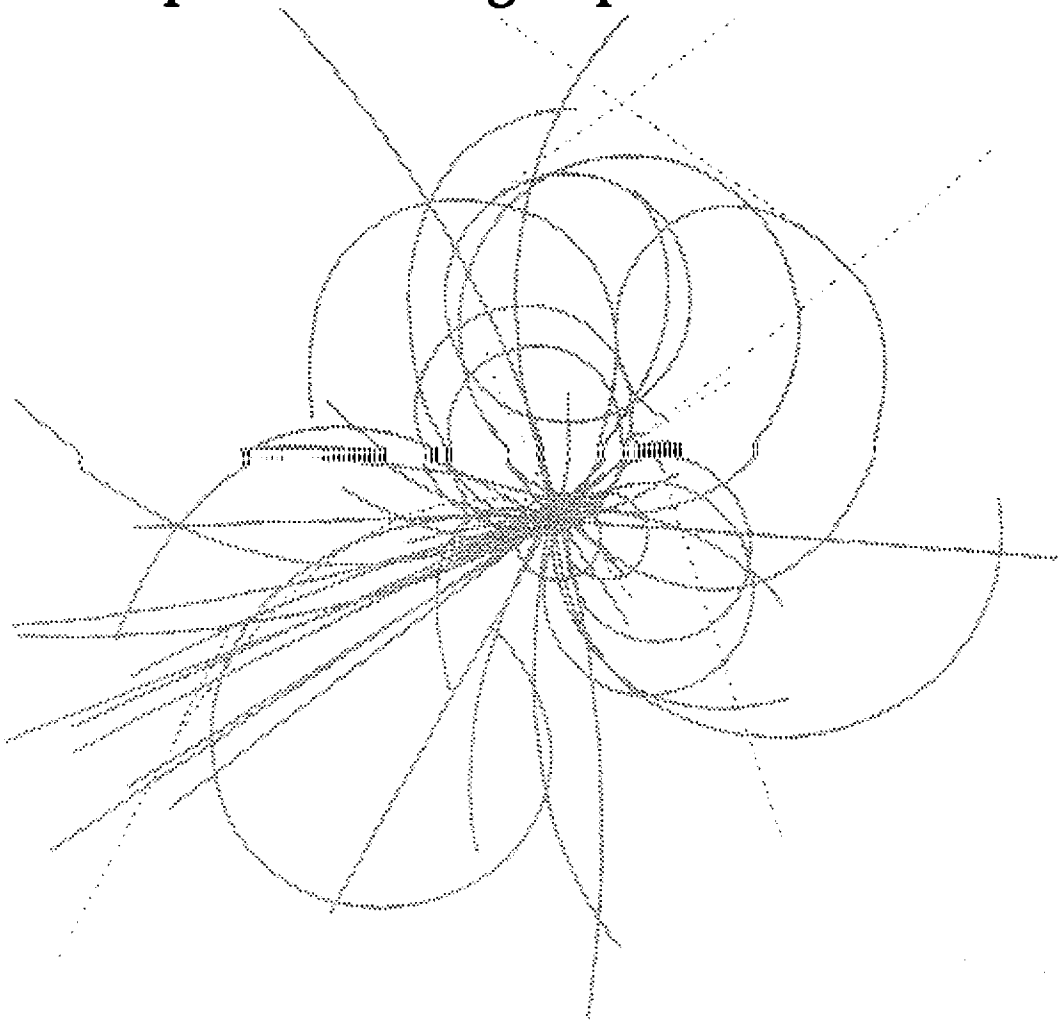
SSC-119

SSC-119

Superconducting Super Collider Laboratory



A Very Large β^* Interaction Region for the SSC

D. Groom, A. Garren, and D. Johnson
SSC Central Design Group

March 1987

SSC-119

A VERY LARGE β* INTERACTION REGION FOR THE SSC*

D.E. Groom, A.A. Garren and D.E. Johnson

SSC Central Design Group†
c/o Lawrence Berkeley Laboratory
Berkeley, California 94720

March 1987

*Submitted to the 1987 Particle Accelerator Conference, Washington, D.C., March 16-19, 1987
†Operated by Universities Research Association for the U.S. Department of Energy

A VERY LARGE β^* INTERACTION REGION FOR THE SSC†

D. E. Groom, A. A. Garren and D. E. Johnson*
SSC Central Design Group‡, Lawrence Berkeley Laboratory 90-4040
Berkeley, California 94720

Abstract

Measurements of the p-p elastic scattering cross section into the Coulomb scattering region are important for determining the total p-p cross section, measuring the slope of the elastic cross section with momentum transfer, and normalising the luminosity of the machine. At SSC energies, Coulomb scattering dominates only below 1 μrad. For the nominal SSC emittance ($\epsilon_N = 10^{-6}$m-rad), the angular spread of the beam is $6.8 \times \sqrt{1\ \mathrm{m}/\beta^*}$ μrad. For the contribution of this spread to be comparable to the desired experimental resolution, we require $\beta^* \approx 4000$ m. Moreover, to scan the cross section at larger angles it is desirable to tune β^* to smaller values. These requirements result in stringent IR design criteria: Crossing angles must be large to avoid disruption at bunch encounters, but not too large, in order to avoid synchro-betatron oscillations. At the same time, beams must pass through common triplet quadrupoles or be sufficiently separated for separate quadrupoles.

Physics Motivation

At sufficiently small angles, p-p scattering is dominated by Coulomb scattering. We might expect little nuclear effect when the classical impact parameter is larger than twice the proton's charge radius. At the SSC nominal beam energy $E_b = 20$ TeV, this occurs only when the scattering angle θ is less than about one microradian. Since the cross section for Rutherford scattering is well known, its measurement calibrates the luminosity of the machine. At larger angles nuclear scattering dominates. It is convenient [1] to parameterize the differential cross section for nuclear elastic scattering as a function of four-momentum transfer t as

$$\frac{d\sigma_n}{dt} = \left.\frac{d\sigma_n}{dt}\right|_{t=0} e^{Bt} \ . \tag{1}$$

For colliding beams at high energies, $t = -2E_b^2(1-\cos\theta) \approx E_b^2\theta^2$. Interesting deviations from this simple exponential behavior occur at large values of t, but not until the cross section has decreased by about two orders of magnitude. When the independent variable is changed from t to θ a Rayleigh distribution obtains, *i.e.* the product of normal distributions in θ_x and θ_y. The slope parameter B increases with energy. With its expected value at SSC energies the width of the distribution ($\langle\theta_x^2\rangle^{1/2}$) will be about 10 μrad.

The optical theorem relates the imaginary part of the forward nuclear scattering amplitude to the total p-p cross section σ_{tot}. With its aid Eq. 1 may be rewritten as

$$\frac{d\sigma_n}{dt} = \frac{(1+\rho^2)}{16\pi}\sigma_{tot}{}^2\ e^{Bt} \ , \tag{2}$$

where ρ is the ratio of the real to the imaginary parts of the elastic scattering amplitude in the forward direction. Since $|\rho| \lesssim 0.1$ at the highest energies so far studied, it does not contribute appreciably to Eq. 2. However, the cross section is quite sensitive to ρ at angles near 1 μrad, where nearly equal Coulomb and nuclear elastic scattering amplitudes interfere. A detailed measurement of elastic scattering at small angles not only calibrates the machine's luminosity monitors but also determines σ_n, σ_{tot}, B, and ρ, all of which are of great physical interest. Because of the large cross section the measurement can be made quickly during the early months of SSC operation, using simple and unobtrusive apparatus . It is of interest to examine machine-associated aspects of the measurement [2].

Method

In the absence of complications due to machine optics, one would place a detector near the beam a distance L from the interaction point (IP). Particles detected a distance y from the beam line would have scattered through $\theta = y/L$. For $\theta = 1$ μrad, scattered particles 100 m from the IP would be 100 μm from the beam line. The matter is further complicated because the angular spread due to the beam emittance at a normal IP far exceeds 1 μrad. Clearly, a different approach must be taken. As in recent experiments of this sort, it is proposed to (a) make β^* large enough that spread due to beam emittance is small compared with the desired measurement uncertainty, and (b) make the position measurement using a miniature detector close to the beam at a place in the machine where the phase advance ψ is an odd multiple of $\pi/2$.

In previous experiments the scattered protons have been tracked with high resolution drift chambers. For the SSC a very tiny arrays of drift tubes with thin metal walls has been proposed [3]. The detectors are pushed close to the beam after colliding beam conditions have been established. The size of the spot at the IP is irrelevant if the betatron phase advance from IP to detector is an odd multiple of 90°, and if α^* (the longitudinal slope of the amplitude function β, evaluated at the IP) vanishes, because any set of parallel rays from the IP is focussed to a point at the detector.

It is desirable to reach $\theta_{min} = 0.8$ μrad, where the Coulomb cross section should be about 2.5 times the nuclear elastic cross section. The contribution of beam divergence to the measurement error should not exceed about 20% of this value, so that of either beam should not exceed 14%, or 0.11 μrad. The rms angular divergence of one beam is given by

$$\theta^*_{rms} = \sqrt{\epsilon_N/\gamma\beta^*} \tag{3}$$

where β^* is the betatron amplitude function at the IP, ϵ_N is the normalized emittance (nominally 10^{-6} for the SSC), and $\gamma = E_b/m$ (21,316 at 20 TeV). Similarly, the rms beam size in a transverse direction is given by

$$\sigma^* = \sqrt{\epsilon_N\beta^*/\gamma} \ . \tag{4}$$

Eq. 3 then yields $\beta^* = 4000$ m for the CDR parameters [4]. As a penalty for a suitably small beam divergence, the beam width (σ^*) becomes 0.43 mm. This presents no problem for the detector, but as will be seen in a later section it produces constraints on the beam crossing angle which do have an impact on interaction region design.

It is convenient to define an effective length $L_1 = y_1/\theta$, where θ is the vertical scattering angle and y_1 is the vertical distance

† SSC Central Design Group Report SSC-119.
* Also Fermilab, Batavia, Illinois 60510
‡ Operated by Universities Research Association, Inc. under contract with the U.S. Department of Energy.

from the beam at detector 1. This effective length plays the role of distance from scatterer to detector in a field-free scattering experiment. It is given by $\sqrt{\beta_1\beta^*}\sin\psi$, where ψ is the phase advance from the IP to the detector[5]. If β_1 at the detector is a typical 300 m, ψ is near a odd multiple of 90°, and β^* is 4000 m, then the effective length is 1095 m. Protons scattering at $\theta_{min} = 0.8$ μrad will then have a displacement $y_1 = 0.88$ mm at the detector. This is a tractable situation if the wall thickness of the detector can be as small as 0.02 mm, as in the design of Ref. 3, where there is only a 2.5% dead space between the smallest detectable angle and the distance of closest approach to the beam. The rms beam spot size at the detector position is $\sqrt{\beta_1\epsilon_N/\gamma} = 0.12$ mm. The detector wall is more than 7 standard deviations from the beam, independently of β^*.

Defining the measured scattering angle at position 1 as the displacement divided by the effective length ($\theta_m \equiv y_1/L_1$) and again following Edwards[5], we find

$$\theta_m \equiv \frac{y_1}{L_1} = \frac{\sqrt{\beta_1/\beta^*}(\cos\psi_1 + \alpha^*\sin\psi_1)y_0}{\sqrt{\beta_1\beta^*}\sin\psi_1} + y_0' \qquad (5)$$

where y_0 is the y-position of the scattering and y_0' is its angle. Then

$$\langle\theta^2\rangle = \frac{1}{\beta^{*2}}(\cot\psi_1 + \alpha^*)^2\langle y_0^2\rangle + \langle y_0'^2\rangle \quad . \qquad (6)$$

If both beams have the same emittance, y_0 is the convolution of two gaussians and $\langle y_0^2\rangle = \frac{1}{2}\langle y^2\rangle = \frac{1}{2}(\epsilon_N\beta^*/\gamma)$. The angular spread of both beams combine to yield $\langle y_0'^2\rangle = 2\langle y'^2\rangle$, or $2\epsilon_N\beta^*/\gamma$. Substitution into Eq. 6 gives

$$\theta_{rms} = \sqrt{\epsilon_N/\beta^*\gamma}[(\cot\psi_1 + \alpha^*)^2/2 + 2]^{1/2} \quad . \qquad (7)$$

For ψ_1 near 90°,

$$\theta_{rms} = \sqrt{(\epsilon_N/\beta^*\gamma)(\alpha^{*2}/2 + 2)} \; . \qquad (8)$$

If α^* cannot be made small, one can use $\theta_m \equiv y_1/2L_1 + |y_2|/2L_2$, and then $\theta_m = (\cot\psi_1 - \cot\psi_2)y_0/2\beta^* + y_0'$, which no longer depends upon α^*. The two scattered protons are assumed collinear at angle y_0. The rms spread in position at the detector is $y_{rms} = L_1\theta_{rms} \approx \sqrt{\epsilon_N\beta_1/\gamma}$ when ψ_1 is near 90°, which at 120 μm is about 2.4 times the measuring accuracy of the detector.

If drift chambers 19 mm high are located 0.8 mm from the beam and $L_{eff} = 1000$ m, the t-range is -0.00025 to -0.16 GeV^2. The same apparatus can be used to measure scattering at larger angles if β^* can be tuned to a smaller value. If β^*, β_1, and β_2 are reduced by a factor of 5 the t-range is from -0.006 to -4.0 GeV^2, which is adequate to explore expected structure in the elastic scattering cross section.

We conclude that no matter how high energy the accelerator and no matter how poor the emittance, it is theoretically possible to measure the cross section in the Coulomb region as long as one has a large enough β^*. The intrinsic angular resolution at the detector position is independent of the local β (β_1) and is best if the phase advance is near an odd multiple of 90°. The main disadvantage of small β_1 is that the distances to be measured become small. This is quite acceptable if the detector can measure such small distances accurately and without introducing any significant dead space. The miniaturized detector described in Ref. 3 meets these conditions, at least for $\beta_1 \gtrsim 300$ m.

An insertion example

Preliminary design work indicates that these experiments are compatible with the machine as presently conceived. An example of a high-β^* insertion is shown in Fig. 1; in this case the starting point was one of the low-β^* IRs. For the tuning shown, $\beta_y^* = \beta_x^* = 4000$ m, and $\alpha_y^* = \alpha_x^* = 0$. β^* is tuneable down to at least 400 m. The detector positions were chosen so that $\sin\psi_y > 0.9$ for 4000 m $> \beta^* >$ 400 m; in fact, it rises from 0.95 to 1.00 as β^* is decreased from 4000 m to 1500 m, then falls to to 0.91 as β^* is further reduced to 400 m.

Unfortunately, $\sin\psi_x$ is quite small over most of this range, so that a circle of constant p-p scattering angle maps into a thin vertical ellipse at the detector, exacerbating the measurement error along the longitudinal axis of the drift tube. The situation could be reversed, but only at the expense of a much smaller y displacement (smaller L_{1y}). Other possible detector locations are 30° apart in ψ, so perhaps a better compromise choice of L_{1x} and L_{1y} can be found.

Other machine considerations

The large transverse beam size (σ) presents several problems, mostly related to the nonlinear interaction of bunches as they pass—for fat bunches, a proton near the passing bunch sees a much larger force than does its fellow traveler on the other side of the bunch. We have already noted that $\sigma^* = 0.43$ mm for $\beta^* = 4000$ m. As a function of distance ℓ away from the interaction point,

$$\sigma = \sigma^*\sqrt{1 + (\ell/\beta^*)^2} \; . \qquad (9)$$

In a low-β^* IR the growth of σ with ℓ is essentially linear in ℓ, but in this case it remains almost constant. The problems engendered by both the beam width and its growth with ℓ have been discussed by Diebold and Johnson[6], Piwinski[7], and Groom[8]. We briefly summarize the main points:

1. The encounter of nearby bunches is disruptive. While there is no sharp criterion for a minimum safe separation, we adopt 7σ. In any realistic situation, only the first encounter at 2.4 m matters. The lower solid curve in Fig. 2 shows this lower limit on the crossing angle α[†] as a function of β^* ($\alpha \propto \sqrt{\beta^*}$ for large β^*, $\propto 1/\sqrt{\beta^*}$ for small β^*).
2. p-p collisions in partially interpenetrating beams occur, resulting in "satellite" interaction points. The luminosity falls off as $\exp(-y^2/4\sigma^2)$, and is 0.1% of the central luminosity for $y = 5\sigma$. This constraint imposes the same $\alpha \propto \sqrt{\beta^*}$ dependence as does bunch disruption, but is less restrictive.
3. Interaction between bunches produces a tune shift in the machine. Such a shift presents no problem in itself, but its *spread* does. If we adopt the criterion that this "long-range" tune shift be no larger than that produced in the head-on collision at the interaction point, we obtain the lower limit on crossing angle shown by the dashed curves in Fig. 2. For large β^* it is less restrictive than beam-beam disruption effects.

These considerations all result in *lower* limits on crossing angle. However,

4. If the crossing angle is too big, head-tail interactions between passing bunches couple synchrotron and betatron

† To be confused with the derivative of β, which is to be confused with v/c.

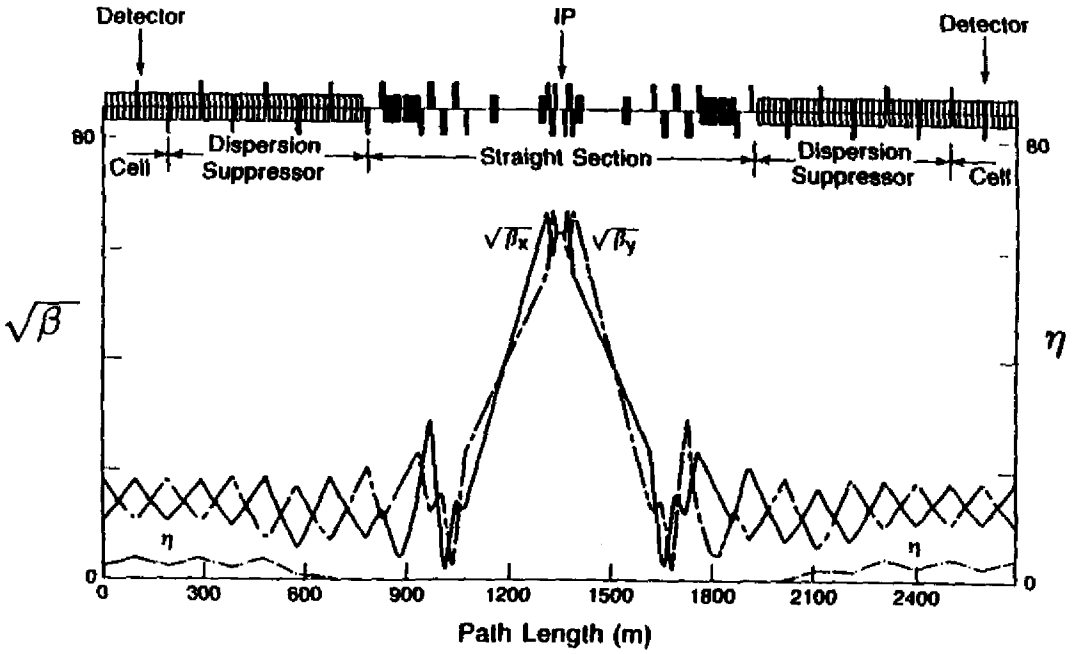


Figure 1. Modification of a standard ±20 m insertion for very large β^* experiments. In this case $\beta_x^* = \beta_y^* = 4000$ m, and $\alpha_x^* = \alpha_y^* = 0$. Detectors will be placed at short warm sections near the indicated position (at a short spool piece). Here $\beta_x = 110$ m and $\beta_y = 310$ m.

oscillations. These become serious for crossing angles large compared with the transverse beam size divided by the bunch length, or, for the SSC parameters, angles large compared with 6.19 mrad$\times\sqrt{\beta^*/1000\text{ m}}$. This upper limit is indicated by the solid line labeled "α_0" in Fig. 2.

For $\beta^* = 4000$ m, we see that the crossing angle must be larger than 1.2 mrad but smaller than 13 mrad. In the absence of dipoles between the triplets, the large minimum angle precludes both beams going through common triplet elements. For the beams to be far enough apart to go through separate elements (say 35 cm), either we must approach the upper limit at α_0 or the triplet quadrupoles must be moved back substantially. In any case, tuning to a smaller β^* means moving to the left in Fig. 2 at constant α—so that running into the α_0 limit determines the minimum β^*.

The easiest way out of this dilemma is to use "straightening" dipoles between the IP and the first triplet elements, so that the beams are allowed to cross at the minimum acceptable angle but are parallel in the triplet. The insertion shown in Fig. 1 incorporates this feature: 5 meter long dipoles start 6 meters from the IP, and the triplets are shown starting at ±12 m. The beams pass through common triplet elements, each 0.6 cm off axis. The increase in the long-range tune shift turns out to be minor, as shown by the dashed line labeled "parallel beams" in Fig. 2.

Our proposed insertion is not unique, and it is certainly not yet optimal. For example, some changes will be necessary to allow room for the small-angle calorimeters for the diffractive studies. The main point is that viable large-β^* IRs are possible within the existing SSC framework, and there seems to be no obstacle to doing these interesting experiments in the weeks following the commissioning of the SSC.

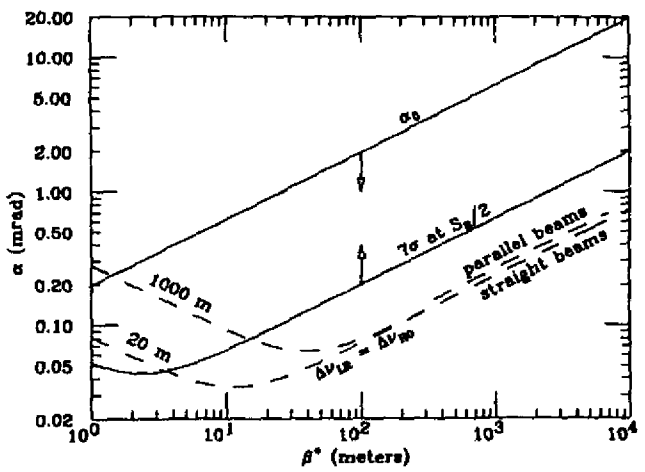


Figure 2. Limits on the crossing angle α as a function of β^*. To satisfy all the conditions, the chosen crossing angle in an IR with $\beta^* > 20$ m must like between the two solid lines.

References

1. M. M. Block and R. N. Cahn, *Rev. Mod. Phys.* **57**, 563 (1985).
2. R. DeSalvo, J. Orear, M. M. Block, A. Donati, A. A. Garren, D. E. Groom, and D. E. Johnson, Low $p_\perp$ Physics Group Report, *Proc. of the 1986 Summer Study on the Physics of the Superconducting Supercollider*, R. Donaldson and J. Marx, eds. Div. of Part. and Fields, Am. Phys. Soc. (to be published, 1987).
3. R. DeSalvo, R. Maleyran, and J. Orear, A Small Angle Elastic Scattering Detector for the SSC, *Proc. of the 1986 Summer Study on the Physics of the Superconducting Supercollider*, R. Donaldson and J. Marx, eds., Div. of Part. and Fields, Am. Phys. Soc. (to be published, 1987).
4. *Conceptual Design of the Superconducting Super Collider*, SSC Central Design Group (J. D. Jackson, ed.), SSC Central Design Group Report SSC-SR-2020 (March 1986).
5. D. A. Edwards, An Introduction to Circular Accelerators, Fermilab (Unnumbered and undated report).
6. R. Diebold and D. E. Johnson, *p-p* Interaction Regions, *Proc. of the 1984 Summer Study on the Design and Utilization of the Superconducting Super Collider*, p. 399, R. Donaldson and J. G. Morfín, eds., Div. of Part. and Fields, Am. Phys. Soc. (to be published, 1987).
7. A. Piwinski, Computer Simulation of Satellite Resonances Caused by Beam-Beam Interaction at a Crossing Angle in the SSC, SSC Central Design Group Report No. SSC-57 (1986).
8. D. E. Groom, Long-Range Tune Shifts and Other Matters in High-β^* IRs, SSC Central Design Group Report SSC-88 (1986).