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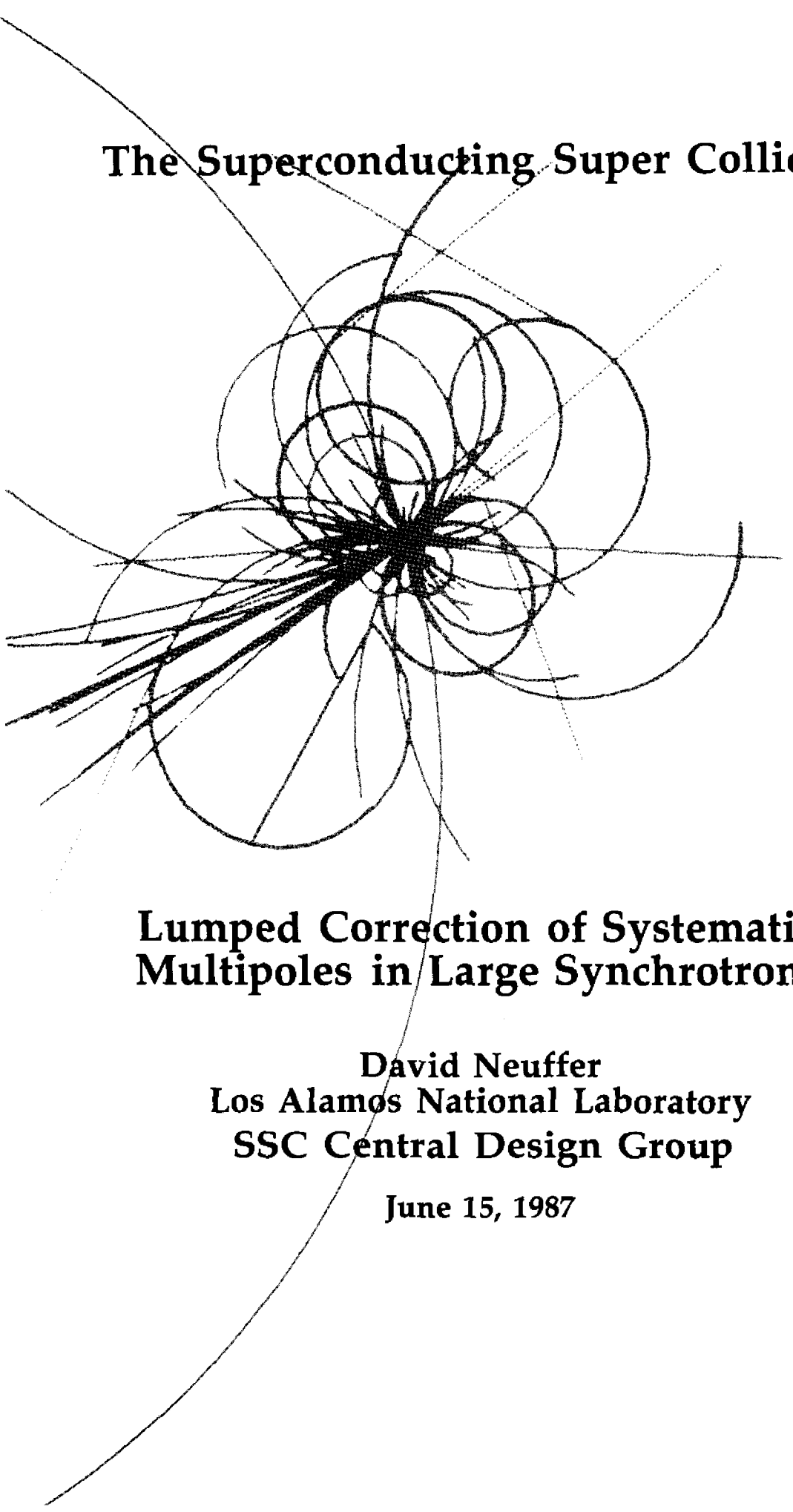
The Superconducting Super Collider



Lumped Correction of Systematic Multipoles in Large Synchrotrons

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LUMPED CORRECTION OF SYSTEMATIC MULTIPOLES IN LARGE SYNCHROTRONS

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Lumped Correction of Systematic Multipoles in Large Synchrotrons

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Abstract

Uncorrected systematic multipole components of the dipoles in synchrotrons can cause unacceptably large amplitude and momentum dependent tune shifts. The problem is exacerbated by the relatively weak focussing (long focussing cells) in large synchrotrons and by the relatively high multipole content of high-field superconducting magnets. Correction of these tune shifts by lumped correction elements is described. A particularly effective method of correcting first-order tune shifts is a "three-lump" system with an additional correction element placed at the center of each half-cell as well as correctors at the ends of the half-cells near the F and D quads, with the relative strengths per half-cell of the correctors following Simpson's Rule for three-point integration. The method is compared with end corrector and distributed correction methods and its limitations in higher order sextupole correction are considered. Application to the Superconducting Super Collider (SSC) is discussed.

Introduction

A large synchrotron such as the Superconducting Super Collider (SSC) requires an adequate linear aperture for reliable operation. Linear motion is required over a working region in amplitude and momentum space sufficient to include the beam size and momentum spread plus closed orbit deviations and injection errors. For the SSC, linearity tolerance guidelines have been

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set on amplitude and momentum dependent tune shifts of $\Delta\nu \leq \pm 0.005$ for $\Delta p/p \leq \pm 0.001$ for amplitudes A_x, A_y up to 0.5 cm in the SSC arcs.[1] The linearity tolerances severely restrict the allowable systematic multipole content of SSC dipoles. At current parameters and expected magnet properties, active corrections for the low order multipoles (sextupole, octupole, and decupole) are required.[2] The initial SSC Conceptual Design considered including trim coil multipoles within each dipole to provide the desired correction; however, this greatly complicates SSC dipole design and construction.

In this paper the possibility of using lumped multipole correctors installed within the lattice is considered. Initial SSC design includes spool pieces near each focusing and defocusing quadrupole. (see fig. 1) The spool pieces include dipole, quadrupole, and sextupole components, and they may be modified to include other components for systematic multipole correction. Additional corrector elements may also be inserted in the lattice. A particularly effective scenario places a correction spool piece at the center of each half cell in addition to elements near the quads, with the relative strengths of the three correctors given to first order by Simpson's Rule for three-point integration. (see fig. 2) This particular three-lump method corrects octupole and decupole systematic multipole tune shifts by a factor of ~ 100, as is discussed in detail below. The three-lump scheme also can correct systematic sextupole effects to an acceptable level at current SSC parameters. The three-lump scheme is compared with the two-lump end correctors scheme. The correction of high order effects is considered; second order effects set the limits on the correction of systematic sextupole effects.

Calculation of Tune Shifts

The transverse magnetic fields in the SSC dipoles may be represented by the complex expression:

$$B_y + iB_x = B_0 \left[1 + \sum (b_n(s) + ia_n(s))(x+iy)^n\right] \tag{1}$$

where $b_n(s), a_n(s)$ are the normal and skew multipole components of the dipoles. The transverse motion may be described by a Hamiltonian which includes the linear focussing:

$$H = \frac{I_x}{\beta_x(s)} + \frac{I_y}{\beta_y(s)} + \Re \sum_n \frac{B_0}{B\rho} \frac{(b_n(s) + ia_n(s))(x+iy)^{n+1}}{n+1} \tag{2}$$

where I_x, I_y are the action coordinates and $\beta_x(s), \beta_y(s)$ are the Courant-Snyder[3] betatron functions determined by the linear focussing. The coordinates x and y of particle motion are represented in action-angle variables by:

$$x = \sqrt{2\beta_x I_x}\cos(\phi_x) + \eta\delta$$

$$y = \sqrt{2\beta_y I_y}\cos(\phi_y)$$

where ϕ_x, ϕ_y are the angle variables (betatron phases) and the off-momentum orbit displacement at $\delta \equiv \Delta p/p$ determined by the dispersion function $\eta(s)$ is included. $A_x = \sqrt{2\beta_x I_x}$ and $A_y = \sqrt{2\beta_y I_y}$ are the amplitudes of the transverse orbits. The x and y perturbed tune shifts are obtained to first order by averaging the phase advance due to the magnetic field perturbation:

$$\Delta\nu_x = \frac{1}{2\pi}\int \frac{d\phi_x}{ds}\,ds = \Re\left\langle \frac{dH}{dI_x}\right\rangle \tag{3}$$

In first order in the coefficients b_n, a_n only systematic normal multipoles ($\overline{b_n}$) contribute and averaging over phases uses:

$$\overline{\cos^{2m}\phi} = \frac{(2m-1)!!}{2^m m!}$$

$$\overline{\cos^{2m+1}} = 0$$

In initial evaluation the optical complications due to special insertions may be ignored and the synchrotron ring may be approximated by a series of cells forming a complete ring. (This is an excellent first approximation in a large synchrotron.) The tune shifts may then be obtained from averaged values of powers of the betatron functions and the dispersion function, where the averaging may be evaluated in a single cell. The resulting expressions for tune shifts as a function of I_x, I_y and δ for sextupole, octupole, and decupole components are displayed in Table 1.

In first evaluation only horizontal motion may be considered and a tolerance on $\overline{b_n}$ may be obtained by requiring $\Delta\nu < 0.005$ for $A_x < 0.5$ cm, $\delta < 0.001$. The resulting maximum permissible uncorrected values of $\overline{b_n}$ for the dipoles in the SSC Conceptual Design lattice (192 m, 60° cells) and in the current SSC design lattice (230 m, 90° cells) are displayed in table 2.[4] Including vertical motion does not greatly modify the magnitude of these tolerances.

The SSC dipoles are expected to have systematic multipole content from a variety of causes. Due to finite conductor sizes, the conductors cannot be

designed in a perfect $\cos\Theta$ distribution, but will be placed in a geometry which necessarily contains higher order multipoles of non-vanishing magnitude for multipoles allowed in dipole symmetry (b_n with n even). Current design strategy is to split the conductor into a finite number of blocks and set the location of the blocks so as to minimize higher order multipoles. Errors in magnet assembly will introduce both allowed and forbidden harmonics into SSC dipoles. The errors are expected to be predominantly random, but will have significant systematic residues. At high field saturation of the iron surrounding the coils introduces significant allowed multipole components. Also significantly large allowed multipole components are induced by persistent currents induced in the finite-size superconducting filaments. The relative effect is greatest at injection field, where aperture and consequently field quality requirements are also greatest.

The tolerances may be compared with the expected multipole content of SSC dipoles, as shown in table 2. Tevatron construction experience provides a statistical sample of ~ 1000 accurately measured superconducting dipoles, from which the multipole content of similarly constructed magnets may be extrapolated. In table 2 we have included rms estimates for the random multipoles obtained from geometric construction errors, as extrapolated in reference 4 from Tevatron parameters. Systematic multipoles for allowed and unallowed harmonics may also be extrapolated from Tevatron data[5] and these are also shown in Table 2; they are not greatly smaller than the rms random magnitudes. The Tevatron design did not attempt to minimize allowed multipole strengths above decupole (b_4); these systematic multipole strengths are dominated by the coil design which will be changed for the SSC to minimize these harmonics.[6] The persistent current multipoles at injection have also been estimated;[7] Table 2 includes estimates for SSC dipoles with 5 μ filaments. Estimates on multipoles induced by iron saturation at highest fields are also included.[8]

Comparison of expected multipole strengths with the tolerances finds that the lower order multipoles will be significantly above tolerance and some correction will be required. Initial SSC design includes trim coils within each dipole for local correction of sextupole, octupole, and decupole (b_2, b_3, b_4) components.

An initially considered alternative places lumped correction elements near the focusing and defocusing elements in each cell. Since in lowest order in b_2 only two independent terms appear in the tune shift expressions, the lowest order sextupole tune shifts are completely correctable. Higher order sextupole terms (b_2^2) become important if b_2 is greater than 10^{-4}cm^{-2}; the

lumped correction method may then be inadequate (see below). For octupole and decupole effects there are five independent terms in lowest order and these cannot be corrected by a two-parameter end corrector system.

The strength of the end multipole correctors placed near the F and D quads can be varied to reduce the maximum tune shift within the design aperture. In Tables 3A and 3B some results for two-lump correction of octupole and decupole are displayed. Three different procedures for setting the corrector strengths are included: 1) The F and D corrector strengths are set equal to each other at half the integrated multipole strength. 2) The corrector strengths are set to cancel the zero-amplitude chromaticity due to the multipole, as reported in reference (9). 3) The corrector strengths are set by varying the strengths so as to minimize the design aperture tune shifts. Empirically, the attempted correction of octupole and decupole is inadequate for the SSC in all of these scenarios. Two-parameter correction of higher order multipoles will be even more inadequate.

Three-Lump Correction of Systematic Multipoles

In an attempt to obtain an acceptable correction system, a three-lump system is considered with correctors placed at the center (C) of each half-cell and at the half-cell ends near the focusing and defocusing quads (F and D quads), as shown in figure 2. In initial evaluation the correctors may be taken as of vanishingly small length with integrated strengths given by

$$S_{n,i} \equiv B_{n,i} l_i = -f_i B_0 \overline{b_n} L \tag{4}$$

where L is the total dipole length per half cell, B_0 is the dipole field and $\overline{b_n}$ is the systematic multipole component of the dipole field, l_i is the corrector length and $B_{n,i}$ is the corrector field strength. The index $i = F, C, D$ identifies the three independent correctors. f_i sets the strengths of the correctors per half cell in units of the error fields; the sum $\sum f_i$ should be near unity.

In an initial evaluation the small but finite lengths of the spacings between dipoles and the finite-length effects of non-dipole elements can be ignored. Non-dipole elements are then represented as zero-length "kicks" and the dipole field with its multipole content is continuous. This is an excellent first approximation in large synchrotrons. The amplitude and momentum dependent tune shifts from each multipole may be evaluated using the expressions in Table 1, including both dipole and corrector contributions. For example, the first-order momentum-dependent horizontal tune

shift due to sextupole components is given by:

$$\begin{aligned}\frac{\Delta\nu_x}{\delta} &= \frac{1}{L}\int_0^L b_n\beta_x(s)\eta(s)ds \\ &+\frac{S_{2,F}}{L}\beta_x(0)\eta(0)+\frac{S_{2,C}}{L}\beta_x(L/2)\eta(L/2)+\frac{S_{2,D}}{L}\beta_x(L)\eta(L) \quad (5)\end{aligned}$$

In principle, the three parameters f_i can be independently varied to obtain a minimum tune spread over the aperture. For octupole and higher multipoles the three parameters are insufficient to set all the first-order tune shifts to zero. However an empirical optimization quickly converges close to a characteristic solution and this solution reduces the remanent tune shifts by two orders of magnitude from the uncorrected case.[10]

The characteristic solution is:

$$f_D = f_F = \frac{1}{6}, f_C = \frac{4}{6} \quad (6)$$

This is recognizable as the proportions in Simpson's Rule for three-point integration:[11]

$$\int_0^L f(x)dx = \frac{L}{6}(f(0)+4f(L/2)+f(L)) - \frac{(L/2)^5}{90}f^{(4)}(\zeta) \quad (7)$$

The last term is the error term and an upper bound on the error in integration can be obtained by replacing the fourth derivative at the unknown point ζ by its maximum through the interval. Simpson's Rule is equivalent to approximating the function by a third order polynomial, and the accuracy of correction by the characteristic solution is simply a measure of the accuracy to which the various expressions in the tune shift expressions are approximated by a third order polynomial in half cells and therefore integrable by Simpson's rule. It can be seen by inspection of equation 5 that the lumped correction is equivalent to approximating an integral over the half-cell by a weighted sum of the function evaluated at particular points; Simpson's Rule is an accurate three-point approximation to obtain the integral of any smooth function.

In a large synchrotron the betatron functions $(\beta_x, \beta_y, \eta)$are smoothly varying functions over the half cell, but have a large, effectively discontinuous, change in their derivatives at the relatively short quads. (see fig. 3) The tune shift expressions are simple powers of these functions and are therefore also smoothly varying on the half cell and the terms through decupole order

are very accurately approximated by third order polynomials and their integrals are correspondingly accurately evaluated by Simpson's Rule. Figures 4 and 5 show the terms which appear in octupole and decupole order. The discontinuity at the half cell ends also provides a qualitative reason why a correction system located only on the half-cell ends is ineffective.

The initial basis for correction, therefore, is now Simpson's Rule with the corrector strengths set in terms of the systematic multipole fields by equations (4) and (6). The resulting correction of octupole and decupole tune shifts in the thin lens version of the SSC lattice is displayed in tables 3A and 3B. Octupole tune shifts are reduced by a factor of 100 while decupole tune shifts are reduced by a factor of 30.

Superior correction is obtainable by setting the relative corrector strengths in the ratio set by Simpson's Rule ($\frac{1}{6} : \frac{4}{6} : \frac{1}{6}$), but varying the total strength so as to minimize the chromatic terms in the tune shift parameters. This procedure simulates the actual correction process in a physical SSC where the multipole content is not precisely known, but is inferred from the chromaticity measurements. Varying the corrector strength power supply to minimize measured chromaticity is a realistic operating procedure. The octupole corrector strength obtained is $\sum f_i = 0.99$ and the resulting tune spread is significantly reduced from the initial case. For decupole, third order chromaticity is minimized at $\sum f_i = 0.975$ with corresponding reductions in tune spread (see Table 3B). For decupole effects, the tune shift functions are more skewed toward the focussing quad end of the half cell, and the correction is further improved by slightly reducing the relative strength of the F correctors $(f_F : f_C : f_D) = (.16 : .67 : .17)$; see Table 3B.

As shown in Table 3, correction of tune spreads from octupole and decupole systematic multipoles by a factor of 100 by the three-lump system is easily obtainable, and correction by a factor of a thousand is possible by some tuning of the corrector strengths. For correction by greater than a factor of 100, the thin lens, cells-only lattice evaluations are inadequate, and second-order effects should also be considered. However correction by a factor of 100 is more than adequate for the SSC. Tolerances for systematic decupole and octupole components are increased from the 10^{-6}–10^{-5} to the 10^{-4}–$10^{-3}\mathrm{cm}^{-n}$ level, and are significantly greater than the expected multipole content of SSC magnets.

Sextupole Correction and Second-Order Effects

Superconducting $\cos\Theta$ dipole magnets, such as those in the SSC, have

very large sextupole components. The persistent-current induced sextupole at injection field is particularly large ($b_2 \sim 5 \times 10^{-4}$ cm^{-2} for the SSC). As discussed above, there are only two independent tune shift terms in first-order for sextupoles (b_2), and these can be completely corrected with F and D correctors only. A three-lump, Simpson's Rule based, correction system can also easily correct these terms. However, at SSC parameters second-order tune shifts are nonnegligible and set a linearity limitation which is somewhat more difficult to correct. The limitations on sextupole correction for two-lump and three-lump systems due to second order effects are discussed in this section.

The second-order terms are "octupole-like", producing five independent amplitude and momentum dependent terms. These terms may be identified using canonical perturbation theory.[12,13] In this section some results of perturbation theory are used to identify the characteristics of the second-order terms. The sextupole perturbation potential in equation (2) may be rewritten as

$$\begin{aligned} V &= \frac{B_2(s)}{12}(2I_x\beta_x)^{3/2}[\cos(3\phi_x) + 3\cos(\phi_x)] \\ &\quad -\frac{B_2(s)}{2}I_y\beta_y\sqrt{2I_x\beta_x}[2\cos(\phi_x) + \cos(2\phi_y + \phi_x) + \cos(2\phi_y - \phi_x)] \\ &\quad +B_2(s)\eta\delta I_x\beta_x[1 + \cos(2\phi_x)] - B_2(s)\eta\delta I_y\beta_y[1 + \cos(2\phi_y)] \end{aligned} \tag{8}$$

I_x, ϕ_x and I_y, ϕ_y are the action-angle variables. $B_2(s)$ gives the sextupole strength around the ring including the correctors; in the dipoles $B_2 = B_0 b_2/B\rho$. In this expression only terms which contribute to the tune shift to second order in δ and A_i are retained. In perturbation theory, a canonical transformation is performed using a generating function $G(I_i, \phi_i, s)$ which moves the dependence on the angles ϕ_x, ϕ_y to higher order, leaving a Hamiltonian only dependent on new action variables (J_x, J_y). The generating function may be expressed as a Fourier series in the various harmonics of the angle variables that appear in the Hamiltonian ($\phi_x, 3\phi_x, 2\phi_y + \phi_x, 2\phi_y - \phi_x, 2\phi_x, 2\phi_y$ for sextupoles). The new second-order Hamiltonian may be written as

$$H = H_0 + \Delta H \equiv H_0 + \langle \frac{\partial G}{\partial \phi_x}\frac{\partial V}{\partial J_x} + \frac{\partial G}{\partial \phi_y}\frac{\partial V}{\partial J_y} \rangle \tag{9}$$

The tune shift is then found by taking the derivative of the perturbation hamiltonian ΔH with respect to the appropriate action term and integrating about a full turn; terms from each harmonic are obtained. In this paper the

complete perturbation expression is not included, the references[12,13] may be consulted to obtain complete expressions. We explicitly display the expression for the horizontal amplitude dependent tune shift

$$\Delta\nu_x = -\frac{CJ_x}{32\pi L}\int_0^{2L} ds\,\beta_x(s)^{3/2}B_2(s)\int_s^{s+2L} ds'\,\beta_x(s')^{3/2}B_2(s')$$
$$\times\,[3\frac{\cos(\psi_x(s')-\psi_x(s)-\pi\nu_x)}{\sin(\pi\nu_x)}+\frac{\cos 3(\psi_x(s')-\psi_x(s)-\pi\nu_x)}{\sin(3\pi\nu_x)}] \qquad (10)$$

In this expression, C is the ring circumference, L is the cell half-length, $\psi_x(s)$ is the betatron phase and ν_x is the cell tune. $\Delta\nu_x$ is the full ring tune shift. The periodicity of all functions with cell length in the simplified lattice has been explicitly included by truncating the integrals at the cell length. The complete tune shift expressions can be found by including all harmonics; ΔH is proportional to b_2^2 and contains terms proportional to $J_x^2, J_xJ_y, J_y^2, J_y\delta^2$, and$J_x\delta^2$, similar to octupole terms. The coefficients of the terms are obtained by double integrations over lattice functions and phases similar to those of the horizontal-amplitude case displayed above.

In initial discussion only two of the tune shift terms are evaluated, the on-momentum horizontal-amplitude and vertical-amplitude dependent tune shifts, which are the largest second order terms in the current SSC design lattice and are easily separated from the other terms in numerical evaluation. The amplitude dependent tune shifts are calculated numerically by tracking one-dimensional particle orbits through a series of synchrotron cells. In setting linear aperture criteria the on-momentum tune shifts at the extended amplitudes $A_x = 0.007$ cm and $A_y = 0.007$ cm are considered, with $\Delta\nu < 0.005$ required. (The reduced aperture $A = 0.005$ at full momentum spread $\delta = \pm 0.001$ does not give stricter multipole requirements in the current SSC design lattice.)

In Table 4, some calculations of second order tune shifts for various correcting schemes are displayed. A systematic multipole strength of $\overline{b_2} = 6.0 \times 10^{-4}$ was used as a reference strength in the calculations; this is approximately the acceptability limit after correction in the SSC. The results may be scaled to other strengths by following the b_2^2 dependence. First, second-order tune shifts without any correction, including no chromaticity correction of $\overline{b_2}$, are shown. Under the SSC aperture criteria, the uncorrected second order tune shift would limit acceptable b_2 to the 10^{-4} cm^{-2} level.

In actual operation, the SSC will have sextupoles near the F and D quads and their values will be varied to correct the chromaticity, including

the chromaticity due to $\overline{b_2}$. This provides a "two-lump" correction system for $\overline{b_2}$ in the present notation. This correction system also corrects the second-order tune shifts by an order of magnitude, implying a $\overline{b_2}$ tolerance of $\sim 3 \times 10^{-4}$. Adding a center corrector to obtain "three-lump" correction can reduce tune shifts by another factor of four, increasing the tolerance to $\sim 6 \times 10^{-4}$. Thus the improved correction is not as dramatic as in the first-order calculation and is simply the degree of enhanced correction of an arbitrary function obtainable by doubling the fitting points.

The reasons why the correction is not much more effective can be surmised by considering the analytical expressions for the second order tune shifts, such as equation (10). These expressions are double-integrals. The internal integral has a discontinuity at $s = s'$ along with a localizing phase factor and this discontinuity and phase factor affects the integration more than the discontinuities in the derivatives of the betatron function at the half-cell ends, which biases the first-order tune shift toward Simpson's Rule integration. The enhanced center corrector strength is in fact a liability in this integration. The optimum center corrector strength is in fact an "equal-weight" strength rule which, in the present notation of relative strengths of the correctors per half cell is $\sim$ (1:2:1) rather than (1:4:1). Table 4 displays the degree of second-order correction obtained with the center corrector strength based on Simpson's Rule and on the equal-weights rule. The equal-weights rule obtains slightly smaller tune shifts. The smallness of the improvement indicates that the relative strength of the center corrector does not need to be fine tuned to obtain the desired correction.

While the improvement in $\overline{b_2}$ tolerance by a factor of two from 3 to 6 units is not dramatic, it may be of considerable practical importance in the SSC, where the expected persistent-current induced sextupole component is 4–5 units. The internal trim coils in the dipoles for systematic sextupole correction may then be replaced by the present three-lump scheme with the additional correcting element at the half-cell center in addition to those near the quads.

Comments on Application to the SSC and Other Accelerators

The currently planned SSC cell differs slightly from the symmetric cell shown in figure 2 (see fig. 1). Rather than having separate correction elements symmetrically placed on both sides of the quadrupoles, a single correction spool piece is placed on one side of the quads. In implementation

of the present three-lump scheme, the correctors on opposite sides of the quads would be lumped within that spool piece into a single element at double the separate strengths. The SSC half cell contains 6 dipoles, and a slot for the center corrector would be placed between dipoles at the center of these assemblies. Because of the asymmetric placement of the end spool piece, the center slot will not be precisely in the center of the half-cell. However, the reflection symmetry of the betatron functions about the quads ensures that the correction retains the basic symmetry necessary for third-order accuracy, when averaged over the full cell. The relative strengths of the correctors should be slightly adjusted for the actual geometry.[14] However, the relatively small asymmetry planned for the SSC will not greatly change these values from the Simpson's Rule values and will not greatly change the accuracy of correction obtained.

The actual strengths of the multipole correctors depend on the actual amount of multipole strength in the SSC dipoles to be corrected, and that is currently not precisely known. The strengths may be calculated using equation (4). To correct $\overline{b_2} = 2 \times 10^{-4}$ cm^{-2} (2 units) at 6.6 T (20 TeV) using equal-weights requires a center corrector with an integrated strength of $S_{2,C} = 660$ T/m. Correction of 6 units at injection energy (1 TeV) requires only $S_{2,C} = 100$ T/m. Correction of 0.5 units (10^{-4} at 1 cm) of octupole and decupole at 20 TeV using Simpson's Rule requires $S_{3,C} = 2.2 \times 10^4$T/m^2 and $S_{4,C} = 2.2 \times 10^6$ T/m^3, respectively. Strength requirements at injection field for similar correction are a factor of 20 smaller. These strengths can be obtained in relatively short correction units ($l_C <$ a few m).

At current SSC parameters, the lumped correction scheme presented in this paper with the additional half-cell center spool piece provides adequate correction for systematic sextupole, octupole, and decupole components in the dipoles and is a possible replacement for the initially planned internal trim coils. Dipole design and construction are greatly simplified by eliminating the internal trim coils, possibly with substantial cost savings. The following comments may be added:

- Octupole and decupole tune shifts are correctable by two orders of magnitude to tolerances on $\overline{b_3}$ and $\overline{b_4}$ of several units (10^{-4} at 1 cm). This provides substantial safety margin over expected multipole strengths.

- Sextupole effects including second-order are correctable to the level of 6 units. This is above the expected sextupole strengths of 4–5 units but does not provide a large margin of safety.

- For octupole and higher order correction, only one power supply per multipole is required, with its strength distributed to the elements following Simpson's Rule.

- The correction system may be modified to include more elements (b_5, b_6, b_8, etc.) by including multipoles of the appropriate order within the corrector spool pieces, should their correction be necessary.

- Since Simpson's Rule is accurate for all smoothly varying functions, the correction scheme does not require that the betatron functions be near design values to be accurate. It only requires that they be smoothly varying on the half-cell level. The multipole-induced tune shifts are locally corrected even with very large deviations of betatron functions from design values. In particular, the multipole-induced tune shifts due to the dipoles in the dispersion suppressors can be corrected by continuing the Simpson's Rule correction scheme in the dispersion suppressor cells, even though the betatron functions are greatly different from those in the normal cells.

While the detailed calculations have been specific to the SSC, the same correction procedure will apply to any similar accelerator; that is, any large synchrotron with relatively large multipole content. Within currently planned machines, these would include the 3 TeV UNK machine planned for Serpukhov[15] and the 10 TeV LHC collider planned for CERN[16]. The Simpson's Rule correction procedure will be particularly applicable to the LHC which will have a relatively low injection energy ($\sim$ 0.5 TeV), will use $\cos\Theta$ dipoles similar to the SSC, and will consequently have large persistent-current induced multipoles at injection where aperture requirements are greatest.

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References

1. SSC Conceptual Design Report, SSC Central Design Group Report, SSC-SR-2020, March 1986.

2. A. Chao and M. Tigner, SSC Central Design Group Report SSC-N-182, May 27, 1986.

3. E. D. Courant and H. L. Snyder, Ann. Phys. 3,1 (1958).

4. D. V. Neuffer and J. M. Peterson, SSC Central Design Group Report SSC-113, presented at the 1987 Particle Accelerator Conference, Washington, D.C., March, 1987.

5. R. Hanft, B. C. Brown, W. E. Cooper, D. A. Gross, L. Michelotti, E. E. Schmidt, F. Turkot, I.E.E.E. Trans. NS-30, p. 3381 (1983).

6. S. Caspi, W. Gilbert, M. Helms, L. J. Laslet and C. Taylor, SSC-N-245, presented at the 1986 Applied Superconductivity Conference, Baltimore, MD., October 1986.

7. J. M. Peterson, editor, SSC-7, April 1985.

8. SSC Conceptual Design Report, SSC-SR-2020, ibid., p. 275.

9. A. Jackson, SSC-107, February 1987.

10. D. Neuffer, SSC-N-339, May 19, 1987.

11. M. Abramowitz and I. A. Stegun, Handbook of Mathematical Functions, National Bureau of Standards, Washington, D.C. (1972).

12. R. D. Ruth, A.I.P. Conf. Proc. 153, American Inst. of Physics, New York, p. 152 (1987).

13. S. Ohnuma, A.I.P. Conf. Proc. 123, American Inst. of Physics, New York, p. 415 (1984).

14. D. Neuffer, SSC-N-359, June 1987.

15. V. I. Balbekov, et. al., Proc. 12th Int. Conf. on High Energy Accelerators, Fermilab (1983), p. 40.

16. Large Hadron Collider in the LEP Tunnel, M. Jacob, ed., CERN Report CERN 84-10 (1984).

Table 1

Expressions for first-order tune shifts due to sextupole, octupole, and decupole components of the dipoles (b_2, b_3, b_4).

Sextupole

$$\Delta\nu_x = \langle b_2\beta_x\eta\delta\rangle$$

$$\Delta\nu_y = \langle b_2\beta_y\eta\delta\rangle$$

Octupole

$$\Delta\nu_x = \langle \tfrac{3}{4}b_3\beta_x^2 I_x - \tfrac{3}{2}b_3\beta_x\beta_y I_y + \tfrac{3}{2}b_3\beta_x\eta^2\delta^2\rangle$$

$$\Delta\nu_y = \langle \tfrac{3}{4}b_3\beta_y^2 I_y - \tfrac{3}{2}b_3\beta_x\beta_y I_x - \tfrac{3}{2}b_3\beta_y\eta^2\delta^2\rangle$$

Decupole

$$\Delta\nu_x = \langle 3b_4\beta_x^2\eta I_x\delta - 6b_4\beta_x\beta_y\eta I_y\delta + 2b_4\beta_x\eta^3\delta^3\rangle$$

$$\Delta\nu_y = \langle 3b_4\beta_y^2\eta I_y\delta - 6b_4\beta_x\beta_y\eta I_x\delta - 2b_4\beta_y\eta^3\delta^3\rangle$$

Table 2

Tolerances and estimated strengths of systematic multipole context in the SSC dipoles. All multipole strengths are in units of $10^{-4} cm^{-n}$. The tolerances are obtained from the linearity criteria. Estimated strengths are extrapolated from Tevatron data on calculated from the magnet properties.

Multipole	Tolerance (Conceptual Design)	Tolerance (Current Lattice)	Estimated Random Error (Tevatron)	Estimated Systematic Strength (Tevatron)	Estimated Persistent Current Multipole Strength	Estimated Saturation Multipole Strength
b_2	0.0072	0.0097	2.0	0.45	-4.7	1.2
b_3	0.011	0.017	0.35	-0.14	--	--
b_4	0.016	0.031	0.60	-0.33	0.30	-0.05
b_5	0.024	0.054	0.06	-0.024	--	--
b_6	0.035	0.096	0.08	1.57+	0.07	-0.01
b_7	0.051	0.17	0.16	0.009	--	--
b_8	0.074	0.29	0.02	-2.1+	<0.02	0.02

+ Tevatron magnet design; to be reduced in SSC magnet design.

Table 3A

Correction of Octupole by Lumped Correctors

The current SSC lattice ($L = 115$ m, $\phi = 90°$) and $b_3 = 10^{-4} cm^{-3}$ are used in the tune shift calculations. Maximum tune shift is evaluated within the desired aperture (A_x, $A_y < 0.5$ cm, $|\delta| < 0.001$). Tolerance is obtained by varying b_3 to obtain $\Delta\nu \lesssim 0.005$, and the correction factor is obtained by comparison with the uncorrected case.

Correction Conditions	$\Delta\nu_{max}$ ($b_3 = 10^{-4} cm^{-3}$)	Correction Factor	b_3 Tolerance ($10^{-4} cm^{-3}$)
No Correction	0.28	1.0	0.018
Two-Lump, equal-weight ($f_F = 0.5$, $f_D = 0.5$)	0.195	1.44	0.026
Two-Lump, minimum chromaticity ($f_F = 0.28$, $f_D = 0.70$)	0.15	1.87	0.033
Two-Lump, minimum $\Delta\nu_{max}$ ($f_F = 0.35$, $f_D = 0.40$)	0.072	3.9	0.069
Three-Lump, Simpson's Rule (f_F, f_C, f_D) = ($\frac{1}{6}, \frac{4}{6}, \frac{1}{6}$)	0.0030	93	1.6
Three-Lump, strength optimized ($f_F, f_C, f_D = 0.99(\frac{1}{6}, \frac{4}{6}, \frac{1}{6})$.	0.00075	370	6.7

Table 3B

Correction of Decupole by Lumped Correction

Correction Conditions	$\Delta\nu_{max}$ ($b_4 = 10^{-4} cm^{-4}$)	Correction Factor	b_4 Tolerance ($10^{-4} cm^{-4}$)
No Correction	0.172	1.0	0.029
Two-Lump, equal-weight ($f_F = 0.5$, $f_D = 0.5$)	0.172	1.0	0.029
Two-Lump, minimum chromaticity ($f_F = 0.24$, $f_D = 0.93$)	0.126	1.4	0.04
Two-Lump, minimum $\Delta\nu_{max}$ ($f_F = 0.29$, $f_D = 0.51$)	0.045	3.8	0.11
Three-Lump, Simpson's Rule (f_F, f_C, f_D) = ($\frac{1}{6}, \frac{4}{6}, \frac{1}{6}$)	0.0056	31	0.9
Three-Lump, strength varied, Simpson's Rule (f_F, f_C, f_D) = $0.975(\frac{1}{6}, \frac{4}{6}, \frac{1}{6})$.	0.0018	95	2.8
Three-Lump, fine-tuned (f_F, f_C, f_D) = 0.99(0.16, 0.67, 0.17)	0.00021	800	24.0

Table 4

Correction of Second Order Sextupole Tune Shifts by Lumped Correction

Correction Conditions	$\Delta\nu_x$ ($A_x = 0.7$cm, $\delta = 0$ $A_y = 0$, $b_2 = 6 \cdot 10^{-4} cm^{-2}$)	$\Delta\nu_y$ ($A_y = 0.7$ cm $A_x = 0$, $\delta = 0$, $b_2 = 6 \cdot 10^{-4} cm^{-2}$)	$\langle b_2 \rangle$ tolerance (from second-order δ=0 tune shift only)($10^{-4} cm^{-2}$)
No correction of b_2 (no chromaticity correction)	0.122	-0.117	1.2
End Correctors only, chromaticity corrected ($f_F = 0.339$, $f_D = 0.528$)	0.0144	0.0237	2.8
Three-Lump Correction, Simpson's Rule ($f_F = 0.166$, $f_C = 0.667$, $f_D = 0.165$)	0.0052	0.0030	5.9
Three-Lump Correction equal weights ($f_F = 0.209$, $f_C = 0.50$, $f_D = 0.257$)	0.00401	0.00402	6.7

Figures

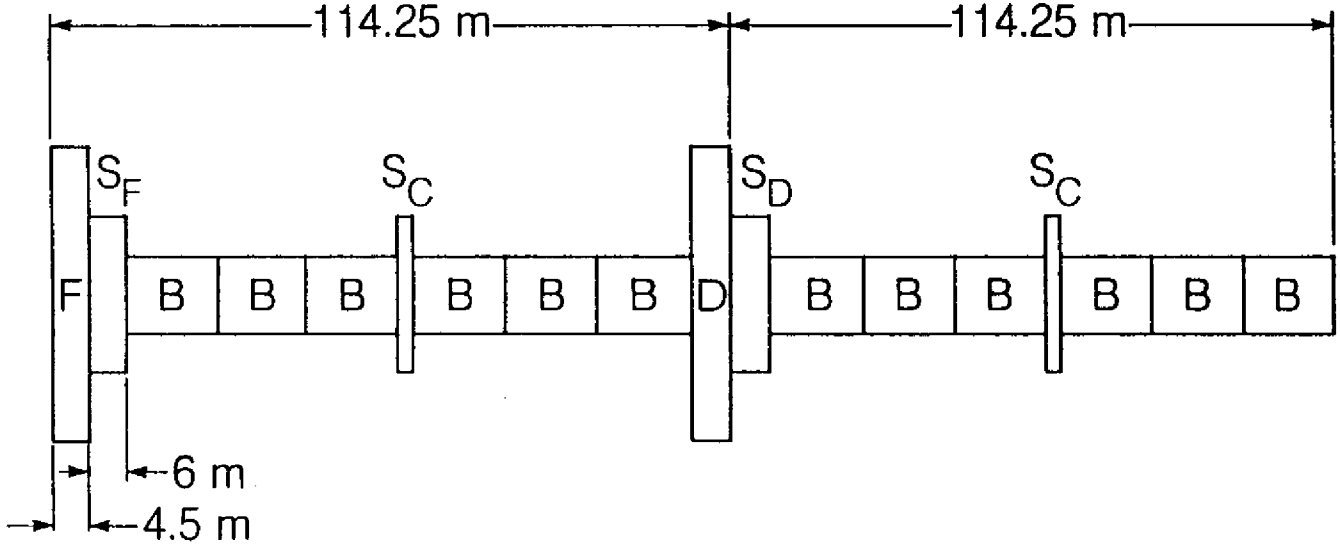
Figure 1: An SSC cell (actual layout, but with added half-cell center correctors)

Figure 2: An SSC cell – symmetrized version to illustrate the correction method with corrector slots split on opposite sides of quads and half-cell center correction slots.

Figure 3: Betatron functions (β_x, β_y, η) for a full SSC cell. The functions which appear in the sextupole induced tune shifts $(\beta_x\eta, \beta_y\eta)$ are also shown. Note the derivative discontinuity and the reflection symmetry at the center quad.

Figure 4: Octupole-induced tune shift functions $(\beta_x^2, 2\beta_x\beta_y, \beta_x^2, \beta_x\eta^2, \beta_y\eta^2)$ on a half cell. (These functions are reflection symmetric about the quadrupoles.)

Figure 5: Decupole-induced tune shift functions $(\beta_x^2\eta, 2\beta_x\beta_y\eta, \beta_x^2\eta, \beta_x\eta^3, \beta_y\eta^3)$ on a half cell. (These functions are reflection symmetric about the quadrupoles.)



B – dipole
F,D – quadrupoles
S_F, S_D – spool pieces
S_C – center corrector

SSC Cell
(Actual layout with added center corrector)

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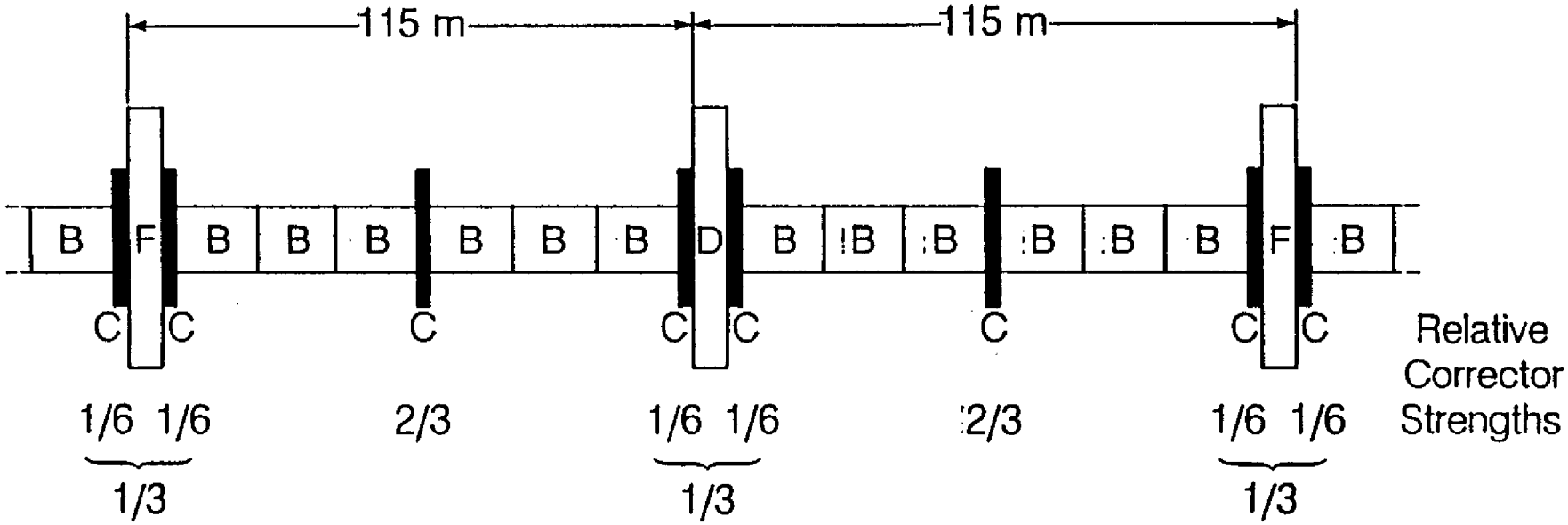
Fig. 1

3-Lump Correction for SSC Systematic Multipoles



B – Dipoles (16 m SSC dipoles)

F,D – Quadrupoles

C – Slots for correctors (correctors on opposite sides of quads can be combined in units on either side)

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Fig. 2

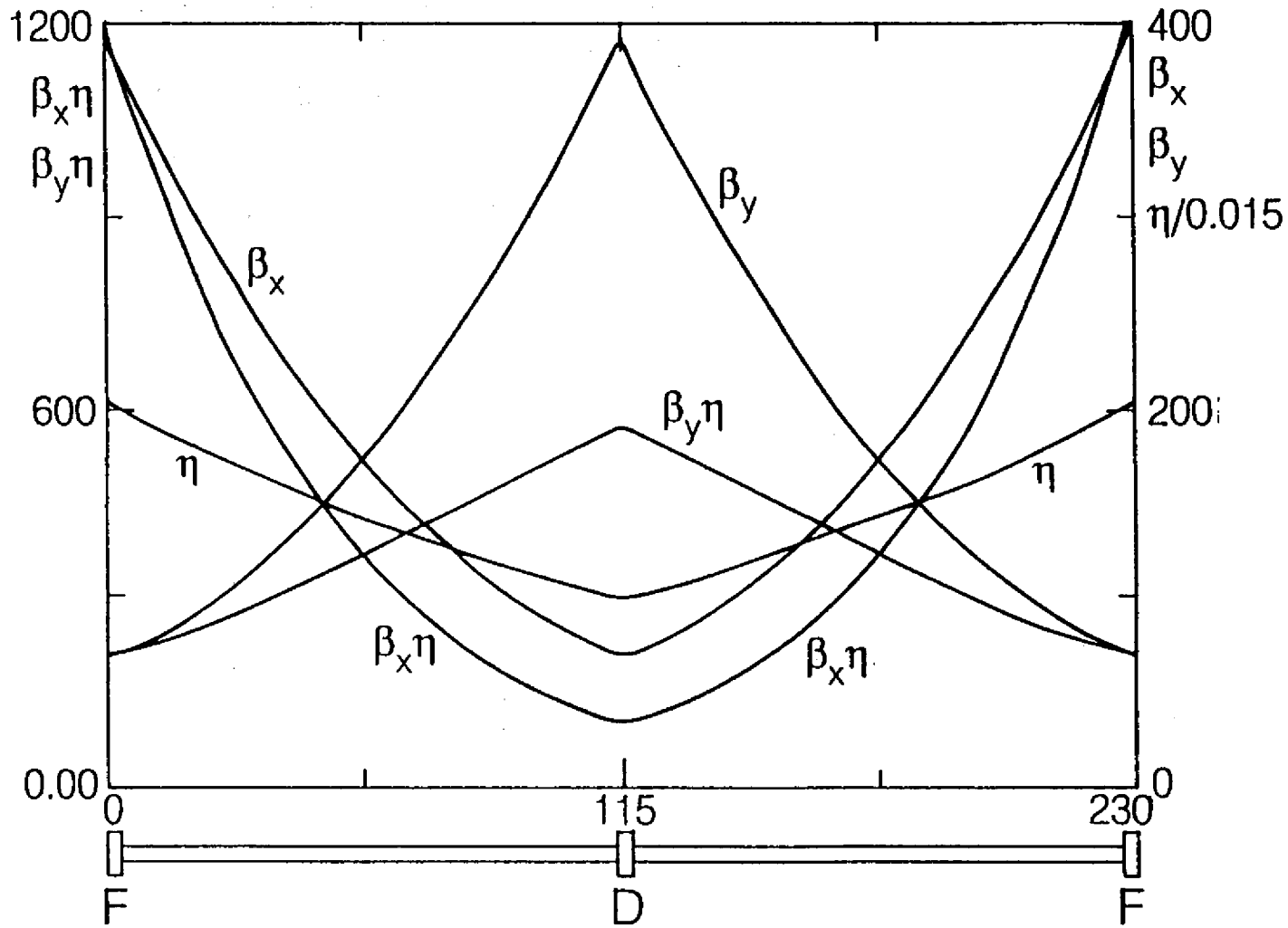

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Fig. 3

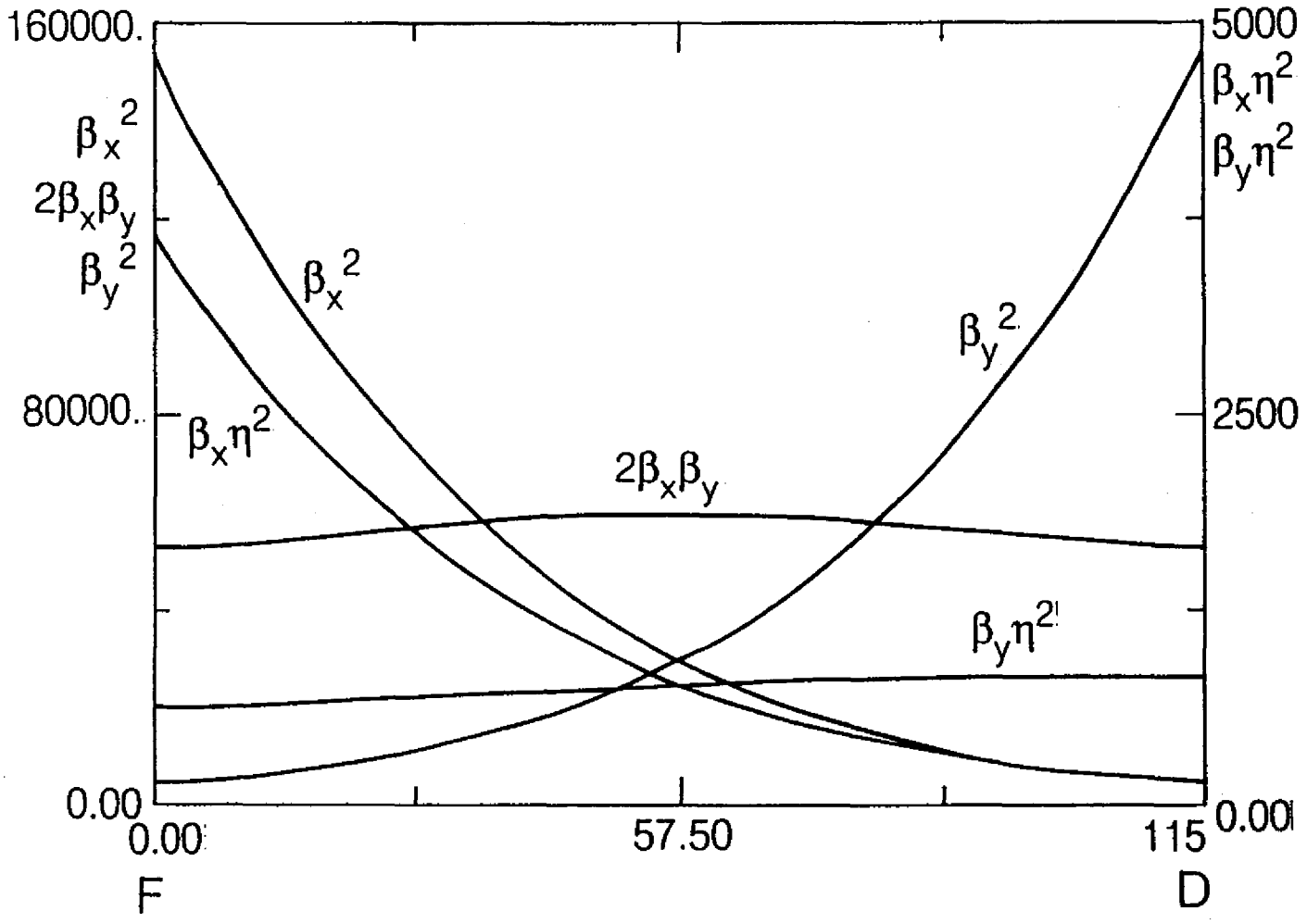

Fig. 4

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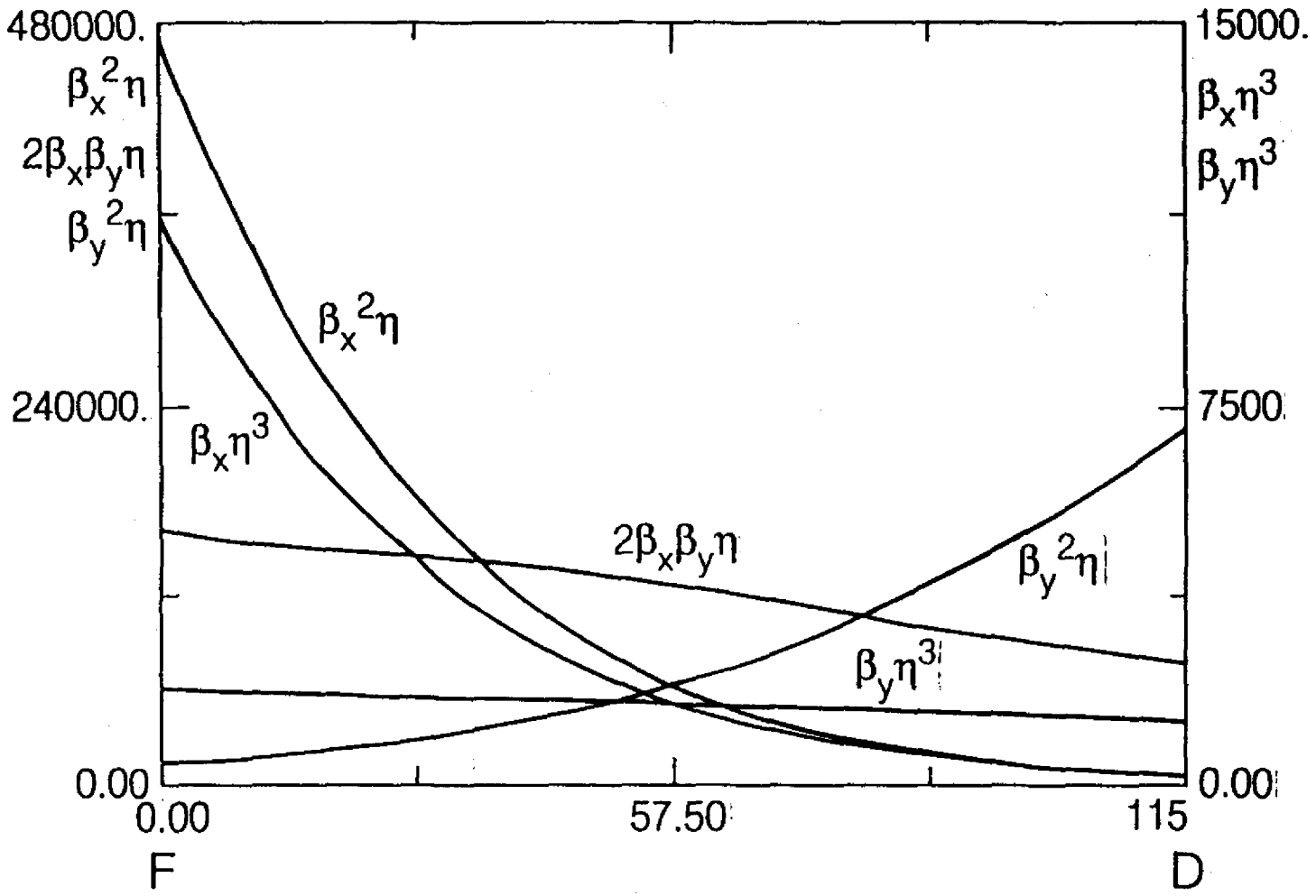

Fig. 5

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