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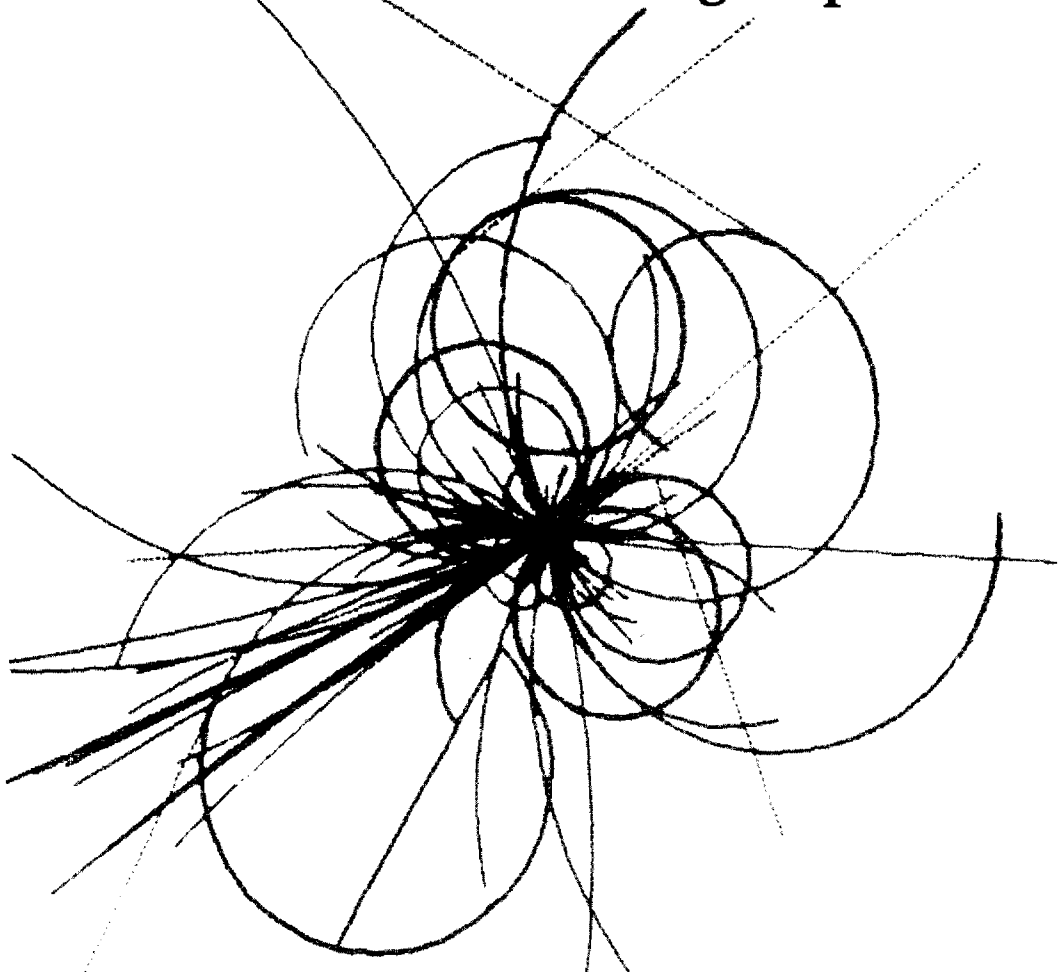
The Superconducting Super Collider



Simulation of Chromaticity Control in the SSC

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ABSTRACT

An operational simulation has been performed to predict the performance of multipole correctors for compensation of linear, quadratic, and cubic momentum dependencies, due to systematic field errors, of the SSC lattice. An iterative scheme, using beams varying over successively larger momentum ranges, is found to be satisfactory. Both distributed bore-tube compensation elements as described in the Conceptual Design Report and "Simpson's rule" lumped correctors (also known as the "Neuffer" scheme) have been successfully employed. To get started a coasting beam with rms energy spread of 10^{-5}, roughly a factor of five smaller than has been achieved in the Tevatron, seems to be required. Realistic assumptions are made about closed orbit errors, about errors in other parameters, and about the precision with which tunes can be measured.

1. Introduction.

The SSC will exhibit strong momentum dependence due to systematic field errors — more so than is true of existing accelerators because of the large radius of the SSC and the small bore size of its magnets. In the Conceptual Design Report (CDR), bore-tube compensation coils have been included for the compensation of this chromatic behaviour. (We will use the word "chromaticity" in a general sense of any functional dependence on momentum as well as in the narrow technical sense of rate of change of tune with respect to momentum.) In this report we describe an operational simulation of the performance to be expected in the use of these coils for chromaticity control. Similar analysis is performed using one of the possible lumped compensation coil schemes.

Even with purely linear elements the accelerator will exhibit chromatic dependence but this will be less important than that due to nonlinear dipole field multipoles. Of these the most important is the b_2 (sextupole) field which is due to persistent superconducting currents. According to CDR plans, there will be coils to compensate for this and coils will also be present for the b_3 (octupole) and b_4 (decapole) multipoles, which would also lead to unacceptably large tune shifts if left uncompensated.[1] The accelerator theory by which these correctors can be set to achieve the required compensation is well understood and has been described in various reports.[2] [3] [4] [5]

Implicit to these calculations is the assumption that the field defects are known from previous measurements. It has been assumed[6] that the dominant b_2 effect can be reduced by roughly a factor of ten by "dead reckoning" based on magnet test facility measurements alone, but that the necessary further improvement of about a factor of one hundred will be based on measurements using the beam itself. In this report we investigate the feasibility of this compensation using an operational simulation technique.

Issues to be studied include the beam quality required (especially when getting started), the anticipated precision of tune measurement, a prescription for

setting correctors (based on measurement), and finally, after doing as well as possible, determination of whether the available correctors and prescriptions are capable of producing the specified performance. The "operational simulation strategy" for studying these questions is to model each aspect which can potentially degrade performance as realistically as practicable and to see how they all "play together". This can be regarded as complimentary to possibly more incisive, but more specialized, analytic investigations, in that it can be expected to treat correctly the all-too-common instances of harmful conspiracy between two or more bad effects. Equally important goals are to help in planning operational procedures and to establish instrumental specifications.

In earlier studies of the effects of field errors it has always been found that orbit distortion as quantified by the variable called "smear" is dominated by random magnet errors while tune shifts are dominated by systematic magnet errors. This report is intended to study tune shifts and we neglect random magnet errors. An example of the above-mentioned possibility of harmful conspiracy is that random orbit displacements taken with systematic magnet errors can cause appreciable smear. We defer investigation of this affect to a later report and concentrate entirely on tune shifts.

Emphasis will be on use of the bore-tube correctors but performance of a lumped corrector scheme will also be analysed. For lumped schemes, since the ultimate performance is almost surely inferior to a continuous scheme, determination of whether the required performance specifications can be met is especially important.

2. Assumed Errors and Available Correctors.

In an ideal simulation every lattice parameter in a fully realistic lattice would be randomly assigned consistent with its specified tolerance. This is clearly too ambitious and we have restricted the analysis in various ways while attempting to retain, and treat faithfully, the most critical features. We have restricted the analysis to single particle motion, ignoring effects like emittance dilution and loss of particles from the tails of the particle distributions. Another important restriction has been to deal only with a lattice having no intersection regions. This is not because we feel there are no important problems in connection with intersection regions but rather that they are being deferred until later.

The "arcs-only" lattice which we have analysed is made up of 320 cells, each essentially identical in all parameters to a regular-arc cell of the SSC. The tunes were fixed at

$$\begin{aligned} Q_x &= 81.285 \\ Q_y &= 82.265 \end{aligned} \tag{2.1}$$

These correspond to phase advances per cell of 92.549° and 91.446° respectively. The integer tunes are separated by one unit in order to reduce sensitivity to systematic coupling effects,[7] but in the present analysis no systematic coupling errors have been included. The fractional tunes are thought to be favorable for colliding beam operation, based on $SP\bar{P}S$ experience.[8] There are almost certainly choices for the fractional tunes which would have simplified some of the manipulations to be described below. Furthermore, in actual operations, it could be sensible to make such tune choices as part of preliminary tune-up. However, we have chosen not to pursue such a strategy.

A discussion follows of the errors that have been incorporated in the simulation (not necessarily all simultaneously). According to the CDR the most important nonlinear systematic dipole field errors, being due to persistent cur-

rents, are, at injection[9]

$$
\begin{aligned}
b_2 &= -\,4.7 \text{ units} \\
b_4 &= \ \ 0.30 \text{ units} \\
b_6 &= -\,0.07 \text{ units}
\end{aligned}
\tag{2.2}
$$

where, according to convention, "units" stands for parts per 10^4 fractional error at a radius of 1 cm. The values entered into the TEAPOT input file are derived from these.

In TEAPOT all elements are replaced by thin elements. It has been shown[5] that replacement of the actual 16 m long SSC dipoles by two thin half-lengths dipoles (call them $dip1$ and $dip2$) causes tune shifts less than ± 0.0002; this was judged to be adequate. The starting nonlinear systematic multipoles used for these magnets were

$$
\begin{aligned}
dip1,\ b_2 &= \ \ 4.2, b_3 = 0.6, b_4 = 0.6 \\
dip2,\ b_2 &= -5.2, b_3 = 0.0, b_4 = 0.0
\end{aligned}
\tag{2.3}
$$

The differences between (2.3) and (2.2) will now be discussed. In the CDR the b_2 bore-tube correction coil extends half the length of the dipole and it was assumed its settings could be "dead-reckoned" from factory magnet measurements to an accuracy of $\pm 10\%$. Suppose a "worst-case" field of -5.2 units in both $dip1$ and $dip2$ is corrected, as regards field integral, entirely in $dip1$, but under the erroneous assumption that the field error is -4.7 units. The b_2 values in (2.3) result and the b_4 errors are the result of a similar argument starting from (2.2). Though there is no systematic b_3 term coming from persistent currents it was judged[10] that the systematic errors in b_3 and b_4 could be comparable, and that was the basis for the b_3 values in (2.3).

The multipole b_6 has been treated differently. An estimate of the importance of this multipole, for the value given in (2.2) and using formulas given below, suggests that it should be negligible for all amplitudes of interest. No correction

elements for this multipole were assumed to be present and in most cases the field error was not included. It was confirmed to be negligible in the few cases for which it was explicitly included.

It has been suggested, especially by Neuffer,[3] that systematic compensation be performed using lumped multipole correctors at the regular F and D locations in the spool pieces situated next to the quadrupoles, as well as one extra lumped corrector at C at the center of each half-cell. The layout is indicated in the sketch at the bottom of Table 2, and the multipole strengths are indicated there. For the present investigation the same systematic errors (2.2) were assumed, with the actual values known only to within 10%. The strengths of D, F, and C multipoles were constrained to be in the ratios $2:2:4$ as indicated in Table 2, except that specially tuned octupoles[5], known to give excellent tune constancy, were used.

It can be feared that the presence of closed orbit errors will hinder the empirical chromaticity compensation. Closed orbit compensation has been described elsewhere,[11] and those methods have been repeated here. Results will be given with and without orbit errors. The rms bend errors used were $\sigma_{a0} = 5.9$ units, and $\sigma_{b0} = 8.5$ units, though these were multiplied by $\sqrt{2}$ to compensate for the fact that each dipole is split in half with random errors being assigned independently to each half. Roll errors of 0.6 mr, similarly increased by $\sqrt{2}$, were assumed. Quadrupole locations were assigned rms uncertainties of 0.5 mm, both horizontally and vertically, and, within the quads, rms location uncertainties of 0.1 mm were used. The combined effect of the various errors causing closed orbit shifts was that the rms orbit deviations both horizontally and vertically were 0.6 mm, after correction. As mentioned above, no coupling errors were included.

3. Analytic Estimates and Numerical Calculation of Chromatic Coefficients.

In this section estimates of various effects will be given in order to assess their relative importance and we give the results of numerical calculations of the tune dependence resulting from particular multipoles.

(i) Dependence of Tune on Momentum. *Consider* N (numerically $12\times320 = 3840$) dipoles magnets each bending the beam through an angle $2\pi/N$. The bend error in a single dipole due to b_2, b_3, and b_4 is

$$[b_2(x+\eta\delta)^2 + b_3(x+\eta\delta)^3 + b_4(x+\eta\delta)^4]2\pi/N \tag{3.1}$$

where, at the dipole, the transverse displacement (assumed for simplicity to be purely horizontal) has a betatron part x and an "energy" part $\eta\delta$ where η is the dispersion and δ is the fractional momentum offset. We are mainly interested in the dependence on δ of the small amplitude tune. For that we need only retain the part of (3.1) which is linear in x. (Strictly the transverse displacement from the off-momentum fixed point should be used but we ignore the distinction.) According to the "Golden Rule" the location dependence brings in one power of the lattice function β and yields tune shift estimates for the two transverse planes given by

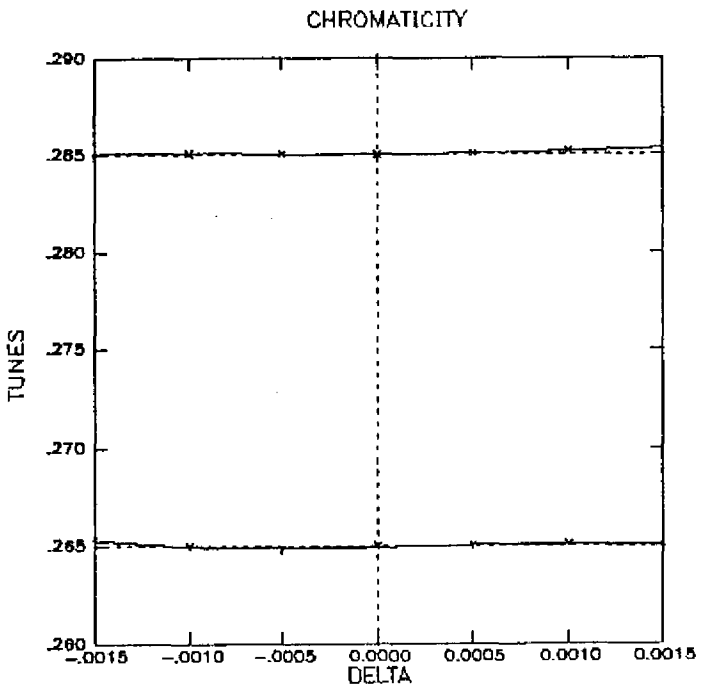
$$\begin{aligned}\pm\Delta Q =& \frac{1}{4\pi}\sum_N \beta[2b_2\eta\delta + 3b_3(\eta\delta)^2 + 4b_4(\eta\delta)^3](2\pi/N)\\ =& b_2 < \beta\eta > \delta + 3b_3/2 < \beta\eta^2 > \delta^2 + 2b_4 < \beta\eta^3 > \delta^3\end{aligned} \tag{3.2}$$

where the summation has been expressed in terms of averages $< \beta\eta >$ etc., which will be similar, but not identical for the horizontal and vertical planes. For purposes of estimation we take

$$\begin{aligned}< \beta\eta > \simeq & < \beta >< \eta >\\ =& 1.85\times10^4\text{cm}\times2.07\times10^2\text{cm}\\ =& 3.8\times10^6\text{cm}^2\end{aligned} \tag{3.3}$$

Substituting this into (3.2), with $b_2 = 0.5 \times 10^{-4}\text{cm}^{-2}$ and $\delta = 0.001$, the leading term yields a tune shift of 0.19. This is far greater than the specified maximum tune shift of ± 0.005, bearing out the earlier assertion that a large residual chromaticity will remain to be corrected empirically after the main effect has been removed using magnetic field measurements.

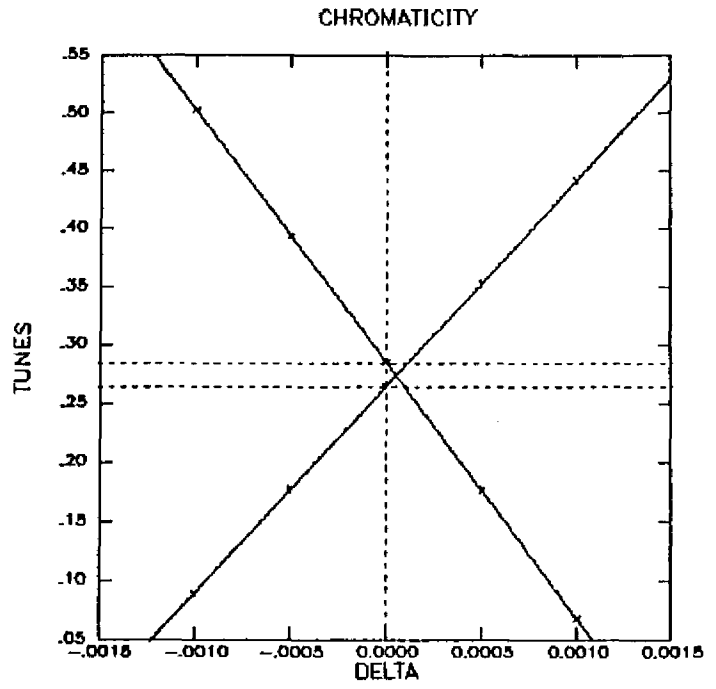
We will call the coefficients of δ, δ^2 etc. chromaticity coefficients. The quadratic and cubic coefficients could be estimated as the linear coefficient was in the previous paragraph, but to obtain any kind of precision in such calculations the averages of the lattice functions would have to be evaluated more accurately than has been done. This would include using a momentum-dependent η function. Refinements such as this would not however be justified, since in the presence of closed orbit errors feed-down from higher multipoles would also destroy the simple proportionality of the linear, quadratic, and cubic coefficients to b_2, b_3 and b_4 respectively. The actual dependencies are calculated using TEAPOT. Thorough comparisons between TEAPOT tune calculations and analytic tune calculations of Neuffer have given excellent agreement for simple lattices.[5] Fig. 1 shows the momentum dependence of the pure lattice (with the linear chromaticities corrected to zero). Figures 2, 3, and 4 show the momentum dependencies that result for the values of b_2, b_3 and b_4 given in (2.3) but with only one turned on at a time, for the otherwise pure lattice having no closed orbit errors. Polynomial fits are given. That the chromatic coefficients in the two planes are not quite equal in magnitude (but opposite in sign) is because the averages enter differently for the two planes in (3.2). The linear coefficients agree to within ten percent with the estimate given just below (3.3). It can be seen, consistent with (3.2), that b_2 mainly causes a linear dependence on δ, b_3 mainly a quadratic dependence and so on.



CHROMPLOT fit on X
coefficients for polynomial in D = 1000.0 * d + 0.0
2.8502e-01 2.1666e-05 8.8777e-05 2.1577e-05

CHROMPLOT fit on Y
coefficients for polynomial in D = 1000.0 * d + 0.0
2.6494e-01 1.8977e-04 1.1111e-04 -1.2022e-04

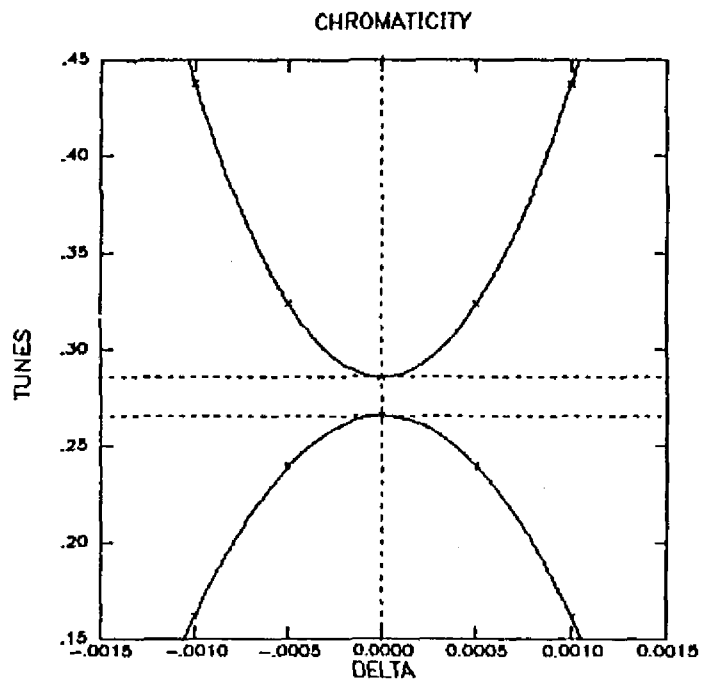
Figure 1. Momentum dependence of pure lattice.



CHROMPLOT fit on X
coefficients for polynomial in D = 1000.0 * d +
2.8500e-01 -2.1652e-01 -1.2692e-04 1.9630e-05

CHROMPLOT fit on Y
coefficients for polynomial in D = 1000.0 * d +
2.6500e-01 1.7567e-01 4.8605e-04 2.1358e-06

Figure 2. Momentum dependence with b_2 only.



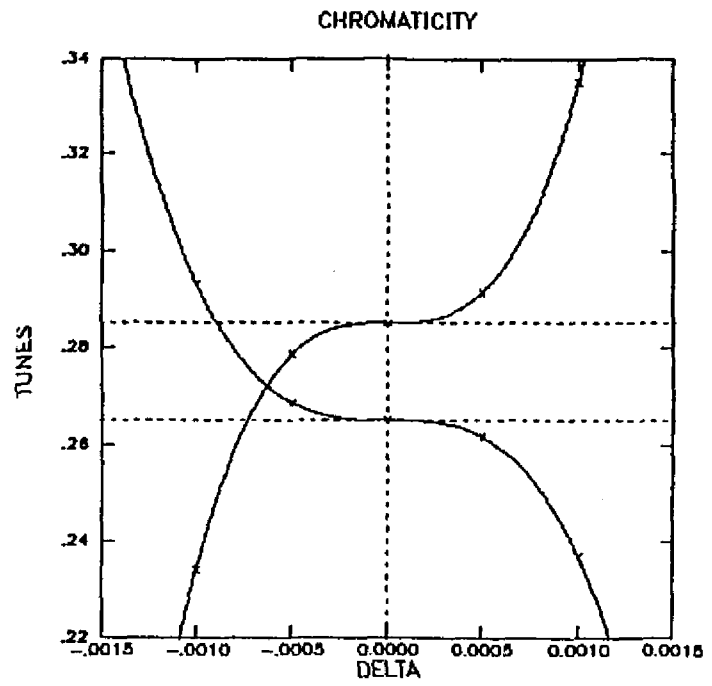
CHROMPLOT fit on X

coefficients for polynomial in D = 1000.0 * d +
2.8535e-01 -3.8594e-05 1.5210e-01 -1.9968e-04

CHROMPLOT fit on Y

coefficients for polynomial in D = 1000.0 * d +
2.6503e-01 1.5296e-05 -1.0299e-01 -7.5132e-05

Figure 3. Momentum dependence with b_3 only.



CHROMPLOT fit on X

coefficients for polynomial in D = 1000.0 * d +
2.8505e-01 1.5298e-04 -4.4003e-04 5.0259e-02

CHROMPLOT fit on Y

coefficients for polynomial in D = 1000.0 * d +
2.6500e-01 2.7525e-05 4.8297e-05 -2.8136e-02

Figure 4. Momentum dependence with b_4 only.

(ii) Incremental Orbit Error Due to b_2 in the Presence of Other Orbit Errors. It is found that chromaticity compensation in the presence of orbit errors is more difficult than in their absence and one can look for the leading cause. One possibility is that the steering effect of the multipole correctors yields undesirable orbit shifts as the correctors are adjusted. Here we estimate, for b_2, the magnitude of the effect and find it to be negligible.

The displacement at point d in the lattice due to bend error $\Delta\theta_i$ at point i is[13]

$$x_d = \frac{\cos(\mu/2 - \phi_{di})}{2\sin\mu/2}\sqrt{\beta_i\beta_d}\Delta\theta_i \tag{3.4}$$

where ϕ_{di} is the phase advance from d to i. Suppose that $\Delta\theta_i$ is due to a closed orbit error x_i at the position of a dipole which bends through $2\pi/N$, where N was defined above. That is

$$\Delta\theta_i = (2\pi/N)b_2 x_i^2 \tag{3.5}$$

Summing over locations i we get

$$x_d = \frac{\sqrt{\beta_d}}{2\sin\mu/2}\frac{2\pi}{N}b_2\sum_{i=1}^{N}\sqrt{\beta_i}x_i^2\cos(\mu/2 - \phi_{di}) \tag{3.6}$$

In the summation β_i and the cosine term are not stochastic while x_i^2 is. On dimensional grounds the sum will come out

$$\sum = b_2\sqrt{<\beta>}\sigma_{co}^2\sqrt{N}C_1 \tag{3.7}$$

where σ_{co} is the rms orbit shift and C_i is a numerical factor of order one. Combining equations we get the estimate

$$\frac{<x_d>}{\sigma_{co}} \simeq \frac{\pi b_2 <\beta> \sigma_{co}}{\sin\mu/2\sqrt{N}} \tag{3.8}$$

Numerical substitution using values introduced above gives the result that the incremental orbit shift from a one unit change in b_2 is about one percent of the preexisting orbit shift. This is presumeably negligible.

(iii) Effect of Closed Orbit Shift on Chromaticity Coefficients. In the presence of closed orbit error x_i at location i, the factors $(x + \eta\delta)$ in (3.1) must be replaced by $(x - x_i + \eta\delta)$. Then the result of keeping only terms linear in x and linear in δ is

$$\Delta\theta_i = 2\pi/N[2b_2 - 6b_3x_i + 12b_4x_i^2]\eta\delta x \tag{3.9}$$

The second term tends to average to zero but the third term does not. Its contribution to the linear chromaticity differs from that due to b_2 by the factor $6(b_4/b_2)\sigma_{co}^2$ which can be several percent. Corrections to the other chromatic coefficients can be estimated similarly.

4. Algorithms for Iterating Measurement and Adjustment.

Two algorithms have been used for setting the multipole correction elements. Both methods assume that a "pencil" circulating beam is available initially having an rms fractional momentum spread of 0.00001. This is a factor of five smaller than energy spreads achieved in the Tevatron.[12] We do not regard this as too serious both because the compensation procedure would probably work adequately (though less quickly) with a larger spread and because the small spread can undoubtedly be achieved by scraping (at the cost of intensity.) The RF frequency stability needed to maintain this energy accuracy does not appear to be difficult. According to reference[12] pulse-to-pulse injection jitter as great as 0.0001 can be expected but that will not adversely affect these procedures, since they use circulating beam. Fractional momentum measurement errors are assumed to be negligible.[13] Tune determinations have been assigned an error of ± 0.0001 consistent with recent experience on the Tevatron during the beam dynamics experiment E778. When this was increased to ± 0.001 in a few cases it impaired the rate of convergence of the adjustment procedure but did not prevent its successful completion.

In each stage of iteration both the horizontal and vertical tunes are measured at five momenta, $\pm 2\Delta\delta, \pm\Delta\delta$, and 0.0, making a full range of $4\Delta\delta$. In successive stages the range is increased, always consistent with restricting the maximum tune shifts to be less than about ± 0.005. In practice the successive measurement ranges used were ± 0.000025, ± 0.00005, ± 0.0001, ± 0.00025, ± 0.00035, ± 0.0005, and ± 0.001.

Success can only be claimed when this condition is met with $2\Delta\delta = 0.001$, for particles with maximum betatron amplitudes as great as 5 mm, since then the CDR specified maximum tune shift tolerance will have been met.

Tunes which would be measured in real operation are calculated in the simulation. Each linear transfer matrix is obtained in TEAPOT by tracking several particles in the near vicinity of each of the off-momentum fixed points. The tunes are obtained from the traces of these matrices. It has often in the past been demonstrated that tracking for many turns (say 256), followed by Fourier analysis, followed by parabolic interpolation, yields accurately ($\simeq \pm 0.0002$) the same tunes. In real operation that would be the procedure employed.

Calculations have been performed on the SUN 3/140 workstation. The lattice is so large that the calculations are very slow—calculation of the tunes for five momentum cases takes 10 minutes. This may be all-too-realistic as the corresponding measurements on the SSC may be very time consuming. It was however demonstrated during the E778 Tevatron experiment that a single chromaticity determination could be performed in as little as five seconds. Based on this, and assuming that the frequency of the accelerator RF can be programmed non-destructively to the coasting beam of the SSC, the entire chromaticity compensation procedure could take as little as a few minutes.

(i) Iterative Polynomial Fitting Procedure. This is a straightforward procedure which has converged satisfactorily both with and without closed orbit errors. Though not exactly true, it is assumed that b_2 gives a pure linear correction, b_3 a pure quadratic correction, and b_4 a pure cubic correction, and that

the coefficients of these correctors are the same with and without closed orbit errors. These coefficients are obtained from the data of Figures 2, 3, and 4. At a given stage of iteration, five measurements both in the horizontal and in the vertical planes are obtained and fit by polynomials. The shifts of b_2, b_3, and b_4 needed to cancel these polynomials term-by-term are calculated and the average of the horizontal and vertical values is used for each multipole. In practice, for fractional momentum less than ± 0.0001, only the b_2 correctors were required or used. Two different error seeds were studied with each giving satisfactory convergence. With no closed-orbit errors a single iteration was sufficient for each of the momentum ranges listed above, but in the presence of orbit errors two iterations were sometimes required before proceeding to the next, wider, momentum range.

(ii) Least Squares Procedure. This is a scheme which, in principle, should converge in several iterations. However it may not be more efficient as it requires accurate determination of each of the chromaticity coefficients for each of the multipole coefficients and, as noted above, these depend on the particular closed orbit errors. For the lattice with no closed orbit errors this scheme converged after five iterations. For a lattice with closed orbit errors, but using the ideal orbit chromaticity coefficients, it did not converge. This method does not naturally take much advantage of the initial use of a beam of very small momentum spread. The formulation of this method will be given both for reference and because minimisation procedures may be be required when the more complicated chromatic behavior due to intersection regions is included, or if the simple averaging of horizontal and vertical corrections proves inadequate, or for analysing not-quite-systematic behavior.

The problem is to obtain the best settings for the b_2, b_3, and b_4 correctors based on the ten "detected" tune offsets, $\Delta Q_x^{(0)}(d)$ and $\Delta Q_y^{(0)}(d)$, $d = 1, 2, ..., 5$ at the five momenta noted above. The solution is set up superficially as a linear least squares method, but in such a way that iteration is natural, which makes it effectively a nonlinear method. This also makes it natural to start with fitting functions calculated for the ideal lattice but then to improve them based on

calculations or observations made with errors present.

The notation to be employed is that the subscript d (for detector) runs over the five momentum values and the subscript a (for adjustor) runs over the three multipole indices, 2, 3, and 4. All summations which appear are over these ranges.

The offsets $\Delta Q_{x,y}^{(0)}(d)$ are due to unknown errors but they are assumed to have been measured. Initially they are measured with the b_a set to zero, but in later iterations they will be obtained with preliminary values b_a present which will be modified to $b_a + \Delta b_a$ as the result of the iteration. Formulas for the x tune offset including the effects of the offsets Δb_a are

$$\Delta Q_x(d) = \Delta Q_x^{(0)}(d) + \sum_a \Delta b_a q_x^{(a)}(d) \qquad d = 1, 2, ..., 5 \tag{4.1}$$

where, for each adjustor, $q_x^{(a)}(d)$ has previously been obtained from data like that in Figures 2, 3, and 4, as the tune offset at the values δ_d due to unit strength change in corrector a. These quantities depend implicitly on all the lattice parameters including errors. Initially they are calculated for the ideal lattice but, as noted above, it may be necessary to improve them operationally in the presence of actual errors. An expression like (4.1) can also be written for $\Delta Q_y(d)$.

The function to be minimized is the "badness" function

$$\begin{aligned} B(\Delta b_2, \Delta b_3, \Delta b_4) &\equiv B(\Delta b_a) \\ &= \sum_d [\Delta Q_x^2(d) + \Delta Q_y^2(d)] \end{aligned} \tag{4.2}$$

The best settings for the Δb_a will be those that minimize $B(\Delta b_a)$; they are obtained by solving the equations

$$\frac{\partial B}{\partial \Delta b_a} = 0 \qquad a = 2, 3, 4 \tag{4.3}$$

Substituting from (4.1) into (4.2) and (4.3), we get three equations which can be

written in matrix form as

$$\mathcal{M}\mathcal{B} = \mathcal{V} \tag{4.4}$$

with solution

$$\mathcal{B} = \mathcal{M}^{-1}\mathcal{V} \tag{4.5}$$

where $\mathcal{B} = (\Delta b_2, \Delta b_3, \Delta b_4)^T$ is a vector of the unknowns,

$$\mathcal{V} = (\mathcal{V}_a) = -\sum_d [\Delta Q_x^{(0)}(d) q_x^{(a)}(d) + \Delta Q_y^{(0)}(d) q_y^{(a)}(d)] \tag{4.6}$$

is the vector of inhomogeneous terms and

$$\mathcal{M} = (\mathcal{M}_{ab}) = \sum_d [q_x^{(a)}(d) q_x^{(b)}(d) + q_y^{(a)}(d) q_y^{(b)}(d)] \tag{4.7}$$

is the coefficient matrix.

After the replacement of b_a by $b_a + \Delta b_a$ new tune offsets can be measured and the process repeated. As described earlier the momentum range must be greatly restricted initially. When the range is expanded, if the fitting functions $q_x^{(a)}(d)$ and $q_y^{(a)}(d)$ can or must be recomputed (or remeasured) it could be done, but at the cost, roughly, of quadrupling the time per iteration. That has not been done.

5. Results.

A selection of initial, intermediate, and final tune dependence curves are shown in the following figures. For Figures 5 through 11 bore tube compensation coils were used. Fig. 5 shows the tune dependence with no closed orbit errors but with the full, uncompensated, systematic errors given in (2.3). Fig. 6 is the same thing except that the linear chromaticity is taken out using the chromaticity sextupoles. Figures 7 and 8 show the result of applying the least squares fitting procedure to these data without any concern for the operational practicality of running the accelerator with such gigantic tune shifts.

The next figures illustrate compensation in the presence of all errors which have been mentioned, using the polynomial fitting procedure. Fig. 9 shows a starting situation, Fig. 10 shows an intermediate situation, and Fig. 11 shows the final results for two different seeds. In both cases the final behavior is fully satisfactory.

The final result of compensation using "Simpson's Rule" compensation as recommended by Neuffer[3] is shown in Figure 12 for two different seeds. In this scheme there is one lumped correction element at the center of each half-cell, constrained to be twice as strong as the lumped corrector at the regular spool-piece location.

Tune variation, both horizontal and vertical, for the distributed coils is given in Table 1 for two different sets of random numbers. Tune shifts away from the small amplitude values are given (in units of 0.001) for points defining the extremes of momentum and betatron variation according to the CDR specifications. In these units, variation by less than ± 5 is defined as meeting the specification. The format of this table is further explained in reference[4] . Table 2 shows similar results for the lumped scheme. Though noticeably inferior to the distributed results the CDR specifications are easily met.

We would like to acknowledge the help of our TEAPOT collaborators Ruth Hinkins, Vern Paxson and Tjet Sun.

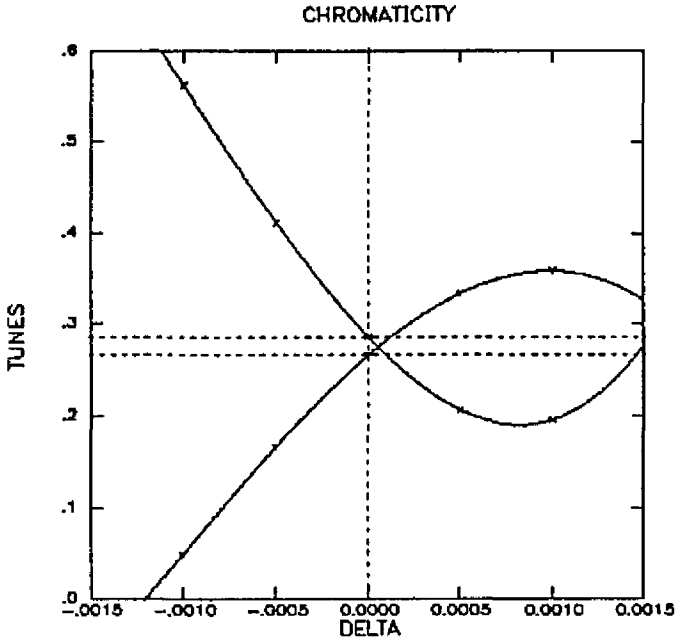


Figure 5. Tune dependence, full systematic errors, no orbit errors, no compensation.

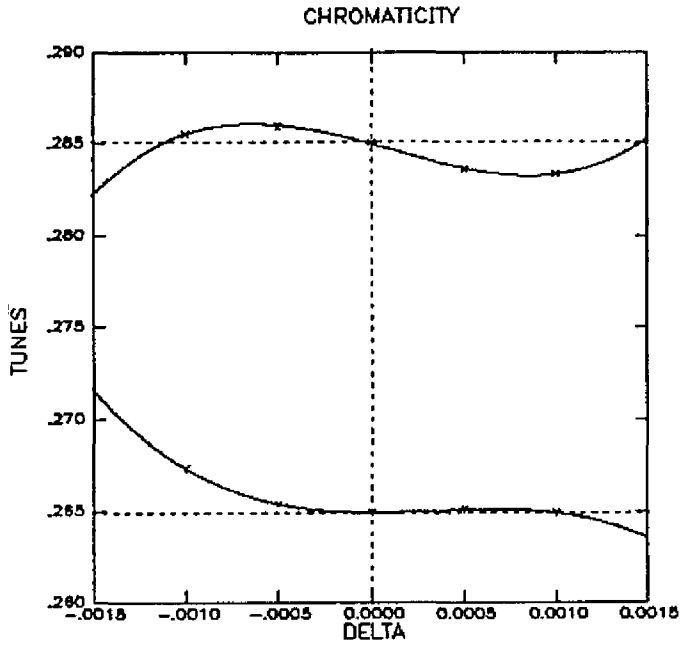


Figure 7. Same as Fig. 5 but with correctors set by least squares method.

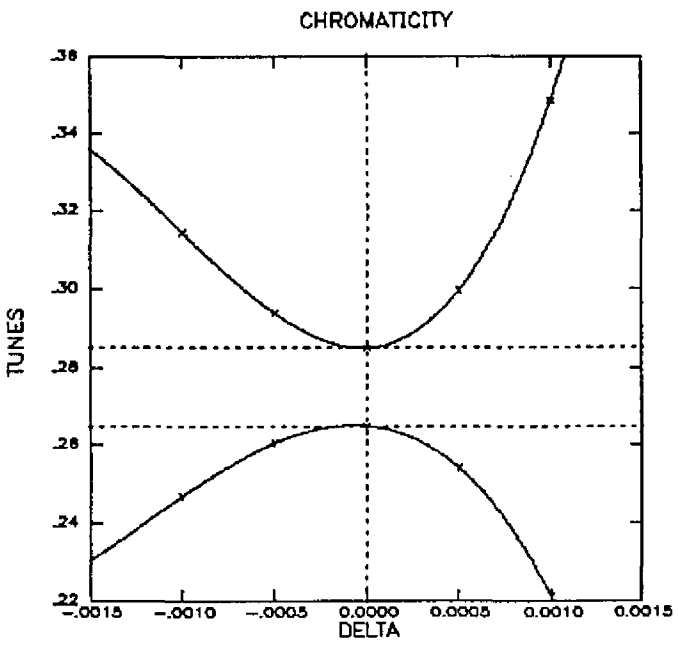


Figure 6. Tune dependence, full systematic errors, no orbit errors, linear chromaticity adjusted to zero.

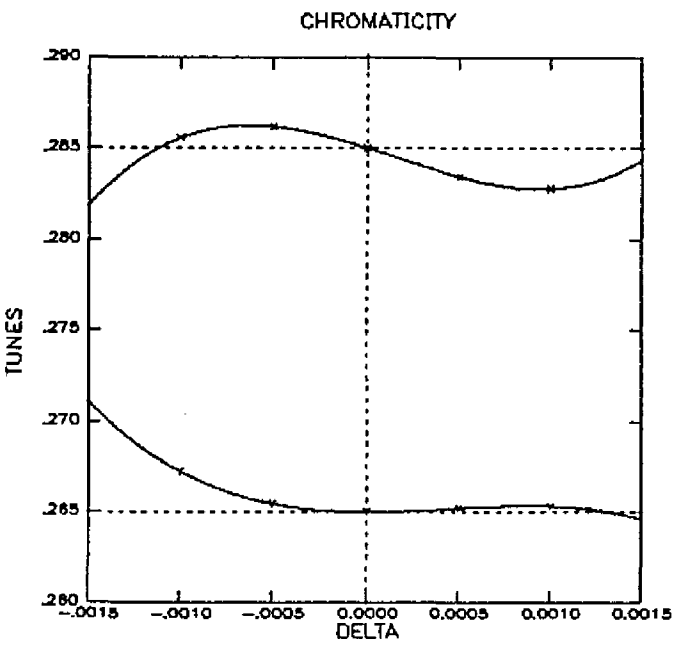


Figure 8. Same as Fig. 6 but with correctors set by least squares method.

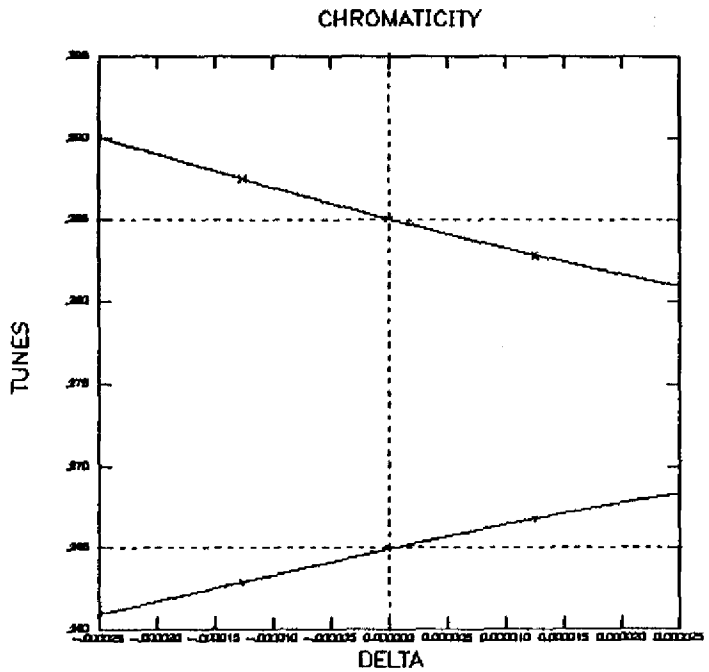


Figure 9. Tune dependence, full orbit errors, starting situation.

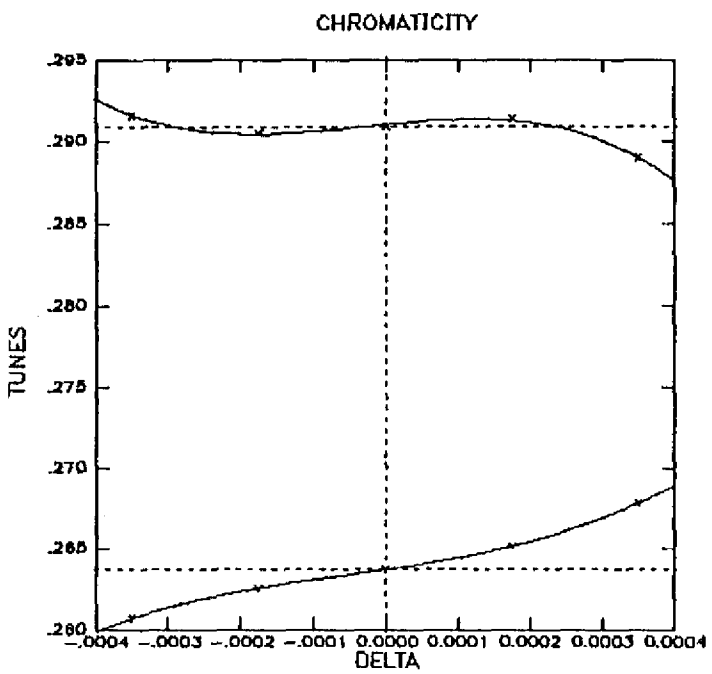
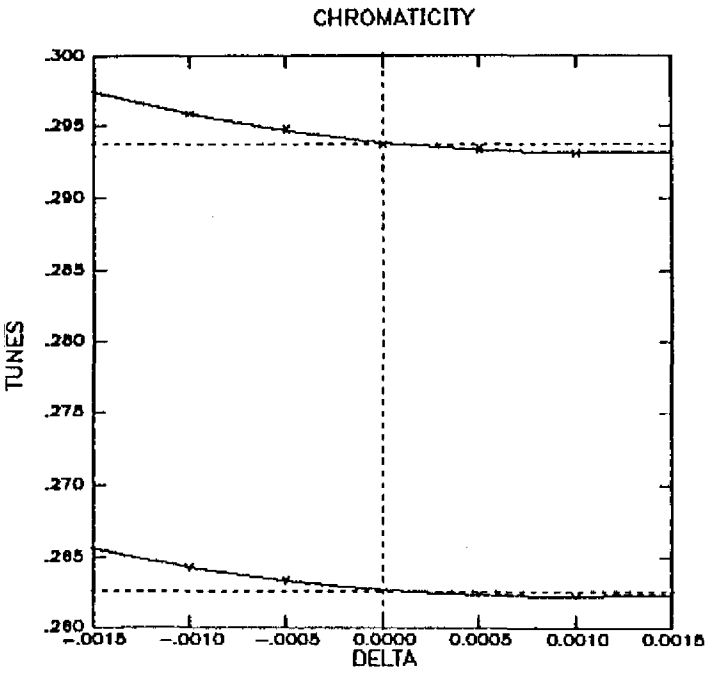


Figure 10. Tune dependence, full orbit errors, intermediate situation.



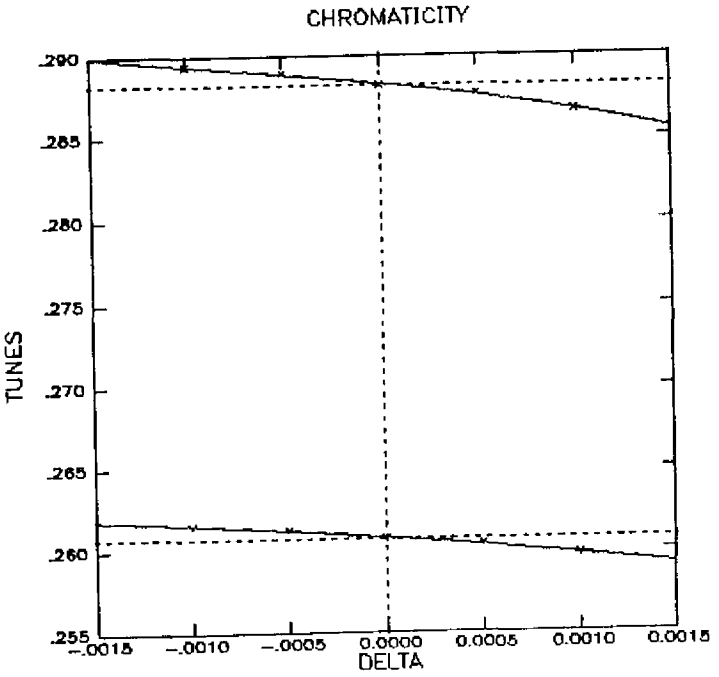
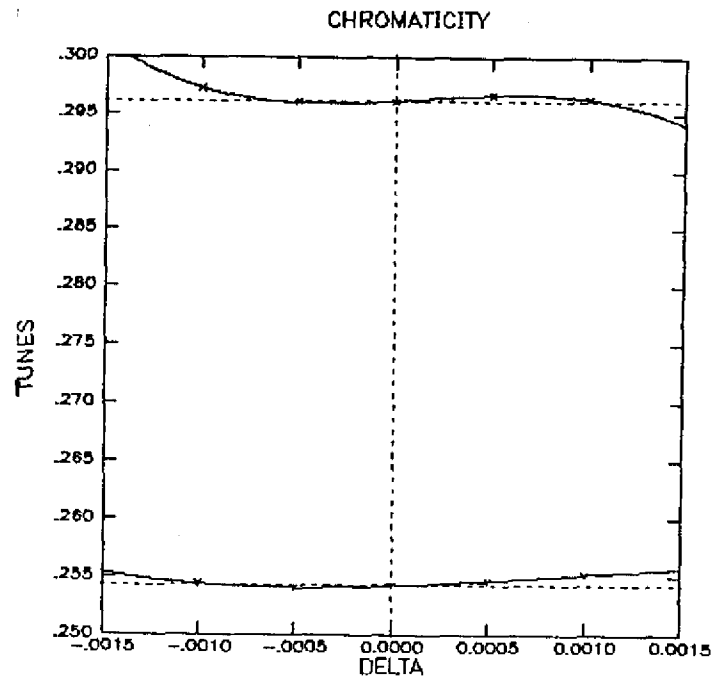


Figure 11. Tune dependences, full orbit errors, final situation after polynomial fitting method, two different seeds.



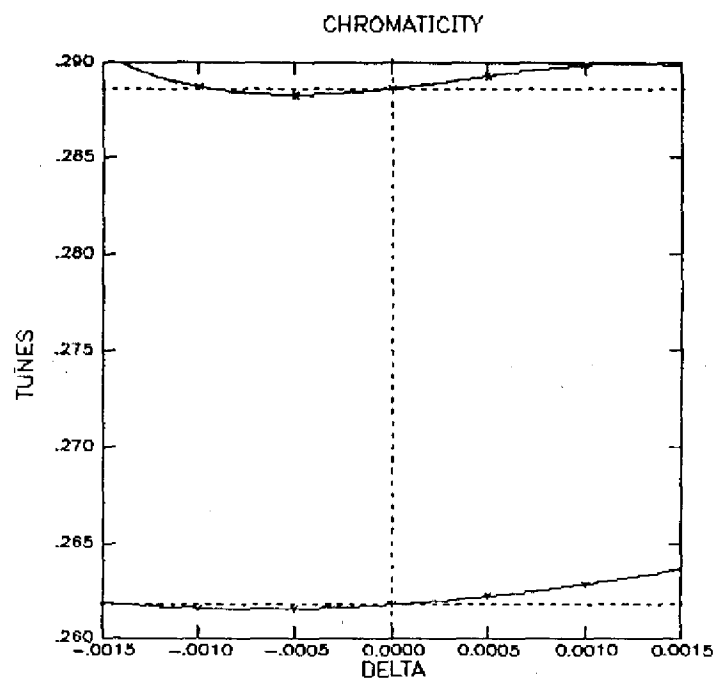
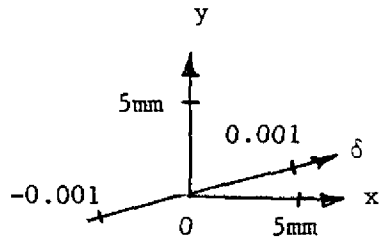


Figure 12. Tune dependence, full orbit errors, final situation after fitting using "Simpson's Rule" or "Neuffer" lumped compensation scheme, two different seeds.

Table 1.

TUNE DEPENDENCE AFTER OPERATIONAL COMPENSATION OF SYSTEMATIC DIPOLE ERRORS

Method: DISTRIBUTED (BORE TUBE) COILS



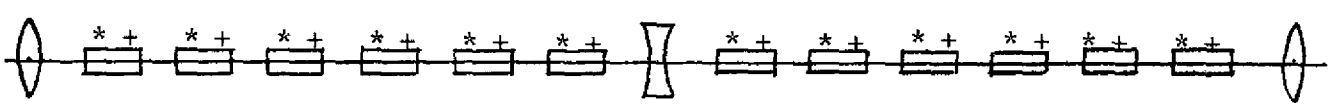
Random seed: 1 2

		0.4	0.1	.			1.0	0.1	.
	0.2	0.1	.			0.4	0.1	.	
0.4	0.4	.			0.4	0.6			
		0.4	0.1	.			0.0	0.1	.
	0.1	0.1	.			0.3	0.2	.	
0.4	0.5	.			0.1	0.6	.		

ΔQ_x ‰

		0.0	0.8	.			0.2	0.6	.
	0.1	0.6	.			0.2	0.4	.	
0.5	0.5	.			0.4	0.4	.		
		0.4	0.6	.			0.3	0.6	.
	0.4	0.4	.			0.4	0.4	.	
0.3	0.4	.			0.3	0.3	.		

ΔQ_y ‰



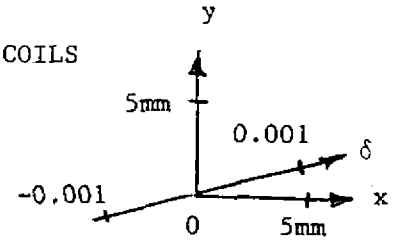
* : b_2 coil + : superimposed b_3 and b_4 coil

		Dipole Error
b_2	see text	$\times \frac{2\pi}{320\times12} \times {}^{4.2}_{-5.2} \times 10^{-4}$
b_3		$\frac{2\pi}{320\times12} \; {}^{0.6}_{0.0} \times 10^{-4}$
b_4		$\times \frac{2\pi}{320\times12} \times {}^{0.6}_{0.0} \times 10^{-4}$

Table 2.

TUNE DEPENDENCE AFTER OPERATIONAL COMPENSATION OF SYSTEMATIC DIPOLE ERRORS

Method: LUMPED (SIMPSON'S RULE ≡ NEUFFER) COILS



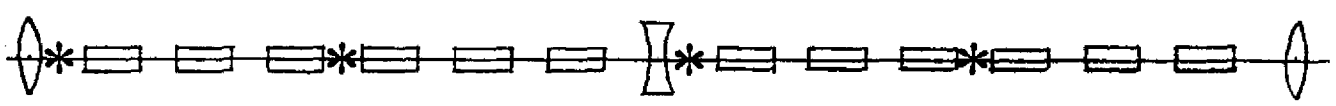
Random seed: 1 2

		0.2	0.5	.			0.5	0.5	.
	0.4	0.2	.			0.6	1.1	.	
1.9	1.3	.			3.1	2.5			
		0.3	0.6	.			0.4	1.3	.
	0.1	0.3	.			0.3	0.2	.	
0.4	0.6	.			0.0	1.3	.		

ΔQ_x $^o/_{oo}$

		0.5	0.0	.			0.6	0.1	.
	0.4	0.8	.			0.8	1.1	.	
0.4	0.8	.			1.6	1.0	.		
		0.4	0.0	.			0.5	0.0	.
	0.4	0.7	.			0.6	1.1	.	
0.2	0.6	.			0.4	1.0	.		

ΔQ_y $^o/_{oo}$



					Dipole Error
b_2	2	4	2	4	$\times \frac{2\pi}{320\times12} \times -4.7\times10^{-4}$
b_3	-0.0122	0.0156	-0.0122	0.0156	0.3×10^{-4}
b_4	2	4	2	4	$\times \frac{2\pi}{320\times12} \times 0.3\ \times10^{-4}$

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8. Private communication from A. Chao.
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10. Private conversation with A. Chao and P. Limon.
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