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Perturbative QCD-Pomeron and High Energy Hadronic Diffractive Cross Sections

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Пертурбативный КХД-померон и дифракционные сечения адронов при высоких энергиях

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Аннотация

С помощью затравочного померона пертурбативной КХД были вычислены дифракционные сечения рассеяния $pp(p\bar{p})$ и πp. Используемая модель базируется на кварковой структуре адронов, цветовом экранировании и s-канальной унитаризации амплитуд рассеяния в эйкональном приближении. Параметры затравочного померона P и трехреджеонная диаграмма PGG (G — реджезованный глюон) фиксировались данными при умеренно высоких энергиях; для сверхвысоких энергий даны предсказания. Величина интерсепта затравочного померона находится в согласии с данными для малых x глубоко неупругого рассеяния.

Abstract

With the bare pomeron of perturbative QCD we calculate the soft diffractive cross sections in pp (or $\bar{p}p$) and πp collisions, exploiting the quark structure of hadrons, colour screening and s-channel unitarization of the scattering amplitudes in the eikonal approach. Parameters of the bare pomeron P and the three-reggeon block PGG (G is a reggeized gluon) are fixed by the data at moderately high energies, while for superhigh energies predictions are made. The intercept of the bare pomeron is found to be in remarkable agreement with the low-x data for deep inelastic scattering.

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In the latest decade the enigmatic structure of the pomeron attracts much attention in the study of diffractive processes at high and superhigh energies. A perpetual item of the agenda is the description of the pomeron in terms consistent with QCD (see, for example, ref. [1] and references therein). Although the growth of the QCD coupling constant at large distances prevents, generally speaking, the direct use of perturbative QCD for the description of soft processes, still it looks as if the pomeron, being almost a point-like object, provides us with an exception. In this case the bare pomeron of perturbative QCD [2,3] seems to be an appropriate object for use as a guide to the description of diffractive processes at high and superhigh energies.

The behaviour of amplitudes for hadronic diffractive processes is determined by singularities of the t-channel partial waves in the complex plane of the angular momentum j. The singularity in the channel with vacuum quantum numbers (pomeron) dominates at high energies. The perturbative QCD-pomeron in the leading logarithmic approximation appears as a composite system of two reggeized gluons while the singularity is a set of regge-poles coming from the righthand side to the point $j = 1$ (BFKL-pomeron [2]). The constraint on the intercept of the leading pole is: $\Delta = j_0(0) - 1 \geq 0.3$ [3].

The BFKL-dynamics reveals itself in the small-x deep inelastic processes: at moderate Q^2 the structure function F_2 should behave as $x^{-\Delta}$. The global fit which is performed in ref. [4] and is based on the new measurements of F^2_{ep} by ZEUS and H1-collaborations [5] provides the value $\Delta = 0.3$. The numerical solution of the BFKL-equation [6] mimics $x^{-0.3}$ behaviour, although it gives a larger value for Δ.

There are reasons to believe that a successful description of soft diffractive processes is possible with a pomeron whose characteristics are close to those of the BFKL pomeron. The point is that the phenomenological pomeron used at moderately high energies for the description of the diffractive processes is a small-size system (see refs. [7,8] and references therein). This is reflected in a small value of α'_P as well as in large masses of the resonances which are candidates for glueballs, $M_G \sim 1.5 \div 2.2$ GeV. As a result the integration in the gluon ladder of the pomeron is carried out over large momenta, where the running coupling constant α_s is not large.

The attempt to work with the BFKL-like pomeron but with small Δ

for the description of πp and pp-scatterings at moderately high energies has been made in ref. [9]. The pomeron, being a gluon ladder (fig. 1a), has two types of couplings with quarks of the scattered hadron: with one quark (fig. 1b) and with two of them (the last type of coupling occurs actually via the three-reggeon vertex PGG, where P and G are pomeron and reggeized gluon, correspondingly, see fig. 1c). These two couplings cancel each other at small interquark distances causing colour screening. The small size of the pomeron reveals itself in a small magnitude for the colour screening radius, r_{cs}. This leads to approximate fulfillment of the additive quark model rules, and the experimental data are fitted reasonably well. Moreover, colour screening effects are able to explain the deviation from additivity in $\sigma_{tot}(\pi p)$ and $\sigma_{tot}(pp)$ at $\sqrt{s} \sim 20$ GeV [8] ($\sigma_{tot}(pp)/\sigma_{tot}(\pi p) \simeq 1.6$). However the growth of the total $pp/\bar{p}p$ cross sections at $\sqrt{s} > 30$ GeV requires the value $\Delta > 0$ [9], and this depends on the necessity to take into account s-channel rescattering processes. This procedure does not converge at small Δ of the order of 0.1: the more rescatterings are taken into account the larger the value of Δ needed for the input pomeron. The description of pp and πp diffractive cross sections $\sigma_{tot}, \sigma_{el}, \sigma_{DD}^{single}$ and σ_{DD}^{double} presented in this paper is performed with the full set of s-channel rescatterings taken into account in the eikonal approximation: it gives $\Delta = 0.29$ just as for the BFKL-pomeron in the deep inelastic HERA experiments.

Below we present first of all the formulae for the diffractive cross sections, then we make some comments on their derivation and the assumptions which are made. The following formulae describe total, elastic and diffractive dissociation cross sections of colliding hadrons A and B:

$$\sigma_{tot}(AB) = 2\int d^2b \int dr'\varphi_A^2(r')dr''\varphi_B^2(r'')\left[1-\exp\left(-\frac{1}{2}\chi_{AB}(r',r'',b)\right)\right], \tag{1}$$

$$\sigma_{el}(AB) = \int d^2b \left\{\int dr'\varphi_A^2(r')dr''\varphi_B^2(r'')\left[1-\exp\left(-\frac{1}{2}\chi_{AB}(r',r'',b)\right)\right]\right\}^2 \tag{2}$$

$$\sigma_{DD(B)}^{single}(AB) + \sigma_{el}(AB) = \int d^2b \int dr'\varphi_A^2(r')dr''\varphi_B^2(r'')d\tilde{r}'\varphi_A^2(\tilde{r}') \tag{3}$$

$$\left[1-\exp\left(-\frac{1}{2}\chi_{AB}(r',r'',b)\right)\right]\left[1-\exp\left(-\frac{1}{2}\chi_{AB}(\tilde{r}',r'',b)\right)\right],$$

$$\sigma_{HD}(AB) = \sigma_{el}(AB) + \sigma_{DD(A)}^{single}(AB) + \sigma_{DD(B)}^{single}(AB) + \sigma_{DD}^{double}(AB) = \tag{4}$$

$$\int d^2b \int dr'\varphi_A^2(r')dr''\varphi_B^2(r'')\left[1-\exp(-\frac{1}{2}\chi_{AB}(r',r'',b))\right]^2.$$

The last expression, $\sigma_{HD}(AB)$, stands for full hadron diffraction. Here $dr\varphi_{A,B}^2(r)$ are the quark densities of colliding hadrons A and B which depend on the transverse coordinates:

$$dr\varphi_\pi^2(r) = d^2r_1d^2r_2\delta^2(\vec{r}_1+\vec{r}_2)\varphi_\pi^2(r_1,r_2), \tag{5}$$

$$dr\varphi_p^2(r) = d^2r_1d^2r_2d^2r_3\delta^2(\vec{r}_1+\vec{r}_2+\vec{r}_3)\varphi_p^2(r_1,r_2,r_3);$$

r_i is the transverse coordinate of a quark, and the wave function squared φ_A^2 has been integrated over longitudinal variables. The profile-function χ_{AB} corresponds to the interaction of quarks via pomeron exchange as follows:

$$\chi_{AB}(r',r'',b) = \int db'db''\delta^2(b-b'+b'')S_A(b',r')S_B(b'',r''). \tag{6}$$

Functions $S_{A,B}$ stand for the pomeron-quark interaction; they are determined by the diagrams with different couplings of the BFKL-pomeron with quarks as is written below:

$$S_\pi(\vec{r},\vec{b}) = \rho(\vec{b}-\vec{r}_1)+\rho(\vec{b}-\vec{r}_2)-2\rho(\vec{b}-\frac{\vec{r}_1+\vec{r}_2}{2})\exp(-\frac{(\vec{r}_1-\vec{r}_2)^2}{4r_{cs}^2}). \tag{7}$$

$$S_p(\vec{r},\vec{b}) = \Sigma_{i=1,2,3}\rho(\vec{b}-\vec{r}_i)-\Sigma_{i\neq k}\rho(\vec{b}-\frac{\vec{r}_i+\vec{r}_k}{2})\exp(-\frac{(\vec{r}_i-\vec{r}_k)^2}{4r_{cs}^2}).$$

The term $\rho(\vec{b}-\vec{r}_i)$ describes the diagram where the pomeron couples to one of the hadron quarks (fig. 1b) while the terms proportional to $\exp(-r_{ij}^2/r_{cs}^2)$ are related to the diagram 1 c with the pomeron which couples to two quarks of the hadron. This diagram is a three-reggeon graph where G is the reggeized gluon. Functions S_π and S_p tend to zero as $|\vec{r}_{ij}|\to 0$: this is the colour screening phenomenon inherent to the BFKL-pomeron. For the sake of convenience, we perform calculations in the cms of the colliding quarks, supposing that hadron momentum is shared equally between its quarks. Then

$$\rho(b) = \frac{g}{4\pi(G+\frac{1}{2}\alpha'_P\ln s_{qq})}\exp\left[-\frac{b^2}{4(G+\frac{1}{2}\alpha'_P\ln s_{qq})}\right], \tag{8}$$

where the vertex g depends on the energy squared of the colliding quarks, s_{qq}:

$$g^2 = g_0^2 + g_1^2 \left(\frac{s_{qq}}{s_0}\right)^{\Delta} . \tag{9}$$

Such a parametrization of g^2 corresponds to the two-pole presentation of the BFKL-pomeron with $j_0 = 1$ and $j_1 = 1 + \Delta$. Here and below $s_0 = 1$ GeV^2.

Now let us make comments on the formulae (1)–(4). Eqs. (1)–(4) in the case of a pion beam are obtained summing the diagrams like fig. 1d where all the possible meson states M_i and M_j are taken into account. The assumption that a full set of meson states M_i corresponds to a full set of the quark-antiquark states leads to the diagram 1e, and just this type of diagrams with the quark intermediate states is reproduced by eqs. (1)–(4). Analogously the diagrams like fig.1 f are taken into account in eqs. (1)–(4) in the case of a proton beam.

The diffractive cross sections (1)–(4) for a meson beam were obtained in ref. [9], while for the proton beam the final formulae were presented in ref. [11]; their derivation will be published elsewhere.

Eqs. (1)–(4) depend on the transverse coordinates of quarks, though the original expressions depend on the fractions of the momenta of the colliding hadrons carried by the quark, x_i. In the functions S we put $x_i = 1/2$ for a meson and $x_i = 1/3$ for the proton, e.g. we assume that hadron wave functions $\varphi_\pi(\vec{r}, x)$ and $\varphi_p(\vec{r}, x)$ select the mean values of x_i in the interaction blocks. A wide range of wave functions is of that type, for example, the wave functions of quark spectroscopy. But the situation with the diagram of fig. 1c is more complicated. One should perform an integration over the part of the energy carried by reggeized gluons and pomeron: this spreads the x_i's of the interacting quarks. However, if the intercept of the reggeized gluon $\alpha_G(0)$ is near 1 (it is actually the requirement of the BFKL pomeron), then x_i's can be considered as frozen. We have checked by numerical calculations that this assumption works at $0.8 < \alpha_G(0) < 1$. for realistic pion and proton wave functions. In due course, in eq. (9) we put $s_{qq} = s/6$ for πp collision and $s_{qq} = s/9$ for pp.

Eqs. (1)–(4) can be used at small momentum transfers where real parts of the amplitudes are small. Hence we neglected the signature factor of the bare pomeron, but it can be easily restored. We shall return to this point

later on.

Eqs. (1)–(4) are written in the eikonal approximation for composite systems. It is well known that certain correlations are missed in this procedure. Let us elucidate what type of correlations is taken into account and what is neglected in the approach being developed. Interactions of quarks of the same hadron and the transitions of these quarks into the excited hadron states are included in the diagrams of type 1*d*, and the completeness condition for the quark states results actually in multiple interactions of the "frozen" quark state (diagram 1 *e*). But the *t*-channel gluon interactions are taken into account only in the simplest form — as a set of non-interacting pomeron exchanges. Including pomeron-pomeron interactions is actually the problem of finding a solution which satisfies simultaneously both *t*- and *s*-channel unitarity conditions. Attempts to solve it have been intensified recently [12].

Let us discuss results of the calculation. Total and elastic pp (or $p\bar{p}$) and πp cross sections are shown in fig. 2*a*, *b*. The parameters of the input pomeron have been obtained in the fitting procedure in a broad energy range ($\sqrt{s} = 23.7 \div 1800$ GeV); they are as follows:

$$g_0^2 = 7.914 \text{ mb}, \quad g_1^2 = 0.179 \text{ mb}, \quad r_{cs} = 0.18 \text{ fm}; \tag{10}$$

$$\Delta = 0.29, \quad G = 0.167\ (\text{GeV/c})^{-2}, \quad \alpha'_P = 0.112\ (\text{GeV/c})^{-2}.$$

The wave functions φ_π and φ_p are chosen to satisfy pion and proton form factors at $|q^2| \leq 1$ GeV^2 in the framework of the conventional quark model.

At high energies total cross sections calculated with the parameters given by eq. (10) can be fitted, within 5% accuracy, by the following expressions:

$$\sigma_{tot}(pp) = 1.75 + 2.27 \ln(s_{qq}/s_0) + 0.32 \ln^2(s_{qq}/s_0), \tag{11}$$

$$\sigma_{tot}(\pi p) = 4.93 - 6.19 \ln(s_{qq}/s_0) + 0.32 \ln^2(s_{qq}/s_0).$$

Numerical coefficients are given in mb.

Elastic cross sections calculated with eq. (2) are shown in fig. 2*b*. At superhigh energies they increase as $\ln^2 s$ as well and can be fitted in the form

$$\sigma_{el}(pp) = -6.13 + 0.797 \ln(s_{qq}/s_0) + 0.16 \ln^2(s_{qq}/s_0), \tag{12}$$

$$\sigma_{el}(\pi p) = 3.05 - 1.38 \ln(s_{qq}/s_0) + 0.16 \ln^2(s_{qq}/s_0).$$

We predict the following values for total and elastic cross sections at LHC energy $\sigma_{tot}(pp) = 131$ mb, $\sigma_{el}(pp) = 41$ mb ($\sqrt{s} = 16$ TeV). The slope B for the elastic $pp(\bar{p}p)$ cross section $d\sigma_{el}/dq^2 \sim \exp(-Bq^2)$ is shown in fig. 2c.

The ratio of the total cross sections $\sigma_{tot}(pp)/\sigma_{tot}(\pi p)$ tends to unity at far asymptotic energies - this is reflected at the same factors in front of $\ln^2 s$ in eq. (11). In this limit $\sigma_{el}(pp)/\sigma_{tot}(pp) \to 1/2$. These limit magnitudes originate because of the disappearance of the colour screening radius at superhigh energies: effective colour screening radius, which is different in pp and πp collisions, tends to zero at asymptotic energies.

As was mentioned above, the restoration of the pomeron signature factor (or crossing symmetry of the amplitude) can be done easily, and this allows one to calculate $\rho = Re\,A/Im\,A$. The result is shown in fig. 2d: the description of the data is quite reasonable.

Now let us discuss the process of the diffraction dissociation. There is a problem of the definition of σ_{DD}^{single} because two mechanisms contribute to the measured diffractive dissociation cross section: one is the dissociation of the colliding hadron, see fig. 3a, and another involves partly dissociating pomeron, fig. 3b. Eqs. (3) and (4) provide the hadron dissociation only, the calculated cross section for the proton dissociation is presented in fig. 4. The difference of the measured value of $\sigma_{DD}^{single}(\bar{p}p)$ and the calculated one provides just the cross section for the partly dissociating pomeron which is determined by the three-pomeron diagram. Unfortunately, the extraction of the process with partly dissociating pomeron faces a difficulty: $\sigma_{DD}^{single}(\bar{p}p)$ calculated by means of eqs. (3) is very sensitive to the inner structure of the colliding hadrons. Actually the separation of these two types of process should be based on the detailed study of the M^2- and t-dependence; it is beyond our present consideration.

In conclusion, the description of σ_{tot}, σ_{el} and ρ in $pp/\bar{p}p$ and πp collisions has been performed in the range from moderately high energies to superhigh energies, using the t-channel multipomeron interactions, where colour screening is taken into account. The intercept of the bare pomeron is close to that of the BFKL-pomeron which was found in the low-x region of deep inelastic scattering.

All calculations have been performed using the program of Monte-Carlo simulation VEGAS [14].

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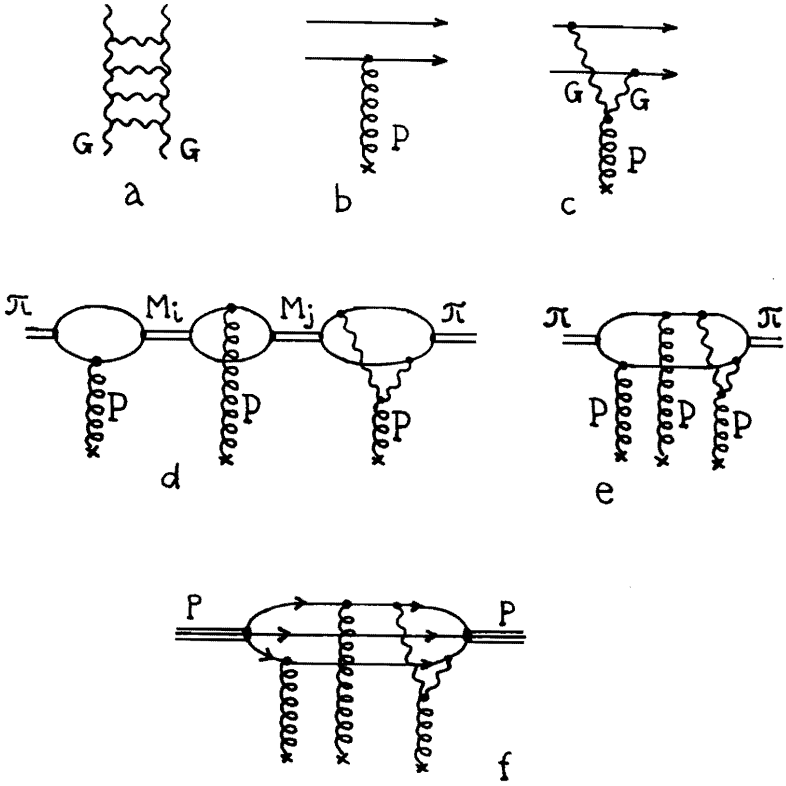

Fig.1. *a*) Gluon ladder diagram of the BFKL-pomeron; *b*, *c*) various types of pomeron-quark couplings; *d*) diagrams of soft multiple rescatterings for the pion beam written using the language of hadron intermediate states; the summation is performed over all allowed meson states M_i and M_j; *e*) the same diagrams as in fig. 1*d* but written in the language of quark rescatterings.

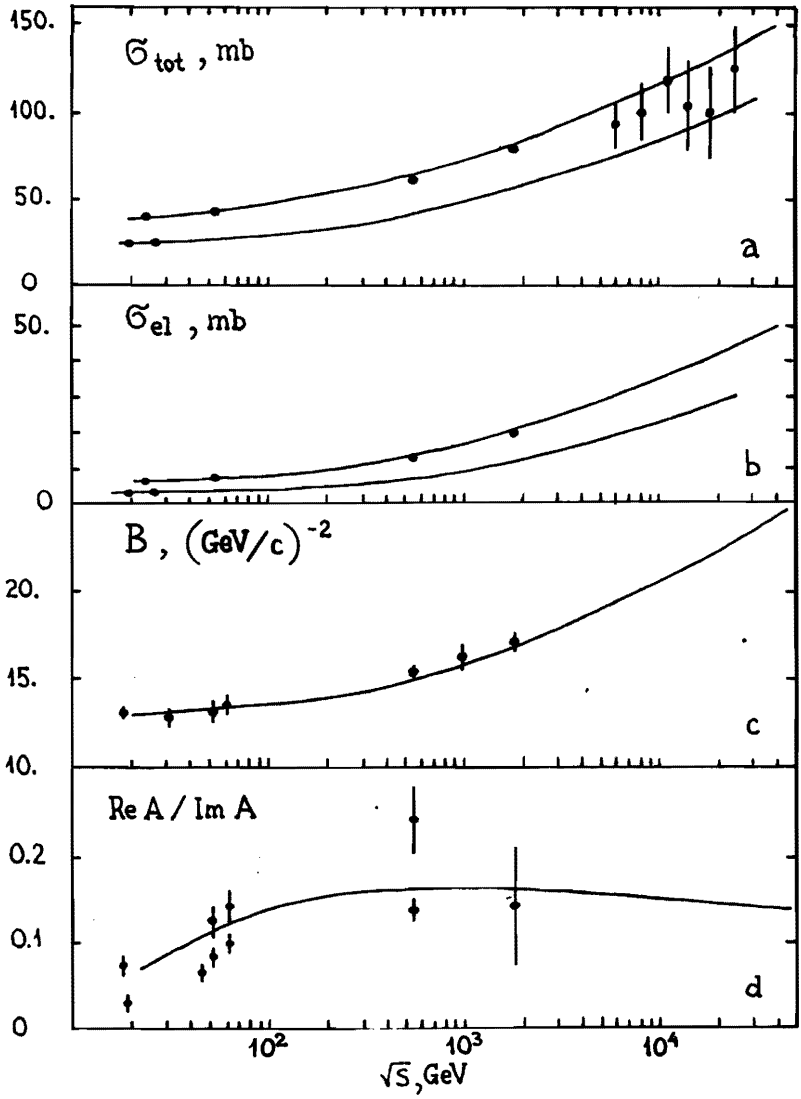

Fig.2. Experimental data and calculated values for $pp(\bar{p}p)$ and πp collisions: *a*) total cross sections, the data at $\sqrt{s} > 5000$ GeV are Akeno cosmic ray experiment; *b*) elastic cross sections; *c*) slope parameter for $pp(\bar{p}p)$ elastic scattering; *d*) $\rho = Re\,A/Im\,A$ for $pp(\bar{p}p)$ scattering.

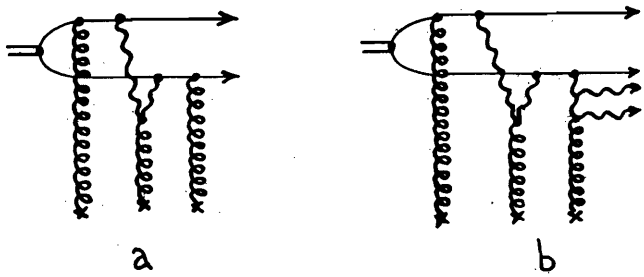

Fig.3. Examples of DD processes which are measured in the experiments: *a*) The dissociation of hadron, *b*) Process with the partly dissociating pomeron - this process has not been taken into account in eqs. (3)-(4).

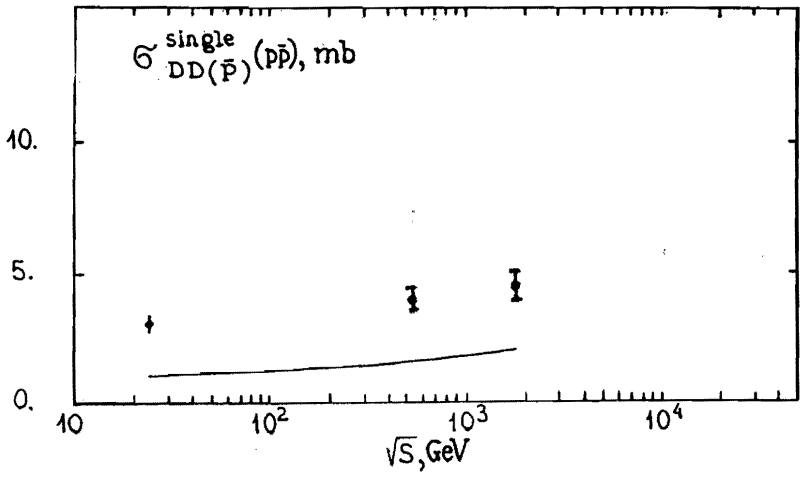

Fig.4. Calculated value of $\sigma_{DD}^{single}(pp)$ for the diffraction dissociation of a proton (the processes like fig. 3a) and experimental data [13] where both processes (like figs. 3 a and b) are measured.

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