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P Haines
SISSA, Strada Costiera 11, Miramare, Trieste 34014, Italy

G F R Ellis
Applied Maths, University of Cape Town, Rondebosch 7700, South Africa

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Observing cosmological perturbations on the past light cone

P Haines[1] and G F R Ellis[1,2]
[1] *SISSA, Strada Costiera 11, Miramare, Trieste 34014, Italy*
[2] *Applied Maths, University of Cape Town, Rondebosch 7700, South Africa*

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ABSTRACT
When determining the state of the universe in the past from conditions on our past null cone, as determined by astronomical data, a crucial issue is whether or not the decaying mode in the density perturbation is zero (within the error bounds of the observations). We demonstrate a method for examining this amplitude, using a Newtonian gauge-invariant formalism. IRAS-QDOT data supply the density, assuming we know the value of the bias parameter, and the POTENT scheme is employed to find the velocity field. While the data are not good enough at present to arrive at a solid conclusion, nevertheless the method is shown to provide a way of testing important assumptions usually made regarding the origin of structure in the universe. An alternative viewpoint is to assume there is no decaying mode, and use this to determine a value for the bias parameter. The value determined in this way, on the basis of present data, is similar to that obtained in other ways.

Key words: large scale structure – general relativity.

1 INTRODUCTION

Modern cosmology is constructed on the basis of a subtle blend of observation and theory, usually each supporting the other but from time to time in confrontation. The theoretical basis of the standard approach is the Robertson-Walker models and their interpretation as average (smoothed-out) models of the physical universe on a large scale (Weinberg 1972; Peebles 1980; Gunn, Longair & Rees 1978; Kolb & Turner 1990). However observational data has not always found the universe to be as smooth as these ideal models suggest. Recent observations have made great strides in determining what is actually there, for example through the Centre for Astrophysics surveys, and have lead to the discovery of voids and walls, large-scale streaming flows, and the great attractor (de Lapparent, Geller & Huchra 1986; Kirshner, Oemler, Schechter & Shectman 1987; Lynden-Bell, Faber, Burstein, Davies, Dressler, Terlevich, & Wegner 1988) – none of them predicted by theory before they were discovered.

The theoretical basis for investigating how the universe really is, rather than comparing observations with the predictions of idealised models, was laid in a fundamental paper by Kristian & Sachs (1966), detailing how null cone observations can be used to determine space-time geometry and the matter distribution in it. The theory of this approach has been developed since in a series of papers (Ellis, Nel, Stoeger, Maartens & Whitman 1985; Stoeger, Ellis & Nel 1992) whose aim and intention is summarised in various reviews (Ellis 1980, 1984). The observational results mentioned above in effect carry out a programme of this kind, on a relatively local scale where a Newtonian-like approach is adequate.

Concomitantly on the theoretical side, considerable progress has been made in recent years in the gauge-free study of perturbations of Robertson-Walker cosmological models, in particular through the major paper by Bardeen (1980) and subsequently through analysis of an equivalent but fully covariant approach (Ellis & Bruni 1989). An interesting issue, then, is how to relate these two studies to each other: specifically, to what extent the initial data for cosmological perturbations can be related to realistic astronomical observations on our past light cone (which is the locus of all events that can be probed by possible astronomical observations), along the lines outlined by Kristian and Sachs, but not necessarily restricted to 'near' our present space-time position (the vertex of the past light cone). Thus there is a need to relate the gauge-invariant analysis of cosmological perturbations to the analysis of possible astronomical observations on our past null cone, in effect amalgamating the perturbation results of Ellis & Bruni and the null-cone observational approach of Stoeger et al. in a strictly observationally based theory of inhomogeneities in cosmology.

This paper represents a testing out of that idea from present data catalogues, using the well-developed POTENT techniques (Bertschinger, Dekel, Faber, Dressler & Burstein

1990) and IRAS data (Kaiser, Efstathiou, Ellis, Frenk, Lawrence, Rowan-Robinson & Saunders 1991) to investigate the initial data for perturbations in the regime near us, where a Newtonian analysis of dynamics is adequate. The results presented are interesting but should be treated with caution, for reasons to be explained in the main text; however some important issues are raised that deserve further study in more systematic long-term analysis of the data, both in the Newtonian regime (studied here) and in the analysis of observations on larger scales where a relativistic analysis is appropriate.

Precisely, then, what we are after is a Newtonian gauge-invariant description (Ellis 1990) of the region that is, cosmologically speaking, close to the Milky Way. This is intended to serve as a slice of initial data from which one could in principle integrate backwards in time. Immediately one encounters the problem of whether the decaying mode is or is not zero, which will be discussed further below. However a preliminary comment is in order: some of our colleagues will doubtless claim that it is obvious this mode is zero nowadays, so there is no need to test this supposition; it is a waste of time doing so. In response to the question of how we know this is 'obvious', the answer has to be that it is predicted by our standard theory of large scale structure formation; if the decaying mode is non-zero, the usual form of this edifice (and in particular, its relation to the cosmic microwave background radiation anisotropy measurements) will have to be altered.

There are essentially two responses to this attitude. The first is to go along with it: why test what is fundamental to our present standard models, which we love and believe in? All we will do is verify what we already know. The second is that it is precisely because of this foundational importance of the result that it should be tested by all means available. The test may verify what we already know; in that case our supposition will be on a sounder footing than before. However there is always the admittedly small but nevertheless non-zero possibility that the standard model may be wrong; in this case the experiment is of importance. If pressed further on why we should query the present standard understanding, the response is that the commonly agreed models have often been wrong or misleading in the past, as fashions have come and gone (see e.g. Ellis 1989), and in particular in recent times the standard understanding has failed to predict the major features already mentioned (walls, voids, streaming motions, great attractors) before they were observationally discovered. The philosophy advocated here is we should try and see what is actually there, rather than assuming we know the answer on the basis of theoretical predilections. At the root is the question of whether we prefer to base cosmology more on observational tests or on philosophically based models - an ongoing debate of fundamental importance (Ellis 1991).

For those not convinced by this argument, one can reverse the reasoning. In order to use the IRAS data to estimate the density, we have to assume a best-fit value for the bias parameter, chosen on the basis of other astronomical observations; this value then underlies our determination of the magnitudes of the growing and decaying modes. One can however instead assume the decaying mode is zero, and use this assumption to determine a corresponding value for the bias parameter. We use our analysis to determine the bias parameter in this way, on the basis of present day data, arriving at a value of b consistent with that determined by other means, but really valid only as a sort of average over our sample, smoothed at a particular scale

For simplicity in discussing the evolution of the perturbation data we consider only the case $\Omega = 1$, but this restriction does not apply to the analysis of the initial data which is the main theme of this paper. Thus our main conclusions are unaffected by this simplification (certainly we would regard observational testing of whether or not Ω is unity, as an important part of the approach advocated; however that is not the concern here).

2 THE NEWTONIAN INITIAL DATA

The evolution of linear Newtonian perturbations of the density and velocity is well known (Peebles 1980). In the linear regime, one finds

$$\delta = \frac{3}{5}\left(\frac{t}{t_i}\right)^{\frac{2}{3}}\left(\delta_i - t_i\frac{\nabla\cdot\vec{v_i}}{a_i}\right) + \frac{2}{5}\left(\frac{t}{t_i}\right)^{-1}\left(\delta_i + \frac{3}{2}t_i\frac{\nabla\cdot\vec{v_i}}{a_i}\right) \quad (1)$$

where the subscript 'i' refers to the initial time, and $a(t)$ is the cosmological expansion factor.

The gauge invariant variables corresponding to density perturbation and its time derivative are the comoving spatial gradients of the density and of the fluid expansion:

$$\mathcal{D}_a \equiv a(t)\,\frac{\rho_{,a}}{\rho} \quad (2)$$

$$\mathcal{Z}_a \equiv a(t)\,\Theta_{,a}, \quad \Theta \equiv \mathrm{div}\,\vec{v}. \quad (3)$$

The evolution of these quantities is given in Ellis (1990): If the perturbations are linear,

$$\dot{\mathcal{D}}_a = -\mathcal{Z}_a \quad (4)$$

where the dot represents the convective derivative, and the second order propagation equation for $\mathcal{D}_a$ is (op. cit., eq. 28):

$$\ddot{\mathcal{D}}_a + \frac{2}{3}\Theta\dot{\mathcal{D}}_a - \frac{1}{2}\kappa\rho\mathcal{D}_a - c_s^2\nabla^2\mathcal{D}_a = 0. \quad (5)$$

in the linear regime. It follows that the comoving fractional spatial density gradient can be written in terms of growing and decaying modes. When $\Omega = 1$ and the pressure is negligible (i.e. $p = c_s = 0$), the solution is exactly the same as that for the usual density contrast in a flat background:

$$\mathcal{D}_a = c_{+a}t^{2/3} + c_{-a}t^{-1}, \quad \dot{c}_{+a} = 0 = \dot{c}_{-a} \quad (6)$$

the first term (with spatially varying coefficient c_{+a}) being the growing mode, and the second (with spatially varying coefficient c_{-a}) the decaying mode. Taking the convective time derivative along the fluid flow lines, we find

$$\dot{\mathcal{D}}_a = \frac{2}{3}c_{+a}t^{-1/3} - c_{-a}t^{-2}. \quad (7)$$

The initial data for (5) is $\mathcal{D}_a$ and $\dot{\mathcal{D}}_a = -\mathcal{Z}_a$; if these quantities can be determined observationally as a function of spatial position at the time t_0, then (6), (7) are a system of two linear equations, from which we can get c_{+a} and c_{-a} at each point within the sampled region. In fact we easily see

$$c_{+a} = \frac{3}{5}t_0^{3/2}(\mathcal{D}_a|_0 + t_0\dot{\mathcal{D}}_a|_0) \quad (8)$$

Figure 1. Radial peculiar velocities in the sample, with $|cz| < 1500$ km/sec. This is the supergalactic plane. Circles mark the positions of galaxies projected onto the SG plane after correcting for Malmquist bias.

Figure 2. Peculiar velocity from POTENT. This is the supergalactic plane. The velocities were smoothed using a gaussian window of width 1200 km/sec. Note the "Great Attractor" near (-4000,2000) and part of Perseus-Pisces near (3000,-3000).

$$c_{-a} = \frac{2}{5} t_0 (\mathcal{D}_a|_0 - \frac{3}{2} t_0 \dot{\mathcal{D}}_a|_0) \tag{9}$$

where the age of the universe t_0 is related to the Hubble constant H_0 by $t_0 = \frac{2}{3} H_0^{-1}$. Substituting these values back into (6) gives the evolution of the density inhomogeneity (in the linear approximation) along the fundamental world lines.

In the following, we use the POTENT analysis to determine $\dot{\mathcal{D}}_a|_0$ in a local region of the universe and IRAS data to determine $\mathcal{D}_a|_0$ in the same region (the reason for these choices will be made clear in the discussion), so determining the initial data needed in (8), (9).

3 THE POTENT ANALYSIS

The POTENT scheme can be summarised as follows: (A) Radial peculiar velocities are observed for a sample of galaxies whose distances are measured independently of redshift. These velocities are smoothed (using a tensor window) on a large-enough scale. After assuming that the velocity field has no vorticity, one can integrate the smoothed radial velocity on radial rays to get the velocity potential as a function of spatial position. The complete velocity field is the gradient of this potential. (B) Now assuming that the velocities arise due to the gravitational attraction of the matter alone (i.e. there were no residual velocities from early processes at the time of structure formation), one can deduce the matter distribution from the velocity field which has arisen through the gravitational effect of matter.

As regards (A), we have been provided with peculiar velocity data by Dave Burstein and Stephane Courteau. The total sample includes roughly 1000 galaxies, of which roughly half are put into groups or clusters in order to reduce the errors. (For a thorough discussion of peculiar velocity measurements, see Faber & Burstein (1988).) Fig. 1. shows a projected slice of the peculiar velocity sample which we employ. We consider the region in a sphere with radius 6000 km/sec centred on the Milky Way. This allows us to find the velocity field and hence the expansion Θ and so the vector $\mathcal{Z}_a$ (where, as we are assuming $\Omega = 1$, we can normalise a by $a(t_0) = 1$ without loss of generality). The weakest part of the scheme is in the estimation of the distances, a notoriously difficult problem; the errors are roughly 20% of the distance. (Also, the way the galaxies are grouped strongly affects the $\log D_n - \log \sigma$ relation. We have followed Bertschinger et al in using the original groupings; for an alternative view, see Weigert & Kates (1991).)

Fig. 2. shows a slice of the reconstructed velocity field from our version of POTENT; it can be compared with

Figure 3. $(\mathcal{Z}^a \mathcal{Z}_a)^{1/2}$ from POTENT.

figure 13 from Bertschinger et al. (1990). Fig. 3 shows $(\mathcal{Z}^a \mathcal{Z}_a)^{1/2}$ for that slice. The values near the origin have been suppressed.

As regards (B), from the velocity field, one can find the density by using the linear solutions of the continuity equation. In the nonlinear regime, Nusser et al. have found a quasi-linear approximation (Nusser, Dekel, Bertschinger & Blumenthal 1991) that recovers the density remarkably well for $-1 < \delta < 4.5$. This method is formally related to the Zeldovich approximation, which explicitly assumes that the decaying mode is zero; for as mentioned already, this deduction of matter density assumes that this (smooth) velocity field is caused by the matter field present, starting from negligible initial velocities (i.e. assuming the homogeneous solution of the equations is zero). If the decaying mode has indeed vanished, then we have the ingredients for constructing the Newtonian gauge-invariant variables, so long as the overdensity is not too large. However by construction the decaying mode will be found to be zero; for that assumption has effectively been the basis of deduction (B).

Thus it is not possible, at least in the linear regime, to compare the relative amplitudes of the decaying and growing modes by using POTENT alone, since they are not, in this approximation, linearly independent. Indeed in this analysis, in terms of the fractional overdensity

$$D_a = a(t) \frac{\delta_a}{1+\delta},$$

to first order we must obtain

$$D_a \approx -aH(t)\mathcal{Z}_a$$

because $\delta = -(\nabla \cdot v)/aH$. Half of the generality of general physical solutions has been lost.

To determine the density gradient vector independently of the velocity field, we need an alternative estimate of the density field, using POTENT to determine the velocity field as in (A) but replacing the density estimate (B) by other observations. This would allow construction of an independent $\mathcal{D}_a$. One method, described below, requires a complete redshift catalogue and estimates of the Mass to Luminosity ratio to determine the density function. The POTENT data cannot be used for this purpose, as it does not comprise a complete catalogue of the relevant part of the sky (the ideal would be a complete catalogue with both redshift and independent distance measures for each galaxy; to obtain such a catalogue would be a very costly exercise). However the IRAS data is in principle suitable for the purpose.

4 THE IRAS DATA

Instead of using POTENT to determine the density distribution, as discussed above, we have estimated $\mathcal{D}_a$ using IRAS-QDOT data. This is a complete infra-red catalogue, with redshifts measured for a large sample of objects. The fluctuation in the number density of galaxies, δ_n, is found following Saunders et al. (1991). The density is then found after assuming a value for b, the linear bias parameter (Kaiser 1987). This assumes that light traces mass in a linear way.

Figure 4. Galaxy number density from *QDOT - IRAS*. This is the supergalactic plane. The smoothing scale is 1200 km/sec.

Figure 5. $(\mathcal{D}^a\mathcal{D}_a)^{1/2}$ for $b = 1$. To correct for a linear bias, divide by b^2.

No correction for peculiar velocity has been made. The number counts are corrected for the IRAS mask using the QDOT luminosity function, and smoothed with a Gaussian window of width 1200 km/sec. $\mathcal{D}_a$ is found numerically by fitting a Chebyshev series to the grid data along each direction separately (supergalactic x,y,z) and then taking the analytic derivative of the fit. This is a noisy procedure because of the numerical derivative.

It is clear that the bias factor affects the gradients $\mathcal{D}_a$ in a critical way. We will assume a constant bias factor here, although recent developments suggest that $b \sim 1$ on very large scales ($r >\sim 100h^{-1}$ Mpc; Kaiser et al. 1991) and $b \sim 3$ on smaller scales ($r <\sim 5h^{-1}$ Mpc; Salucci, Persic & Borgani 1993). However this should not affect us as we are using one smoothing scale; the assumption is that the bias factor should be (spatially) constant on that scale. In the linear theory, one finds not b but the combination $b\Omega^{0.6}$; this is just equivalent to a renormalisation of the bias value.

Fig. 4 shows the number density from QDOT, which is the number density of (mostly spiral) galaxies, in the Supergalactic plane. Fig. 5 shows $(\mathcal{D}^a\mathcal{D}_a)^{1/2}$ for the QDOT slice, for $b = 1$.

A number of technical points arise in the analysis: (a) The distance to each galaxy is assumed to be equal to the redshift in the Local Group frame. There is error involved here but we smooth over large scales.
(b) The number density is found on a grid $25 \times 25 \times 25$ by weighting each galaxy (of 2181) in the sample by the inverse of the QDOT luminosity function (Saunders et al. 1991) and smoothing with a Gaussian window of width 1200 km/sec. The QDOT mask (sections of the sky where no data is available from IRAS due to "incomplete satellite coverage, source confusion, or redshift incompleteness") is taken into account.
(c) The real density will be displaced due to the effects of bulk motions (see Kaiser 1987) So far no correction has been made for this. We do not expect the effect to be large.
(d) The POTENT group compared its results with the 1.936 Jy IRAS results (Strauss et al. 1992) by selecting a random subsample of galaxies and running them through the POTENT machine (Dekel et al. 1992). To do this, they estimated the peculiar gravitational potential due to the galaxy distribution, and then inferred peculiar velocities for the random subsample (it's an iterative procedure). To get these velocities, they assumed the decaying mode is gone. The broad agreement obtained shows it is not outrageous to use the different samples to get the density and the velocity field, as we do here (it would of course be preferable to have one data set that gave all the needed information, as mentioned above; such data is obtainable in principle, but is not at present available).

Figure 6. $|c_-|/|c_+|$ for $b = 1$ and $b = 2$.

5 THE ISSUE OF GROWING AND DECAYING MODES

Now that $\mathcal{D}$ and $\mathcal{Z}$ have been independently constructed, one can use equations (9) and (4) to estimate the amplitude of the decaying mode. Properly scaled (see the appendix), the squared magnitudes are

$$c_-^a c_{-a} = (\frac{2}{5})^2(\mathcal{D}^a\mathcal{D}_a + \mathcal{Z}^a\mathcal{Z}_a + 2\mathcal{D}^a\mathcal{Z}_a); \tag{10}$$

$$c_+^a c_{+a} = (\frac{3}{5})^2\mathcal{D}^a\mathcal{D}_a + (\frac{2}{5})^2\mathcal{Z}^a\mathcal{Z}_a - \frac{12}{25}\mathcal{D}^a\mathcal{Z}_a. \tag{11}$$

Fig. 6 shows the ratio of the magnitudes of c_{-a} and c_{+a}. The ratio is quite sensitive to the value of b. The fundamental point is that we obtain a non-zero value for the decaying mode. What is not clear is the errors involved: is this non-zero value equivalent to zero, on taking the observational errors into account? This issue is crucial.

If one runs the linear evolution equations backwards in time, one finds that any residual decaying mode anywhere carries with it the implication of enormous inhomogeneity at the last scattering surface (Nusser & Dekel 1992), on using the linear analysis to propagate conditions back in time from the observed data to the last scattering surface (see equation (6)). A non-linear analysis would lead to the estimation of greater inhomogeneity at last scattering due to these modes, than estimated from the linear analysis (we are running the equations backwards in time: thus the non-linear gravitational tendency to generate greater inhomogeneity than estimated by linear theory from the initial data we are using, is operative in that direction of time, the equations of course being time symmetric). It is well known that when the fractional overdensity enters the nonlinear regime, the linear theory will underestimate it (Yahil 1988). The decaying mode, run *backwards* in time, will exhibit the same behaviour, i.e. if there is any non-zero amplitude in this mode today, then we obtain a lower estimate of the amplitude at an earlier time on using linear theory. In this sense, it is essential that the decaying mode be strictly zero today, if one is to interpret for example the *COBE* data (Smoot et al. 1992) in the standard way as showing the last scattering surface was very smooth.

Now there are some problems with the analysis above leading to a non-zero estimate of the decaying mode. In particular, as implied above, the use of two data sets rather than one increases the number of hidden biases. The POTENT sample includes a sampling gradient bias which can be reduced by varying the size of the smoothing window but not eliminated; the QDOT sample is biased towards spiral galaxies, giving perhaps less weight to structures rich in elliptical galaxies. The linear bias factor should be modified to include spatial variation. Nevertheless, it is at present impossible to disentangle these errors and biases from the evidence for a decaying mode. The critical issue is to estimate realistically the errors in the data and in the resultant estimation of the density gradient and velocity gradient fields; only this will establish if the value determined should really be taken as an indication of a non-zero decaying mode, or not. One of the interesting issues is to see how stable other structures in-

Figure 7. The Wilcoxon-Mann-Whitney parameter for the samples D^2 and Z^2, varying b

ferred from similar analyses – such as the 'Great Attractor' seen in Figure 2 – are when we allow realistic error estimates, such as may be required to set the decaying mode to zero.

One other possibility to keep in mind is that power is put into this mode via "non-gravitational" processes, e.g. large scale explosions (Ostriker & Cowie 1981) or wakes from relativistic cosmic strings (Vachaspati 1986), at various times t_*, so that the underlying assumption of (6) that the growth is purely due to gravitational effects is revised. If the evidence favours a non-zero mode after rigorous error estimates, and one finds a nonzero decaying mode too onerous despite such alternative sources of power for this mode, then an analysis similar to that given here could perhaps be used to constrain and improve the assumptions leading to the conclusion (implying a need to revise distance indicators, for example, or providing evidence for biased galaxy formation with an effectively spatially varying bias factor).

The alternative procedure is to insist that the decaying mode is zero and use that assumption to constrain the linear bias factor. We take $\mathcal{Z}$ from POTENT and $\mathcal{D}$ from QDOT. Then we apply the non-parametric Wilcoxon-Mann-Whitney test to the samples $\mathcal{D}^2$ and $\mathcal{Z}^2$, varying b. This test produces a combined order statistic by ranking all the values in the combined sample and then comparing the sums of the ranks of each sample (Lloyd 1986). If these sums are very different, the samples are probably not the same. If the statistic $z = (R_1 - \rangle R_1 \langle)/\mathrm{var}(R_1)$ has magnitude greater than 2, where R_1 is the sum of the ranks from one of the two samples, and $\langle R \rangle$ is the expectation value, then the probability is less than 95%. Fig. 7 shows the graph of z vs. b. For comparison, the POTENT-IRAS collaboration finds $b_{IRAS} = 0.7^{+0.6}_{-0.2}$ (Dekel et al. 1992), which is not inconsistent with Fig. 7 (that figure does not strongly indicate a particular value of b, rather suggesting broad ranges that are compatible with the observations).

6 FURTHER DEVELOPMENT

The first deirable further development would be to proceed with the present analysis but better data (as that becomes available), trying to run the argument both ways (as in the present article).

Secondly, it would be interesting to extend this work to much larger scales, which should still be in the linear regime. A direct comparison with the observed fluctuations in the CMB would in principle be possible. Unfortunately, the redshift-independent distance measures used to find the peculiar velocities of the test galaxies cannot be extended very far, and since this analysis depends on independent velocity and density information, different methods of obtaining peculiar velocities on larger scales may be required.

In the long term, gauge-invariant variables should be used to numerically examine the evolution of inhomogeneities in nearly FRW spacetimes, in the relativistic domain. Fully general relativistic numerical approaches (Bishop 1993) can then be based on null cone data, when integrating back into the past. In this way it should be possible to consider realistically observable data on the null cone and extrapolate backwards.

The present paper can be taken as testing this approach in the Newtonian (linear and non-linear) regimes. The extension required is to the relativistic, non-linear regime. In using this approach one must recognise the problems of completeness and bias in data analysis (taking seriously selection effects and the existence of dark matter); nevertheless one can insist on attempting to test what is really there rather than relying on theoretical prejudice, at least where it is possible to test such prejudice.

7 CONCLUSIONS

Using methods similar to those of Nusser & Dekel (1992), but estimating the density variation independently of the velocity field, we have found values for Newtonian gauge-invariant density perturbation variables (Ellis 1990) in a region in a sphere 6000 km/sec centred on the Milky Way. From these, we can not rule out the possibility of a non-zero decaying mode amplitude at present; indeed taking the data at face value, we find there is such a mode present. An important feature of our analysis is that it decouples some of the series of assumptions usually made, indeed it is usually assumed that there is a negligible decaying mode. However we still have had to follow the POTENT analysis in assuming zero vorticity, which is certainly something one would like to test observationally rather than taking for granted.

If there is in reality a non-zero decaying mode present, this implies problems for the standard picture of growth of inhomogeneity in the universe. It is not possible to draw definite conclusions from the data without reliable error estimates; these are not yet available. Nevertheless the analysis above shows how in principle important assumptions underlying our present viewpoint are amenable to checking. In due course it should be possible, by using similar methods on more extensive data, either to verify that the standard view – that the decaying mode is essentially zero today – is acceptably within the error limits; or to show it is untenable. This procedure depends on an assumed value for the bias parameter; one can alternatively run the procedure the other way, assuming a zero decaying mode and determining a value for the bias parameter. Perhaps the best view is that one is testing the series of assumptions as a whole, including the value of the bias parameter; our point then is that one can explicitly formulate the analysis so as to test the hypothesis that the decaying mode is zero, and this is a fundamental feature which should indeed be checked.

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Appendix: Dimensionless scaling relations for $\mathcal{D}$ and $\mathcal{Z}$

We define a dimensionless time $\bar{t} = t/t_0$ where the subscript "0" denotes the present. Then $a(\bar{t}) = k\bar{t}^{2/3}$ is the dimensionless expansion and $\overline{H} = 2/3\bar{t}$ is the dimensionless Hubble parameter. The gauge-invariant variables $\mathcal{D}$ and $\mathcal{Z}$ have dimensions L^{-1} and $(LT)^{-1}$, respectively. We choose a typical length scale x_0 and then take the present Hubble parameter H_0 and then construct dimensionless variables,

$$\overline{\mathcal{D}}_a = \bar{c}_{+a}\bar{t}^{2/3} + \bar{c}_{-a}\bar{t}^{-1} \tag{A1}$$

$$\overline{\mathcal{Z}}_a = -\bar{c}_{+a}\bar{t}^{-1/3} + \frac{3}{2}\bar{c}_{-a}\bar{t}^{-2} \tag{A2}$$

where $\bar{c} = x_0 c$, $\overline{\mathcal{D}} = x_0\mathcal{D}$, and $\overline{\mathcal{Z}} = (x_0/H_0)\mathcal{Z}$. At the present, $\bar{t} = 1$, and we have

$$\bar{c}_{-a} = 2/5\left(\overline{\mathcal{D}}_a + \overline{\mathcal{Z}}_a\right); \tag{A3}$$

$$\bar{c}_{+a} = 3/5\,\overline{\mathcal{D}}_a - 2/5\,\overline{\mathcal{Z}}_a. \tag{A4}$$

If $v = 0$ then $\mathcal{Z} = 0$ and one finds $\mathbf{c}_-/\mathbf{c}_+ = 2/3$, in agreement with Peebles (page 68, 1980).

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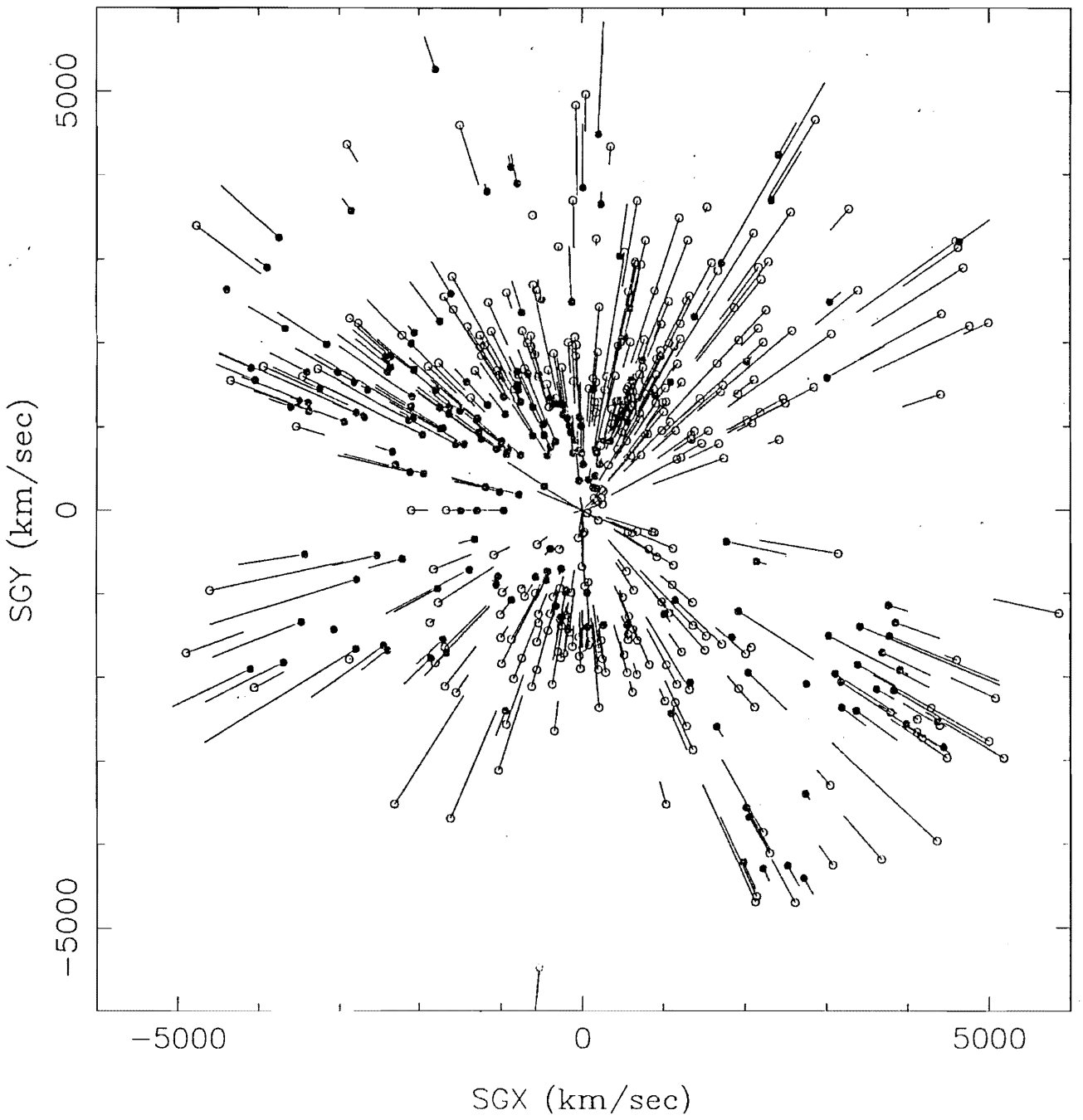


Fig 1

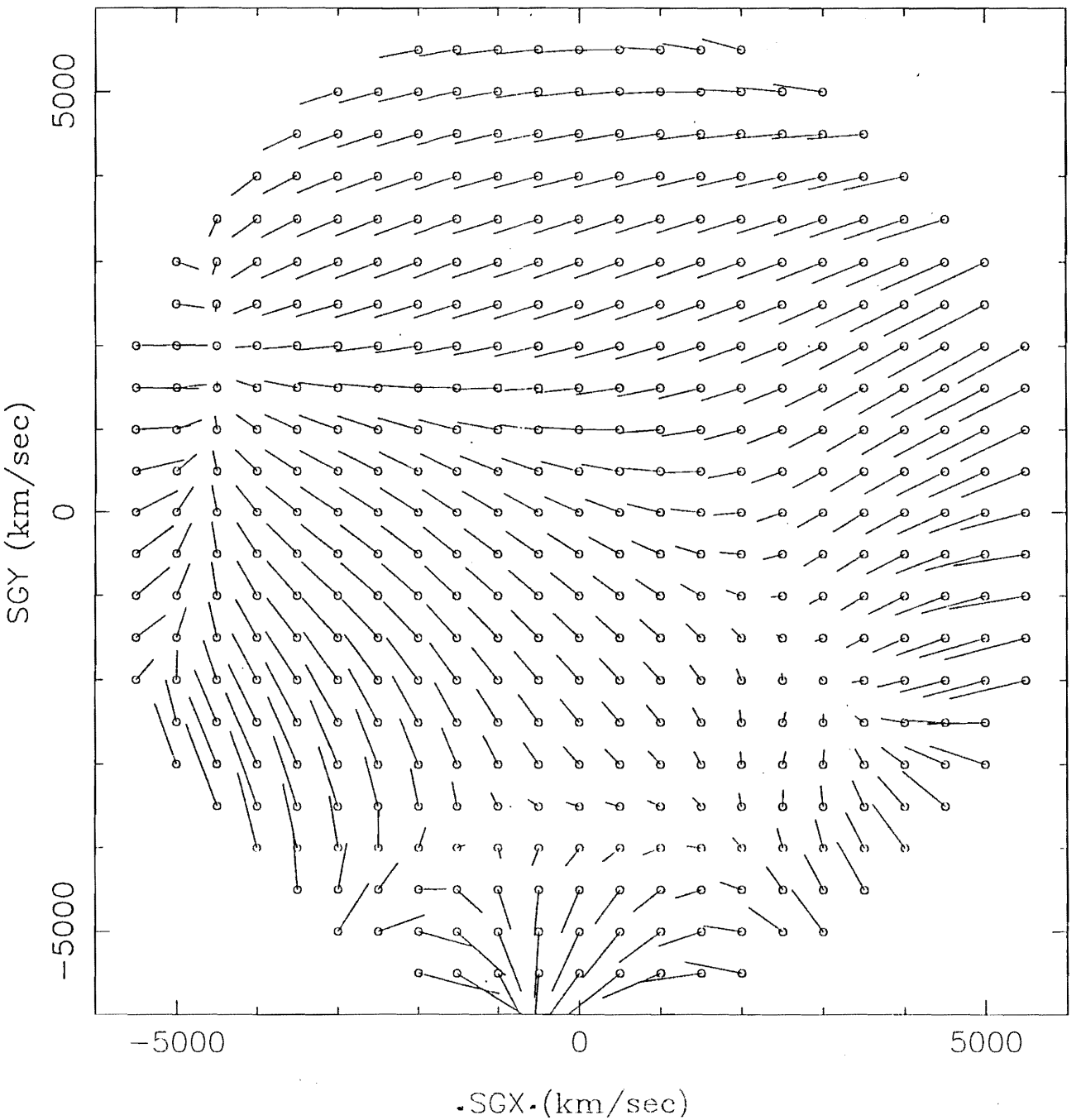


Fig. 2

Fig. 3

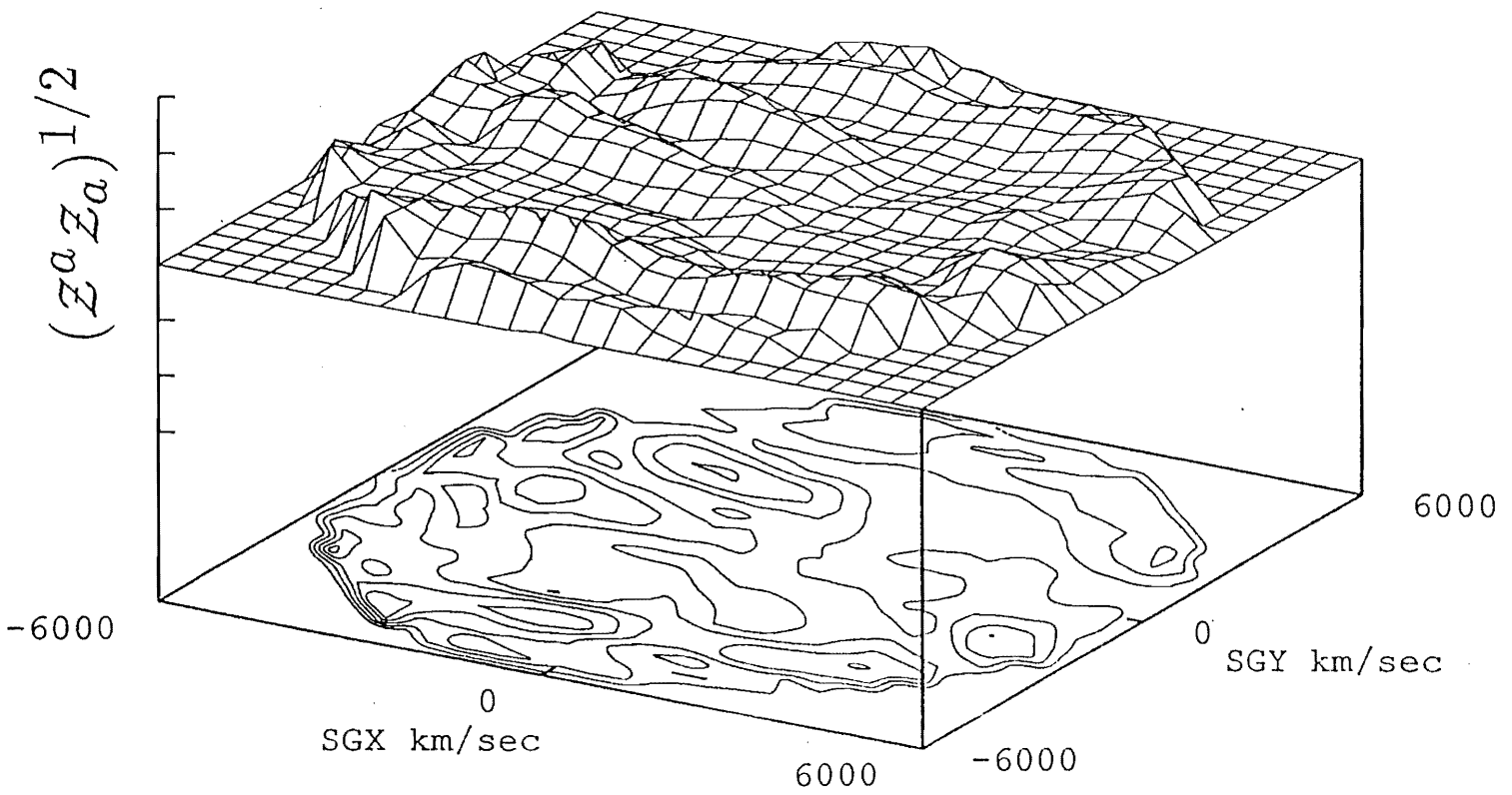


Fig 4

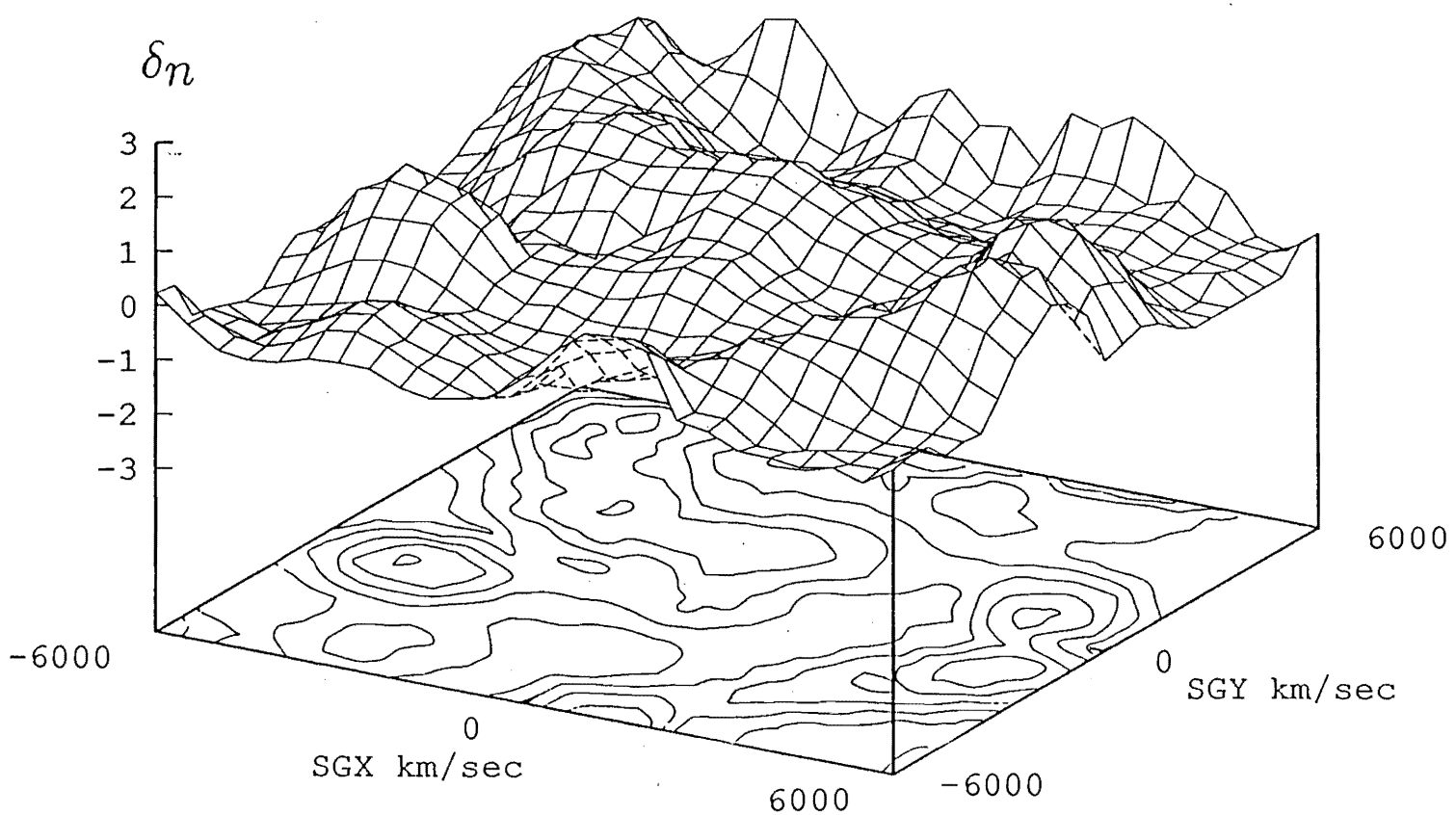


Fig 5

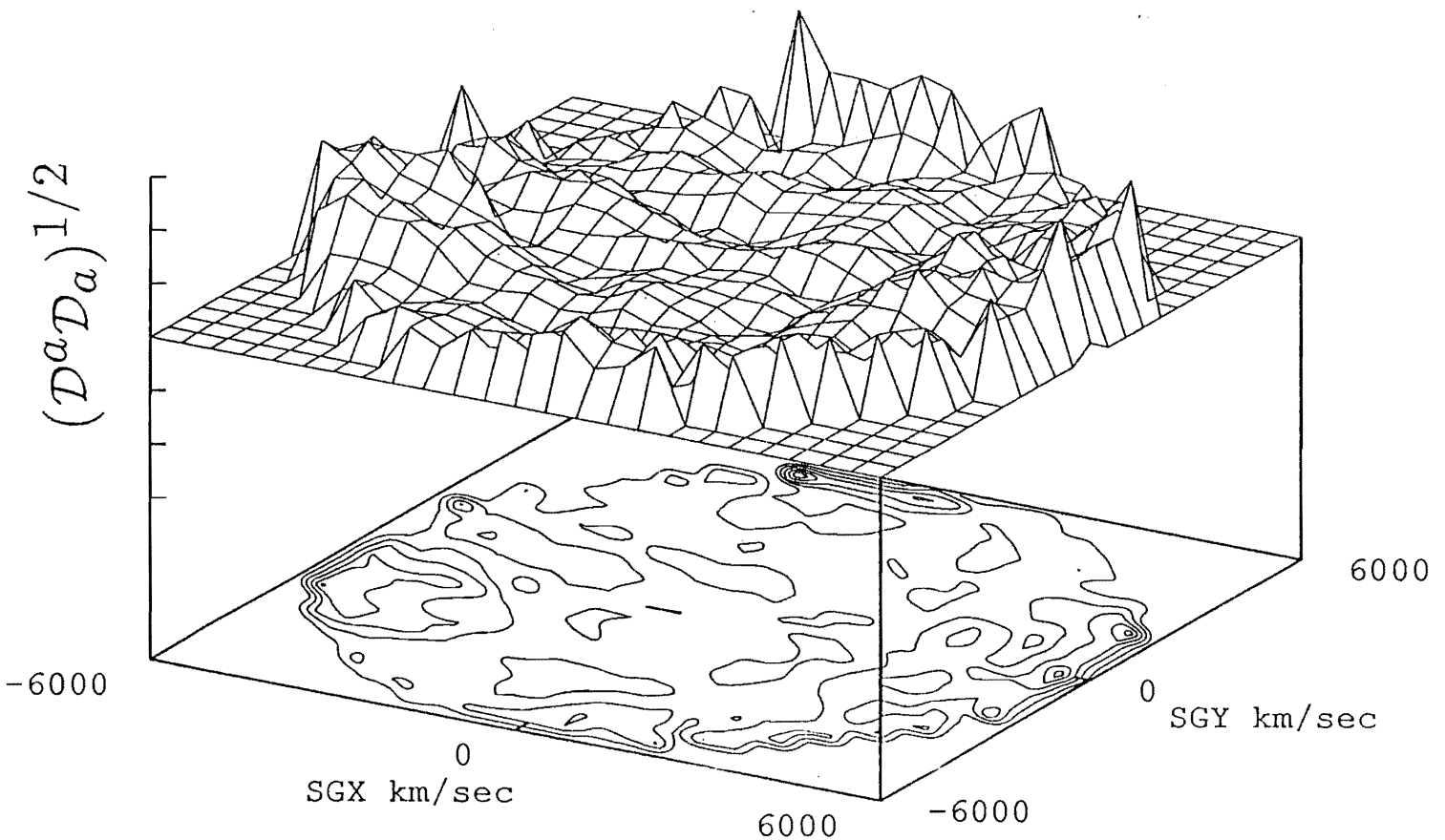

$(\mathcal{D}^a\mathcal{D}_a)^{1/2}$
-6000
0
SGX km/sec
6000
-6000
0
SGY km/sec
6000

Fig 6a

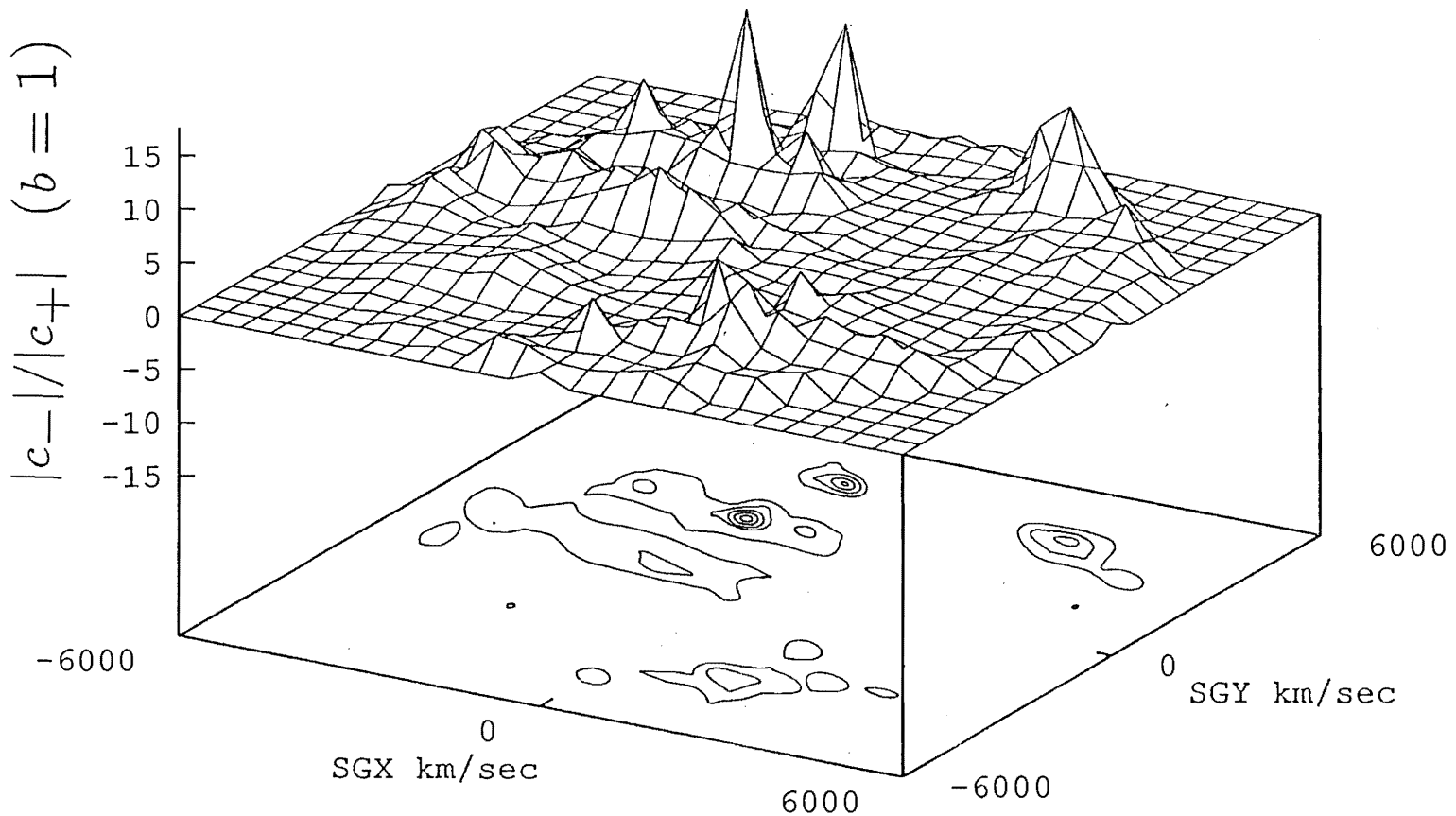


Fig 6 b

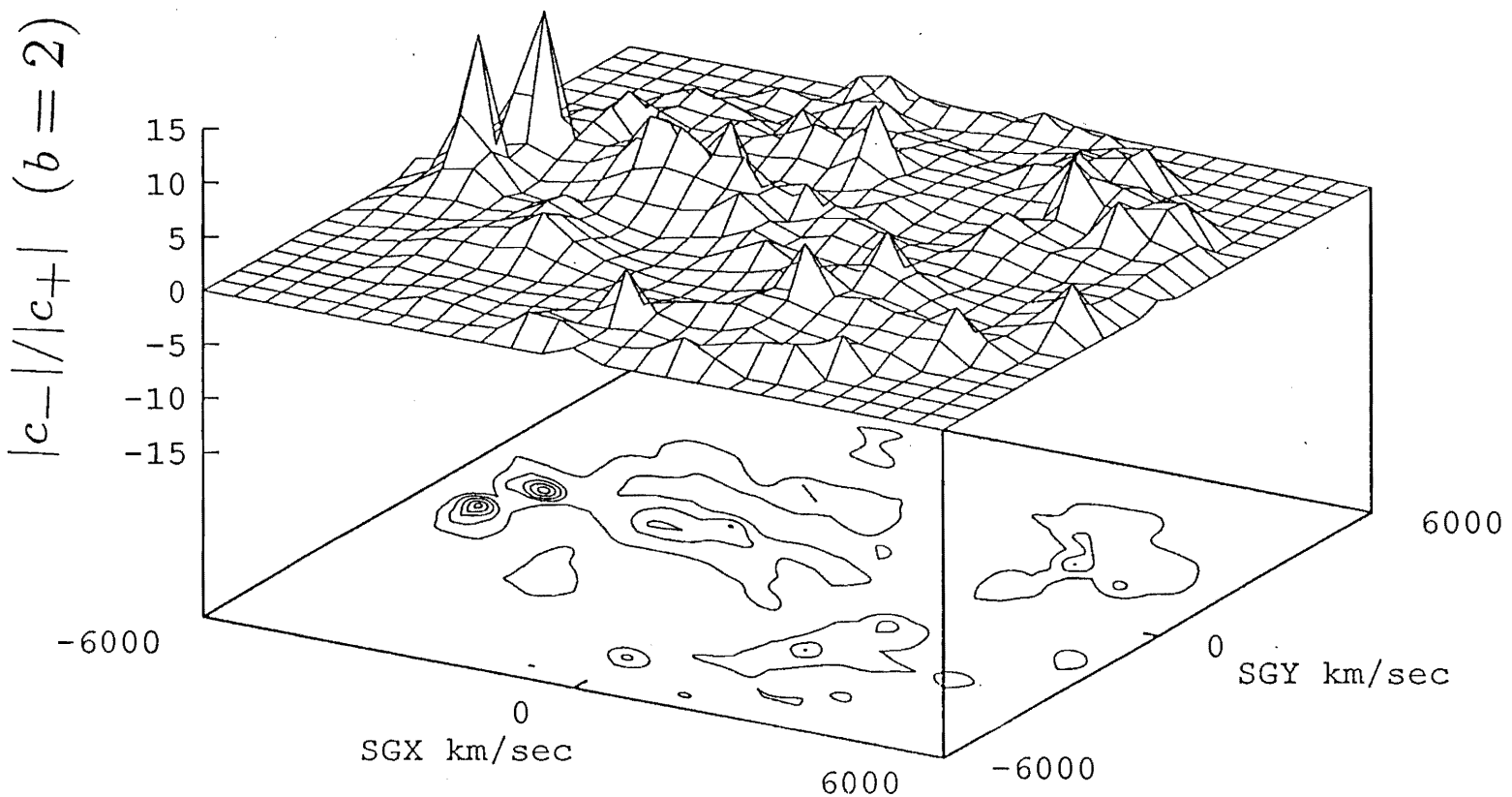
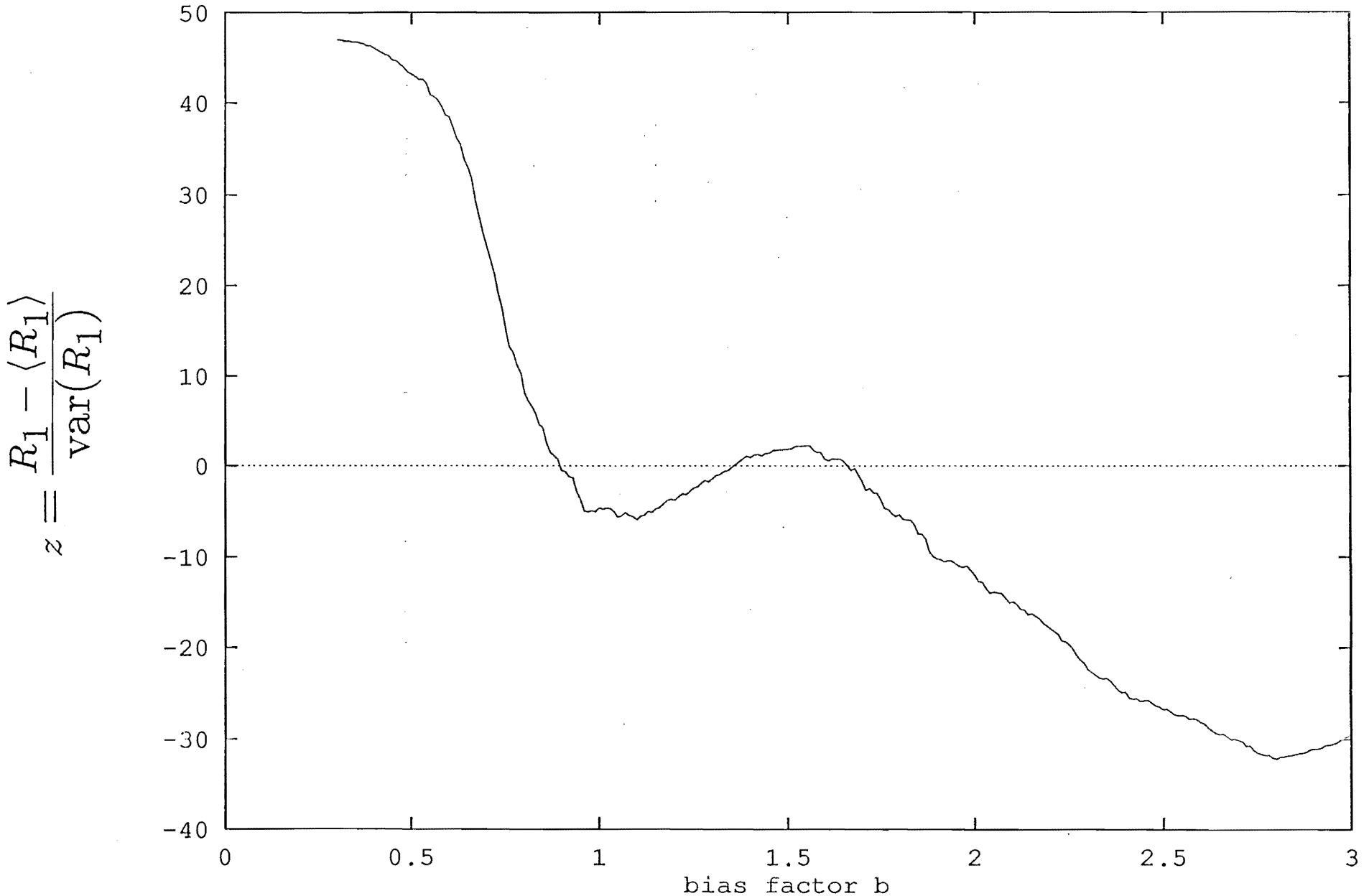


Fig 7


$z = \frac{R_1 - \langle R_1 \rangle}{\mathrm{var}(R_1)}$
50
40
30
20
10
0
-10
-20
-30
-40
0
0.5
1
1.5
2
2.5
3
bias factor b