(^1\), \(^2\), \(^28\)

**Different aspects of the problem**

A complete study of the propagation of the normal zone along the layer of conductors of Figure 1 must take into account the following four influences:

- **Influence of the current-sharing zone**
- **Influence of the temperature dependence of the specific heat and of the thermal conductivity of materials.**
- **Influence of electromagnetic diffusion in the super-stabilizer.** In effect, the conductor used by the ALEPH solenoid consists of a conventional multi-filament composite enclosed in a large section of aluminum. When such a conductor switches to the normal resistive state, the current, initially confined to the composite, needs a certain amount of time to be diffused in the super-stabilizer. The propagation front thus carries with it a wake of electromagnetic diffusion along which the dissipated power density per unit volume varies. Thus the question is posed of the role played by this power in calculating propagation velocity.
- **Influence of thermal diffusion in the layer insulation.** During a transition of the layer of conductors in Figure 1, most of the dissipated power serves to heat the conductor and to make it switch to the normal resistive state and the residue is transferred to the layer of insulation. As we have described in Section I.1, given the weak thermal conductivity of the insulator, the heat, like the current, needs a certain amount of time to be diffused in the insulation. The propagation front carries with it a wake of thermal diffusion. Thus the question is likewise posed of the role played by the insulation layer in calculating propagation velocity.

**Method**

The above review of influences on propagation velocity shows that the latter is a blend of thermal and electromagnetic phenomena, mainly non-linear, and that these phenomena occur on different scales. The difference between scales renders all numerical approaches to the problem difficult. We have thus adopted an entirely different method of which we present here the principal results. This method consists, first, of a study of each of the aspects of the problem independently from each other, and, second, of an attempt to reconstitute the mechanics of the entirety. Sections II-V present the conclusions of the studies of each of the previously described influences. In each case, we have analytically solved the equations of thermal and electromagnetic behavior and we have established an implicit equation for propagation velocity. This implicit equation was then put into dimensionless form, which has allowed the reduction of the numerous related parameters to one or two, as the case may be. It is this dimensionless form that we present here. In Section VI, we finally show how to combine these parameters to estimate the propagation velocity along the layer of conductors in Figure 1. The details of all these studies can be found in Reference 36.

**Influence of the current-sharing zone**

Let us consider the current-sharing phenomenon. First, we introduce an average power density per unit volume \(P_j\), variable in relation to temperature \(T\), and defined\(^37\) by:

\[
P_j = \begin{cases} 
0 & \forall T, T \leq T_{ci} \\
\frac{P_j}{T - T_{ci}} & \forall T, T_{ci} \leq T < T_c \\
P_j & \forall T, T \leq T_c
\end{cases}
\]

where:

- \(T_c\) is the critical temperature at the given field and zero current (K).
- \(T_{ci}\) is the critical temperature at the given field and the given current (K).
- \(P_j\) is the average power density per unit volume dissipated by the elementary winding pattern in the normal resistive state (Wm\(^{-3}\)).

Let us call the dimensionless parameter \(m_j\) the current-sharing parameter, defined by:

\[
m_j = \frac{4}{m} \frac{\Delta H_s}{\Delta H_t}
\]

where:

- \(\Delta H_s\) is the variation between \(T_0\) and \(T_{ci}\) of the enthalpy per unit volume of the elementary winding pattern (Jm\(^{-3}\)).
- \(\Delta H_t\) is the variation between \(T_{ci}\) and \(T_c\) of the enthalpy per unit volume of the elementary winding pattern (Jm\(^{-3}\)).
- \(T_0\) is the operating temperature (K).

We thus establish, for all values of \(m_j\), the existence of a constant-velocity asymptotic shift of the temperature profile, for which the velocity \(v\) is given by:

\[
v = v(m_j) \sqrt{-\frac{D_t}{P_j \Delta H_t}} \quad (ms^{-1})
\]

where:

- \(\Delta H_s\) is the variation between \(T_0\) and \(T_t = (T_c + T_{ci})/2\) of the enthalpy per unit volume of the elementary winding pattern (Jm\(^{-3}\)).
- \(D_t\) is the equivalent longitudinal thermal diffusivity of the elementary winding pattern, at \(T_t\) (m\(^2\)s\(^{-1}\)).
The current-sharing corrective factor, $v_I$, is determined versus $m_I$ by the system of equations:

\[
\begin{align*}
v_I &= \sin \left[ \sqrt{\frac{m_I}{v_I}} \ln \left( \frac{m_I}{m_I^*} \right) \right] = \frac{2v_I^2}{m_I^2} - \frac{m_I^2}{v_I^2} - 1 \\
\cos \left[ \sqrt{\frac{m_I}{v_I}} \ln \left( \frac{m_I}{m_I^*} \right) \right] &= 1 - \frac{2v_I^2}{m_I^2} \\
0 &\leq \sqrt{\frac{m_I}{v_I}} \ln \left( \frac{m_I}{m_I^*} \right) \leq \pi
\end{align*}
\]

The curve $v_I = f(m_I)$ is presented in Figure 2.

Influence of the temperature dependence of physical characteristics

Let us consider the temperature dependence of the characteristics of the elementary winding pattern. First, we introduce a specific heat $C_t$ and a thermal conductivity $k_t$, variables in relation to $T$ and defined by:

\[
C_t = \beta_t T^3 + \gamma_t T \\
k_t = \kappa_t T
\]

where $\beta_t$, $\gamma_t$, and $\kappa_t$ are real, positive constants, determined so as to obtain the best approximation of the real curves $C_t = f(T)$ and $k_t = f(T)$ alongside $T_t$.

Let us call the dimensionless parameter $m_{II}$ the temperature dependence parameter, defined by:

\[
m_{II} = \frac{4\beta_t T^2 \Delta H_t}{C_t(T_t)^2}
\]

where $C_t(T_t)$ is the specific heat per unit volume of the elementary winding pattern at $T_t$ ($J/m^3K^{-1}$).

We thus establish, for all values of $m_{II}$ ($0 \leq m_{II} \leq 1$), the existence of a constant-velocity asymptotic shift of the temperature profile, for which the velocity $V_{II}$ is given by:

\[
v_{II} = v_I(m_{II}) \sqrt{\frac{D_{I} P_{II}}{\Delta H}} \quad (m^{-1})
\]

where $v_{II}$ is the temperature-dependence corrective factor determined versus $m_{II}$ by the implicit equation:

\[
K_{1/3} \left[ \frac{1}{3} \frac{(1-m_{II})^{3/2}}{m_{II}} \right] = \sqrt{1 - m_{II}}
\]

$K_{1/3}$ being the modified Bessel function of the second kind and of order $\frac{1}{3}$.

The curve $v_{II} = f(m_{II})$ is presented in Figure 3.

Influence of electromagnetic diffusion in the super-stabilizer

The model we have developed in order to examine this influence is presented in Reference 39; here we are satisfied with a reexamination of the principal results, generalized to any conductor geometry.

Let us call the dimensionless parameter $m_{III}$ the first super-stabilization parameter, defined by:

\[
m_{III} = \frac{\rho_{II} A_{II}}{\rho_{I} A_{I}}
\]

where:

- $\rho_1$ ($\rho_2$ respectively) is the equivalent longitudinal resistivity of the composite in the normal resistive state (of the super-stabilizer, respectively)($\Omega m$).
- $A_1$ ($A_2$ respectively) is the composite section (the super-stabilizer, respectively)($m^2$).
A criterion of super-stabilization is given by:

\[ m_{III} \ll 1 \]  \hspace{1cm} (12)

Let us call the dimensionless parameter \( p_{III} \) the second super-stabilization parameter, defined by:

\[ p_{III} = \frac{\nu_{0}L_{2e}}{D_{e}} \]  \hspace{1cm} (dimensionless)  \hspace{1cm} (13)

where:

\[ L_{2e} \] is the equivalent depth of the super-stabilizer defined by:

\[ L_{2e} = \frac{A_{2}}{P_{d}} \]  \hspace{1cm} (m) \hspace{1cm} (14)

\( P_{d} \) is the composite perimeter in contact with the super-stabilizer (m).

\( D_{e} \) is the equivalent diffusivity defined by:

\[ D_{e} = \frac{1}{\sqrt{2D_{m}D_{t}}} \]  \hspace{1cm} \((m^{2}s^{-1})\) \hspace{1cm} (15)

\( D_{m} = \frac{D_{2}}{\mu_{0}} \) is the electromagnetic diffusivity of the super-stabilizer \( (m^{2}s^{-1}) \).

\( v_{0} \) is the propagation velocity calculated assuming that the current remains confined to the composite:

\[ v_{0} = \sqrt{\frac{D_{m}P_{0}}{A_{1}}} \]  \hspace{1cm} \((ms^{-1})\)  \hspace{1cm} (16)

\( P_{0} \) is the average power density per unit volume dissipated by the elementary winding pattern assuming that the current is confined to the composite in the normal resistive state \((Wm^{-3})\).

We thus establish, for all parameter values, the existence of a constant-velocity asymptotic shift of the temperature profile and of the magnetic induction profile, for which the velocity \( v_{III} \) is given by:

\[ v_{III} = \frac{m_{III}p_{III}}{v_{0}} \]  \hspace{1cm} \((ms^{-1})\) \hspace{1cm} (17)

where \( v_{III} \) is the super-stabilization corrective factor, for which the result changes depending of the geometry studied.

For a layer of conductors such as that presented in Figure 1, \( v_{III} \) is determined versus \( m_{III} \) and \( p_{III} \) by the implicit equation:

\[ v_{III} = \frac{m_{III}p_{III}}{\tanh(p_{III}v_{III}) + m_{III}p_{III}v_{III}} \] \hspace{1cm} (18)

Curves of \( v_{III} = f(m_{III}p_{III}) \) are presented in Figure 4.

Influence of thermal diffusion in the layer insulation

General features

In a layer of conductors such as that in Figure 1, we distinguish two types of insulation: the insulation between conductors, of typical width \( e_{1} = 100 \mu m \), and the layer insulation, which surrounds the layer of conductors. A characteristic time of thermal diffusion in the insulation between conductors is given by:

\[ \tau_{i} = \frac{e_{1}^{2}}{D_{4}} = 10 \text{ ms} \] \hspace{1cm} (19)

It seems therefore that \( \tau_{i} \) is of the same order as \( \tau_{p} \), but is smaller than \( \tau_{a} \), where \( \tau_{p} \) and \( \tau_{a} \) are the characteristic times previously defined. During \( \tau_{p} \), the heat is thus largely diffused in the insulation between conductors, but only penetrates a small portion of the layer insulation.

For the calculation of velocity, we therefore suppose that temperature is constant through each cross-section of the system (conductor and insulation between conductors), which we regard as a homogeneous, isotropic medium.

In the following, we designate by medium 3 the system (conductor and insulation between conductors) and by medium 4 the layer insulation. We have conducted a separate study for the layers of insulated super-stabilized conductors, in which both electromagnetic and thermal diffusion phenomena interfere.

Case of a layer of insulated multi-filament superconductive composites

Let us call the dimensionless parameter \( m_{IV} \) the first insulation parameter, defined by:

\[ m_{IV} = \sqrt{\frac{k_{3}C_{3}}{\tau_{0}L_{3}}} \]  \hspace{1cm} \((dimensionless)\) \hspace{1cm} (20)

where:

\( C_{3} \) \((C_{4} \text{ respectively})\) is the equivalent specific heat per unit volume of medium 3 \((\text{medium} 4 \text{ respectively})\) at \( T_{1} \) \((Jm^{-3}K^{-1})\).

\( k_{3} \) \((k_{4} \text{ respectively})\) is the equivalent longitudinal thermal diffusivity of medium 3 \((\text{medium} 4 \text{ respectively})\) at \( T_{1} \) \((Wm^{-1}K^{-1})\).

\( D_{3} = k_{3}/C_{3} \) \((L_{3} \text{ respectively})\) is the equivalent longitudinal thermal diffusivity (half-width, respectively) of medium 3.

\( \tau_{0} \) is the propagation velocity calculated assuming that the layer of conductors is bare (without layer insulation):
\[ v_0 = \sqrt{\frac{D_3 P_{j0}}{\Delta H_3}} \quad (ms^{-1}) \tag{21} \]

\( P_{j0} \) is the average power density per unit volume dissipated by the elementary winding pattern without the layer insulation (Wm\(^{-3}\)).

\( \Delta H_3 \) is the variation between \( T_0 \) and \( T_t \) of the enthalpy per unit volume of medium 3 (Jm\(^{-3}\)).

Let us call the dimensionless parameter \( p_{IV} \) the second insulation parameter, defined by:

\[ p_{IV} = \frac{v_0 L_4}{\sqrt{D_3 D_4}} \quad \text{(dimensionless)} \tag{22} \]

where \( D_4 = k_4/C_4 \) (\( L_4 \) respectively) is the thermal diffusivity (the width, respectively) of medium 4.

We thus establish, for all values of \( m_{IV} \) and of \( p_{IV} \), the existence of a constant-velocity asymptotic shift of the temperature profile, for which the velocity \( v_{IV} \) is given by:

\[ v_{IV} = \frac{V_{IV}}{\sqrt{1 + m_{IV} \tanh(p_{IV} V_{IV})}} v_0 \quad (ms^{-1}) \tag{23} \]

where \( V_{IV} \) is the insulation corrective factor, determined versus \( m_{IV} \) and \( p_{IV} \) by the implicit equation:

\[ \tanh(p_{IV} V_{IV}) + 3 \frac{\tanh(p_{IV} V_{IV})}{p_{IV} V_{IV}} = \left[ 2 - \frac{1}{m_{IV} p_{IV}} \right] \]

\[ = \frac{2}{m_{IV} p_{IV} V_{IV}^2} \tag{24} \]

Curves of \( V_{IV} = f(m_{IV}, p_{IV}) \) are presented in Figure 5.

**Case of a layer of insulated super-stabilized conductors**

We designate by medium 1 the multi-filament superconductive composite, by medium 2 the super-stabilizer, by medium 3 the system (composite + super-stabilizer + insulation between conductors), and by medium 4 the layer insulation.

Let us reintroduce the parameters \( m_{III}, p_{III}, m_{IV}, \) and \( p_{IV} \):

\[ m_{III} = \frac{D_2 A_1}{\rho_1 A_2} \quad \text{and} \quad p_{III} = \frac{v_0 L_2 e}{D_3} \]

\[ m_{IV} = \frac{\sqrt{k_4 C_4}}{k_3 C_3} \quad \text{and} \quad p_{IV} = \frac{v_0 L_4}{\sqrt{D_3 D_4}} \]

in defining the equivalent diffusivity \( D_e \) by:

\[ D_e = \sqrt{2 D_m D_3} \quad (ms^{-1}) \tag{25} \]

and in taking for \( v_0 \) the velocity calculated assuming that the layer of super-stabilized conductors is bare (without layer insulation) and the current is confined to the composite:

\[ v_0 = \sqrt{\frac{D_3 P_{j0}}{\Delta H_3}} \quad (ms^{-1}) \tag{26} \]

where \( P_{j0} \) is the average power density per unit volume dissipated by the elementary winding pattern without the layer insulation and assuming that the current is confined to the composite in the normal resistive state.

We thus establish, for all values of \( m_{III}, p_{III}, m_{IV}, \) and \( p_{IV} \), the existence of a propagation velocity for the normal zone \( v_{IV} \) given by:

\[ v_{IV} = \frac{v_Y}{\sqrt{1 + m_{IV} \tanh(p_{IV} V_{IV})}} v_Y \quad (ms^{-1}) \tag{27} \]

where \( v_Y \) is the super-stabilization and insulation corrective factor determined versus \( m_{III}, p_{III}, m_{IV}, \) and \( p_{IV} \), by the implicit equation:

\[ \tanh^2(p_{IV} V_{IV}) + 3 \frac{\tanh(p_{IV} V_{IV})}{p_{IV} V_{IV}} + \left[ 2 - \frac{1}{m_{IV} p_{IV}} \right] \]

\[ = 2 \frac{m_{III} p_{III}}{m_{IV} p_{IV}} \frac{1}{V_{IV} [\tanh(p_{III} V_{IV}) + m_{III} p_{III} V_{IV}]} \tag{28} \]

**General method of calculating propagation velocity**

After having presented the results of the studies of each of the four influences, it remains to explain how to integrate these individual studies in order to arrive at a general method of calculating propagation velocity. Let us proceed in progressive steps.

**Case of a layer of multi-filament superconductive composites without layer insulation**

In this case, only two influences must be taken into account: the current-sharing zone and the temperature-dependence of the physical characteristics.
We must remember that for the current-sharing zone:

\[ v_I = v(m_I) v_0 \]

where \( v_0 \) is the velocity calculated neglecting the current-sharing zone.

To consider the temperature-dependence of the physical characteristics, it seems logical to replace \( v_0 \) by the velocity calculated taking into account this influence. Thus we see that the velocity \( v_{I-II} \) along a layer of composites without layer insulation is given by:

\[ v_{I-II} = v_I(m_{II}) v_2(m_2) \sqrt{\frac{D_I P}{\Delta H_I}} \quad (29) \]

Case of a layer of super-stabilized conductors without layer insulation

Now three influences must be taken into account: the current-sharing zone, the temperature-dependence of the physical characteristics, and the electromagnetic diffusion in the super-stabilizer.

We must remember that for the electromagnetic diffusion:

\[ v_{III} = v_{III}(m_{III}, m_{III}) v_0 \]

where \( v_0 \) is the velocity calculated assuming that the current is confined to the composite.

To consider the influence of the current-sharing zone and the temperature-dependence of the physical characteristics, it seems logical to replace \( v_0 \) with the velocity calculated taking these influences into account. Thus we see that the velocity \( v_{I-II-III} \) along the layer of calculations is given by

\[ v_{I-II-III} = v_I(m_I) v_{II}(m_{II}) v_{III}(m_{III}, m_{III}, p_{III}) \sqrt{\frac{D_I P}{\Delta H_I}} \quad (30) \]

where \( m_I, m_{II}, \) and \( m_{III} \) are the parameters defined by Equations (4), (8), and (11), respectively, and \( p_{III} \) is the second super-stabilization parameter generalized in:

\[ p_{III} = v_I(m_I) v_{III}(m_{III}) \sqrt{\frac{P_{III(I-2e) ^2}}{2 D m \Delta H_I}} \quad (31) \]

The other definitions remain the same.

To illustrate these results, we present, in Figure 6, the curve \( v = f(I) \) calculated along a layer of bare ALEPH conductors (without layer insulation), and in Figure 7, the curves \( v = f(I) \) for each of the influences.

Case of a layer of insulated conductors

Through reasoning similar to the above, we see that the propagation velocity along a layer of insulated composites is given by formula (23), and along a layer of insulated super-stabilized conductors by formula (27). by replacing \( v_0 \) with the velocity calculated taking into account the current-sharing zone and the temperature-dependence of the physical characteristics (\( v_0 \) is also to be replaced in the definitions of \( p_{III} \) and \( p_{IV} \)).

Conclusion

The previous equations allow us to consider all cases of indirectly-cooled windings, where the heat exchanges with helium are negligible at the scale of the propagation phenomenon. However, our modelization has a weakness. We see that all the interface resistances, either electrical (between the composite or the super-stabilizer) or thermal (between the layer of conductors and the layer of insulation) equals zero. It can occur that in certain configurations these become dominant.
References


