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Gauge CP Symmetry, Wormhole Effects, and Strong CP Problem

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ABSTRACT

The recent observation of four dimensional CP as a discrete gauge symmetry in $8k+1, 8k+2$ and $8k+3$ Minkowski dimensions calls for a serious reconsideration of various solutions of the strong CP problem. In these possible higher dimensions, the relevant one is a ten dimensional string theory in which a possibility of light fermions exists. We point out that the model independent axion in the heterotic string theory solves the strong CP problem. We also point out that the wormhole effects to low energy global and discrete symmetries are negligible.

The recent observation of four dimensional CP as a discrete gauge symmetry in $8k+1, 8k+2$ and $8k+3$ Minkowski dimensions [1] calls for a new understanding on the weak CP violation and the strong CP invariance. Among the possible higher dimensions, the relevant one is 10 because there exists a phenomenologically viable heterotic string theory [2]. The other extra dimensions not utilized by nature are only of academic interest. In the heterotic string theory, it has been known that the four dimensional CP must be a gauge symmetry since the theory does not have any discrete symmetry except $E_8 \leftrightarrow E_8'$ [3]. The discrete symmetry $E_8 \leftrightarrow E_8'$ cannot contain the four dimensional CP of light fermions and anti-fermions since they are singlets of E_8'. In string theories, therefore, the required weak CP violation must arise through spontaneous breaking [4]. The Kobayashi-Maskawa weak CP phase which looks like a hard CP violation at low energy may in fact have descended down from string scale physics through spontaneous breaking mechanism [5]. Inflation after the spontaneous CP violation might have removed the dangerous domain walls from our observable universe. The 'calculability' solutions of the Nelson-Barr type [6] to the strong CP problem have a rationale ($\theta_{QCD} = 0$) *in higher dimensions* due to CP as a discrete gauge symmetry of Lagrangian.

However, there are two questions to be cleared regarding the strong CP problem. Firstly, should we consider extra dimensions? Second, what is the effect of gravity on the four dimensional low energy symmetries? In this Letter, we consider both of these questions from Planck scale physics.

Regarding the first question, an extension of spatial dimensions should not be in contradiction with low energy physics. Phenomenologically, there is no reason to extend the spatial dimensions. However, one may try to include more spatial dimensions to solve theoretical problems of the standard model. So in consideration of discrete and global symmetries in higher dimensions, at present the ten dimensional heterotic string is the only one to be considered. Thus, let us consider the θ parameter in the heterotic string theory in ten dimension. The heterotic string has a model-independent axion a_{MI} [7] whose decay constant is

$\sim 10^{15}$ GeV [8]. Even if CP may be a discrete gauge symmetry, the θ parameter problem should be tackled in this framework only.

The Nelson-Barr mechanism introduces the weak CP phase before the completion of the inflationary epoch (presumably at $T \sim 10^{14}$ GeV), not to introduce a cosmological domain wall problem. After completion of the inflation, θ_{QCD} will remain at 0. Even if Yukawa couplings obtain complex phases after inflation, θ_{QFD} is not determined yet because quarks still remain massless until the electroweak symmetry breaking. After the electroweak symmetry breaking, the Nelson-Barr type mass matrix will render $\theta_{QFD} = 0$ at tree level, leading to $\bar{\theta} \simeq 0$. However, the model independent axion changes the above argument. After the inflation, one of the vacuum values of the model independent axion is chosen, $\langle a_{MI} \rangle = \tilde{\theta} F_a$. Thus, $\bar{\theta}$ must be

$$\bar{\theta} = \theta_{QCD} + \theta_{QFD} + \tilde{\theta} \tag{1}$$

In low energy physics, $\theta_{QCD} + \tilde{\theta}$ has been called before θ_{QCD}. There is no reason (not even an anthropic principle) that $\tilde{\theta}$ should be chosen at a narrow band $|\tilde{\theta}| < 10^{-10}$. However, the axion field evolves in an expanding unverse and eventually will settle to zero. Then the basic mechanism for solving the strong CP problem is the model independent axion. But the details depend on the compactification schemes.

If the compactification gives only one confining group $SU(3)_c$ below the compactification scale, the model independent axion is the usual invisible axion solving the strong CP problem but with the axion decay constant problem [8]. If we disregard the axion decay constant problem, there is no need for the quark mass matrix to take a specific form because the model independent axion solves the strong CP problem any way. On the other hand, if a hidden sector confining group survives below the compactification scale, the model independent axion settles one θ to zero. This works as follows. Suppose two confining groups, quantum chromodynamics $SU(3)_c$ and hidden sector confining group $SU(N)_h$. We

can write the $F\tilde{F}$ terms as

$$\frac{1}{32\pi^2}\left([\theta + \frac{a_{MI}}{F_a}]F^a_{\mu\nu}\tilde{F}^{a\mu\nu} + [\theta' + \frac{a_{MI}}{F_a}]F'^h_{\mu\nu}\tilde{F}'^{h\mu\nu}\right) \tag{2}$$

where F and F' are field strengths of QCD and hidden sector gluons, respectively. The vacuum expectation value of $a' \equiv a_{MI} + \theta' F_a$ settles at zero at the hidden sector scale, but then the coefficient of $F\tilde{F}$, $\theta - \theta' + a'/F_a$, becomes $\theta - \theta'$. If CP is a discrete gauge symmetry, $\theta = \theta' = 0$. However, weak interactions introduce θ_{QFD} at the electroweak scale and in this regard the Nelson-Barr type mass matrix is required to give a sufficiently small $\bar{\theta}$. Due to possible CP violations at the hidden sector scale, θ' (*i.e.* θ'_{QFD}) will be generated and it must be proven for each compactification scheme θ' is also sufficiently small or more generally $|\theta - \theta'|$ is sufficiently small, $< 10^{-10}$. The easiest scenario is that the hidden sector is CP invariant and the observable sector introduces the weak CP violation *a la* Nelson and Barr. But this scenario seems to be too contrived and a better compactification scheme is the one leading to a QCD invisible axion as successfully introduced in Ref. [9].

The second question on the effect of gravity at low energy is a difficult one because of our ignorance of quantum gravity at present. From the beginning of the axion solution to the strong CP problem, it has been assumed that gravity respects the Peccei-Quinn (PQ) symmetry [10]. The classical Einstein-Hilbert action does not break the PQ symmetry, and hence it is not expected that gravity breaks the global symmetry perturbatively. Thus the recent surge of interest [11] on the gravitational effects to low energy symmetries has a root to nonperturbative effects such as gravitational instantons, wormholes, *etc.* Let the global transformation be

$$\Psi \to e^{\alpha Q}\Psi \tag{3}$$

where Ψ is a collection of complex fields and Q is the global charge operator. If gravity breaks the global symmetry, one may expect a nontrivial α dependence of free energy after integrating out $g_{\mu\nu}$ and fermion fields. However, the fundamental difference between QCD and gravity on the confinement and chiral symmetry breaking invalidates the comparison.

This leaves the wormhole [12] physics as the best known nonperturbative gravitational effects to low energy symmetries. The effect of wormhole on the breaking of the PQ symmetry has been considered before [13]. The argument is the following. A baby universe separating out from the parent one through a wormhole is a closed universe. The closed baby universe cannot carry a gauge charge, but is not forbidden from carrying global or discrete charges. In particular, if the classical wormhole solution is an axionic one, it must carry a PQ charge. Then the parent universe will not carry a vanishing PQ charge, and the effect of separating out the baby universes will appear to an observer in the parent universe as an apparent violation of the PQ symmetry [13],

$$\delta_w \mathcal{L} = M_w^4 \sum_{n=1}^{\infty} \left[\omega_n \alpha_n e^{-ina/F_a} Y_n + \text{H.c.}\right]. \tag{4}$$

where $\alpha_n = |\alpha_n| \exp(i\theta_n)$ is a dimensionless complex variable. The operators Y_n are PQ singlets. The mass parameter M_w is the wormhole scale, ω_n ($\sim$ the probability amplitude for n units of PQ charge to be drained) is $\sim e^{-nS_0}$, where S_0 is a half of wormhole action, in the dilute gas approximation, and α_n depends on the degree of violation of the PQ symmetry in the parent universe. Since we are interested in the axion mass, we set $Y_n = 1$. For large n's the relative probability to drain n units of PQ charge compared to drain one unit of PQ charge is extremely small.

Even if the above symmetry argument predicts the violation of the PQ symmetry, but the symmetry argument alone does not give any information on the magnitude of α_n, and we will argue below that α_n's are sufficiently suppressed in our universe.

For ease of discussion, let us consider a $U(1)$ gauge symmetry first. We have the global symmetry in mind, but the magnitude of symmetry breaking in terms

of a gauge boson mass is better to tackle, and hence let us proceed to discuss the gauge symmetry first. The classical gauge field equation is

$$\partial_\mu F^{\mu\nu} = j^\nu \tag{5}$$

where j_μ is the conserved current. The total charge carried by the closed universe vanishes

$$Q = \int j^0 d^3x = \int \partial_i F^{i0} d^3x = \int_\Sigma \vec{F}^0 \cdot d^2\vec{\sigma} = 0 \tag{6}$$

because the boundary Σ vanishes in a closed universe. We, however, argue that the condition (5) is an unnecessarily strong *constraint* in an evolving universe. Quantum wormholes are not required to satisfy the classical equation of motion. Then baby universes may not be forbidden from carrying gauge charges. This possibility arises at the quantum level for gauge charges. Of course, then Eq. (6) implies that we cannot use Eq. (5) and gauge symmetry must be broken in the baby universe as well as in the parent universe [14]. The strong belief that wormholes do not carry gauge charges is based on requiring Eq. (5). Thus violating Eq. (5) with baby universes carrying gauge charges seems to be a logical possibility. For global charges, even the classical wormholes can carry them. If baby universes take out gauge charges, one effect of the gauge symmetry breaking in the parent universe can be parametrized in terms of the gauge boson mass. The effect of the nonvanishing charge on the gauge boson mass in a closed universe has been given before [14]

$$m^2 = 8\pi G \frac{J^0 J^0}{R - 2\Lambda} \tag{7}$$

where G is Newton's constant, J^0 is the charge density, R is the Ricci scalar and Λ is the cosmological constant. We present our discussion for a large and closed $\Lambda = 0$ universe because for a closed universe Eq. (7) is derived. For $\Lambda = 0$, the gauge boson mass in the baby universe of size l carrying one unit of charge is of order $1/M_p l^2$. Thus the Planck scale baby universe carrying one unit of gauge charge will render the gauge boson a Planck scale mass but a large parent universe gives a negligible mass. This makes sense because the size of universe must play a role to see how bad the effect of the symmetry violation. The vanishing gauge boson mass is a manifestation of unbroken gauge symmetry. For a nonlinearly realized global symmetry, the vanishing Goldstone boson mass is a manifestation of the global symmetry. Therefore, we propose, for the PQ symmetry breaking also, that the degree of the symmetry breaking due to the effect of the wormhole can be parametrized by the following axion mass,

$$m_a^2 = 8\pi G \frac{J^0 J^0}{R} \tag{8}$$

where J^0 is the PQ charge density. The axionic wormhole whose size is $\sim \sqrt{1/M_p v_{PQ}}$ will give an axion in the baby universe a mass of order v_{PQ}. However, the size of parent universe is enormous, and drainage of one unit of PQ charge from the parent universe result in a negligible axion mass in the parent universe. In the parent universe, the PQ symmetry looks like almost unbroken by the axionic wormhole. If a large number of PQ charges are drained out to the baby universes, the explicit PQ symmetry breaking by wormholes will be more noticeable. But the probability for this to happen is exponentially suppressed, $\sim e^{-nS_0}$.

Eq. (4) has a correct form (taking out the probability factor ω_n) to account $(J^0)^2$ factor in Eq. (8). Thus, comparing Eq. (8) with Eq. (4) without this probability factor, we obtain

$$|\alpha_n| \sim \frac{9GF_a^2}{4\pi M_w^4 R l^6} \tag{9}$$

As a guide, we take $R \sim l^{-2}, l \sim 10^{11}$ lys, $F_a \sim 10^{12}$ GeV, and $M_w \sim 10^{16}$ GeV, and obtain $|\alpha_n| \sim 10^{-249}$. For this wormhole contribution to the axion mass to match the QCD contribution, the drained PQ charge must be enormous, $n \sim 10^{90}$, but in this large n region ω_n is extremely small, $\omega_n \sim \exp(-10^{90} S_0)$.

Thus, we can practically neglect the effect of the drained global charges on the axion mass in our universe. Similarly, the effect of wormholes on the discrete symmetry breaking can be neglected in our universe.

In conclusion, we observed that the strong CP solution *a la* Nelson and Barr in the heterotic string theory makes sense if there survives an extra confining gauge group. But possible CP violations at the hidden sector scale must be taken into account to solve the strong CP problem. The invisible axion type solution of the strong CP problem through compactification scheme discussed in Ref. [9] seems to be simpler and nicer. More importantly, we observe that the wormhole effects on the violation of low energy global and discrete symmetries are negligible in our universe. Thus, the strong CP solution in $D = 4$ through the invisible axion [15] remains as a good solution of the strong CP problem.

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