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F.Matteucci

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The chemical evolution of galaxies

F. Matteucci

Dipartimento di Astronomia, Università di Trieste and SISSA/ISAS, Via Beirut 2-4, I-34014 Trieste, Italy

Abstract. The basic principles and the main recipes for constructing models of galactic chemical evolution are critically reviewed together with the most common adopted models. The importance of using abundance ratios as cosmic clocks and to infer the past history of galaxies is discussed. The importance of knowing stellar evolution and nucleosynthesis as fundamental tools to understand the evolution of galaxies and ultimately the evolution of the universe is emphasized.

1. Introduction

In 1980 Beatrice Tinsley wrote a fundamental review containing all the basic principles of the chemical evolution of galaxies. Tinsley showed that galaxy evolution, in particular photometric evolution, can be used to infer the deceleration parameter q_o, since it allows us to compute the evolutionary effects in the magnitude-redshift relation for galaxies.

The study of galactic chemical evolution is an attempt to reconstruct the history of the chemical composition of gas in galaxies through consideration of the processes of galaxy formation, star formation, stellar evolution, nucleosynthesis and possible gas flows. In the past recent years a great deal of work in the field of galactic chemical evolution has appeared, as indicated by many reviews on the subject (Tayler 1990, Matteucci 1991, Rana 1991, Wilson and Matteucci 1992, Pagel 1995, Shore and Ferrini 1995, Matteucci 1996).

In this series of lectures I will concentrate the attention on the main ingredients necessary to built models of galactic chemical evolution and discuss them critically. Analytical and numerical models of galactic chemical evolution will be also presented and finally a comparison of model results and observational constraints for our own Galaxy will be presented. The evolution of external galaxies will be only briefly mentioned.

The aim of these lectures is to emphasize the importance of the stellar physics in order to understand galaxy evolution and cosmology. The chemical abundances, in fact, can represent a very important tool to impose constraints both on nucleosynthesis and galaxy formation mechanisms. In particular, abundance ratios which do not depend on many model assumptions but mostly on nucleosynthesis and stellar initial mass function can be used as cosmic clocks, thus providing extremely important information on the time scale of formation of galaxies.

The basic parameters intervening in the chemical evolution of galaxies are:

- Initial conditions:

  the model can be either closed (all the gas already present at the beginning and constant total mass during the galactic lifetime) or open (slow accretion of matter and/or galactic winds); the gas which forms galaxies can be pure gas from Big Bang or pre-enriched gas by a pre-galactic stellar generation.

- Stellar evolution and nucleosynthesis.

- Birthrate function:

$$B(m,t) = \psi(t)\varphi(m) \tag{1}$$

  where $\psi(t)$ is the star formation rate (SFR) and $\varphi(m)$ is the initial mass function (IMF)

- Supplementary parameters:

  a) infall

  b) radial inflow

  c) galactic wind

The lectures are organized as follows: in section 2, 3 and 4 we will discuss the basic parameters of galactic chemical evolution, then in section 5 we will discuss both analytical and numerical models. In section 6 the main observational constraints for the Galaxy will be presented. In section 7 the chemical evolution of the Galaxy will be discussed and finally, in section 8, the chemical evolution of external galaxies will be briefly reviewed.

2. Stellar evolution and nucleosynthesis

The amount of material nuclearly processed inside the stars and subsequently ejected in the form of a specific chemical species into the interstellar medium (ISM) relative to the amount of material which remains locked up in low mass stars and remnants is usually called "yield" of that specific element (Tinsley, 1980 and section 5.1) The yields of the chemical elements depend on a few fundamental parameters:

- M_α, the mass of the He-core (where H is turned into He)

- M_{CO}, the mass of the C-O core (where He is turned into heavier elements)

- M_R, the remnant mass

- $(M_{ej})_i$, the mass ejected in the form of an element i

It is worth noting that in order to calculate the chemical evolution of the ISM we need also the mass ejected in the form of an element i without being processed, as we will see in section 5.1.

2.1. The evolution of single stars

We will remind here the stellar evolution results which are relevant to the calculation of the stellar yields, in particular the various contributions to the chemical enrichment of the ISM from stars of different masses.

- $M \leq 0.08\ M_\odot$, if they exist, will never ignite H. They are called **brown dwarfs**. They do not contribute to galactic enrichment except for locking up material. Their lifetimes, τ_m, correspond to several Hubble times.

- $0.08 \leq M/M_\odot < 0.5$, they ignite H in the center but their He-core becomes electron degenerate before getting hot enough to burn. They will never ignite He and will end up as He-white dwarfs. They also contribute only to lock up material, their $\tau_m >> 15 \cdot 10^{10}$ years.

- $0.5 < M/M_\odot < M_{HeF}$, ignite He in an electron degenerate core (He-flash) and end their lives as C-O white dwarfs. The precise value of M_{HeF} depends on the treatment of convection: the classic (no mass loss, no overshooting) value is $\simeq 2.2\ M_\odot$, with convective overshooting is lower, $\sim 1.85\ M_\odot$ (Maeder and Meynet, 1989). Only for $M \geq 1\ M_\odot$ these stars contribute to galactic enrichment in ^4He, ^{14}N and s-process elements such as Ba and Sr. Their τ_m go from $>> 15 \cdot 10^{10}$ to $\sim 10^9$ years.

- $M_{HeF} < M < M_w$ (M_w is the limiting mass for the formation of a C-O white dwarf). They ignite He in a He-non-degenerate core but develop an electron degenerate C-O core. They end their lives as C-O white dwarfs and contribute to ^4He, ^{12}C, ^{13}C, ^{14}N, ^{17}O and s-process elements.

- $M_w < M < M_{up}$ (M_{up} is the limiting mass for the formation of an electron degenerate C-O core or the minimum initial mass for an off-center C-ignition proceeding up to central C-exhaustion). Depending on whether $M_w \sim M_{up}$ (mass loss and/or overshooting) these stars should ignite C in a degenerate core when their mass reaches the Chandrasekhar limit ($\sim 1.4\ M_\odot$) with subsequent explosion (type I1/2 supernovae?). Empirical estimates give $M_w \sim 8\ M_\odot$ (Weidemann 1987, Koester and Reimers 1988). Theoretical estimates of M_{up} from models with mass loss and overshooting give $M_{up} \sim 6.6\ M_\odot$ (Maeder and Meynet, 1989) and $M_{up} = 4 - 5\ M_\odot$ (Marigo et al. 1997) against the classic value of $8 - 9\ M_\odot$. This means that type I1/2 supernovae (SNe) are very unlikely. The lifetimes of stars in this mass range go from $\tau_m \sim 10^9$ to several 10^7 years.

- Stars in the mass range $M_{HeF} - M_{up}$ eject their nuclearly processed material into the ISM during the asymptotic giant branch phase (AGB) by means of quiescent mass loss and during the planetary nebula phase (PNe). Episodes of dredge-up in conjunction with thermal pulses bring to the surface freshly synthetized material (H and He-burning products). Envelope burning will then transform this fresh material, e.g. ^{12}C, into ^{14}N and ^{13}C. These two elements in this case have a *primary* origin (Renzini and

Voli, 1981). A chemical element is called *primary* if it is synthesized directly from H and He, whereas is called *secondary* if it is synthesized from metals originally present in the stars and not produced "in situ". Stars in this mass range, if in binary systems, can give rise to type Ia SNe and be responsible for substantial iron production.

- $M_{up} < M/M_{\odot} < 10 - 12$, these stars ignite C non-degenerately and those which have He-cores between 2.2 and 2.5 $M_{\odot}$ ignite O in a degenerate Ne-O core. For those with He-cores from 2.5 to 3.0 $M_{\odot}$ all six burning stages, H,He,C,Ne,O and Si are ignited non-degenerately and an Fe-core in hydrostatic equilibrium is eventually formed. The Main Sequence (MS) masses of these stars in classic models are 8-12 $M_{\odot}$, whereas in models with overshooting are 6.6-10 $M_{\odot}$. All of these stars end their lives as SNe of type II; they contribute to galactic enrichment in ^4He and only very little in heavier elements. Their lifetimes are of the order of several times 10^7 years.

- $10 - 12 < M/M_{\odot} < M_{SNII}$, they are responsible for producing the bulk of heavy elements such as ^{16}O, ^{20}Ne, ^{24}Mg, ^{28}Si, ^{32}S, ^{40}Ca..... and r-process elements. They end their lives as type II SNe. When they die are still in possession of their H-rich envelopes. M_{SNII} is the limiting mass for the formation of Wolf-Rayet stars ($M_{WR} \geq 40$ $M_{\odot}$ although depending on Z, Maeder 1992) Type II SNe leave a neutron star or a black hole after explosion. The explosion mechanism in massive stars has not yet been well understood, so it is highly uncertain how the shock wave forms and propagates through the outer layers.

  The most recent and complete compilation of metallicity dependent yields for massive stars ($M > 10$ $M_{\odot}$) can be found in Maeder (1992), Langer and Henkel (1996) and Woosley and Weaver (1995). The effect of mass loss in massive stars on the stellar yields has been investigated by Maeder(1992), Langer and Henkel (1996) and Woosley et al. (1993,1995).

- $M > M_{SNII}$ these stars are Wolf-Rayet stars, characterized by a strong mass loss. They will eventually explode but lacking their H-envelope their light curves and spectra are different from those of type II SNe. They are possibly the progenitors of type Ib SNe although recently other candidates such as massive stars ($M > 10$ $M_{\odot}$) in close binary systems have been proposed (Nomoto et al. 1995). Wolf-Rayet stars contribute to ^4He, ^{12}C, ^{22}Ne, ^{14}N, (^{18}O ?) through stellar winds. Their lifetimes are $\tau_m \sim 10^6$ years.

- $M > 100$ $M_{\odot}$, following the exhaustion of He the central regions contract and the star goes directly to O-burning. For a certain value of the He-core, ^{16}O is not ignited in a stable fashion. This instability is known as "pair instability". It occurs because a large portion of the energy from gravitational contraction goes, at $T \sim 2 \cdot 10^9$ K, into creation of pairs (e^+, e^-). This reduces Γ below 4/3. This kind of explosion destroys completely the star. These stars, if they exist, contribute mostly to the enrichment in

oxygen. Models with overshooting and mass loss give a range of initial masses for these SNe of 100-200 $M_\odot$. Their lifetimes $\tau_m \leq 10^6$ years.

- **Supermassive objects** are those which collapse directly to black holes or suffer total disruption due to explosive H-burning. The minimum initial mass for these objects is $M \sim 410\ M_\odot$ (Appenzeller and Tscharnuter, 1973). Masses $M > 7.5 \cdot 10^5\ M_\odot$ would end up as massive black holes (Appenzeller and Fricke, 1972). Stars in the range $410 < M/M_\odot < 7.5 \cdot 10^5$ suffer total disruption during H-burning but the results are very sensitive to the initial stellar metal content. In particular, the explosion is possible only for solar metal content. They produce mostly ^4He and traces of ^{15}N and ^7Li (Woosley et al. 1984).

2.2. The evolution of binary systems

Binary systems are important since they can give rise to supernova explosions producing a relevant amount of heavy elements, in particular to SNe of type Ia.

C-deflagration in C-O white dwarfs (WDs) whose mass exceeds the Chandrasekhar limit can reproduce the majority of observed features in type Ia SNe, since produces the right amount of $^{56}Ni \rightarrow^{56} Co \rightarrow^{56} Fe$ to power their light curve and allows for the formation of intermediate mass elements (from C to Si) observed in type Ia SN spectra. In particular, model W7 from Nomoto et al. (1984) predicts 0.86 $M_\odot$ of Fe-group elements, 0.58 $M_\odot$ are initially in the form of ^{56}Ni and 0.27 $M_\odot$ of Si-Ca elements. White dwarfs as progenitors can also account for the presence of type Ia SNe in ellipticals.

We will discuss now some particular models leading to supernovae of type Ia:

- **A degenerate C-O WD + a red giant companion** filling its Roche-lobe (Whelan and Iben, 1973). A C-deflagration occurs when the WD reaches the Chandrasekhar limit due to accretion of H-rich material. The expected realization frequency, considering the limits on the initial WD mass, primordial separation, mass ratio is $\sim 0.003 SNyr^{-1}$ (Iben and Tutukov 1984) which is marginally consistent with the observed rate in the Galaxy. van den Bergh (1988) estimated the type Ia SN rate in or Galaxy to be in the range 0.0035-0.014 $SNyr^{-1}$, depending on the Hubble constant. The clock for the explosion in this scenario is given by the lifetime of the red giant which is in the range 0.8-$M_{up}(M_\odot)$.

- **Two degenerate C-O WDs** merging after gravitational wave emission, the so called "double degenerate model" (Iben and Tutukov 1984, 1985). The explosion mechanism is again C-deflagration. The clock for the explosion is given by the lifetime of the primordially less massive component (secondary) plus the gravitational time delay which can vary from 10^8 years to several Hubble times. The masses of the two components of such a binary system are assumed to be almost the same and in the mass range $5 - M_{up}(M_\odot)$.

- **Sub-Chandrasekhar WD in binary systems**

The fact that a sub-Chandrasekhar mass can still give rise to a SN explosion is only a recent theoretical development (Limongi and Tornambè 1991, Woosley and Weaver 1994, Arnett 1997). There is, however, still a general agreement on the fact that the explosion mechanism should be the C-deflagration.

2.3. Observational features of SNe

In general, type I SNe differ from type II for the absence of H lines, indicating that these SNe have lost their H envelope at the time of explosion or they originate from compact objects. They are divided in type Ia, Ib and Ic.

Type Ibc SNe have a luminosity at the maximum lower by 1.5 to 2.0 magnitudes with respect to type Ia. Type Ia, Ib and Ic show an exponential decay in their light curves which could be explained by the $^{56}Ni \rightarrow ^{56}Co \rightarrow ^{56}Fe$ decay, but type Ib and Ic require a smaller amount of Ni than type Ia. Type Ic SNe, in turn, differ from type Ib and type Ia since they show weak Si and He lines.

In general, only SNe of type Ia occur in elliptical galaxies whereas SNe of type Ia, Ibc and II are all observed to occur in spiral galaxies. In early-type spirals the frequency of SNe Ia per unit luminosity is higher than that of SNe II whereas SNe II have a higher specific frequency in late-type spirals than do SNe Ia. SNe II are found to be more concentrated to spiral arms than are SNe Ia (Maza and van den Bergh 1976). The degree to which SNe Ibc and SNII are associated with HII regions is similar. This suggests that both SNe Ibc and SNe II have progenitors in the same mass range. Originally, it was suggested that Wolf-Rayet stars ($M_{WR} \geq 40\ M_{\odot}$) are the progenitors of type Ib SNe (Gaskell et al. 1986, Filippenko and Sargent 1986). Recently, it has been suggested that both type Ib and Ic originate from the core collapse of massive ($\geq 10\ M_{\odot}$) stars in close binary systems (Nomoto et al. 1995).

All of these informations lead to suggest that, as we have discussed already, normal SNe II are thought to originate from core collapse of single massive stars ($M > 10\ M_{\odot}$), whereas SNe IIb, II-L and IIn (peculiar type II) have probably the same progenitors as type Ibc SNe. The supernova rates for the Galaxy, as suggested by van den Bergh (1988), are: type II $\sim 2.2\ h^2 century^{-1}$, type Ib $\sim 0.8\ h^2 century^{-1}$ and type Ia $\sim 0.6\ h^2 century^{-1}$ ($h = H_o/100 Kmsec^{-1} Mpc^{-1}$). The supernova rates in elliptical galaxies are: SNIa $\sim (0.25 \rightarrow 0.44) h^2$SNu (SNe $century^{-1} 10^{-10} L_{B_{\odot}}$) (Turatto et al. 1994)

2.4. The influence of mass loss and overshooting on the yields

The mass M_{α} (mass of the He-core) and M_{C-O} (mass of the C-O core) are mostly controlled by mass loss (stellar winds) and overshooting from convective cores.

Mass loss

It is still controversial whether the evolution of massive stars with mass loss and no overshooting leads to smaller or larger M_{α} masses than in the conservative case (no mass loss, no overshooting). Chiosi et al. (1978,1979) found a smaller M_{α} as a consequence of mass loss. Maeder (1981,1983) did not find any sensitive difference with respect to conservative models. Bressan et al. (1981), including mass loss and overshooting in their stellar models, found that due to

the competing effects of mass loss and overshooting, M_α was again like in the conservative models. Maeder and Meynet (1987,1989) recomputed models with mass loss and moderate overshooting and found that M_α is larger than in the conservative case. More recently, Maeder (1992) showed that mass loss depends on metallicity and large mass loss rates imply a lower production of metals and a larger production of He.

The effect of mass loss in low and intermediate mass stars is that the stellar yields increase with decreasing mass loss rate, since the lifetime of AGB is longer and thus more thermal pulses are occurring, as recently shown by van den Hoeck and Groenewegen (1997).

Convective overshooting

Usually the extension of a stellar convective core is set at the layer where the acceleration of the fluid elements is zero. However, zero acceleration point does not coincide with the zero velocity point which should be the real limit of the convective core. Consideration of this effect leads to overshooting (see Chiosi 1987). Most of the formulations for overshooting are based on the mixing-length theory and use the mean free path of convective elements expressed as $\alpha_{over} = 1/H_p$ (H_p is the pressure scale height and is a free parameter). The net effect of convective overshooting in stars of all masses is to increase M_α and M_{CO} with respect to the case with no overshooting. The increase of M_α and M_{CO} in massive stars leads to an increased production of heavy elements, whereas in intermediate mass stars ($M > M_{HeF}$) the situation is more complicated. Serrano (1986) pointed out that the effect of overshooting in intermediate mass stars results in a reduction of primary ^{14}N and ^{13}C as well as ^{4}He produced during the envelope burning. This is due to the bigger core mass in presence of overshooting for the same final mass which leads to a smaller mass, in the envelope, available for nuclear processing during the third dredge-up and envelope burning.

The effect of overshooting is also to decrease the value of M_{up} due to the increased core mass for the same initial mass. Recent nucleosynthesis calculations for intermediate mass stars ($0.7 - 4.0\ M_\odot$) in presence of strong overshooting and for two different stellar metallicities are from Marigo et al. (1997). They found a value of $M_{up} \sim 4 - 5\ M_\odot$. van den Hoeck and Groenewegen (1997) also calculated new yields for low and intermediate mass stars ($0.8 - 8.0\ M_\odot$) for different stellar metallicities and compared them with the Renzini and Voli (1981) (RV) ones. The stellar models they adopted are from the Geneva group and contain moderate overshooting. The comparison with the yields of RV has shown that the new yields differ from those of RV by a factor 2-3. In particular, for high mass AGB stars ($M > 3.5\ M_\odot$) the effect of the hot-bottom-burning on the N yields is much higher for the RV yields. This implies a value of the mixing-length parameter, as defined in RV, lower than 2 to be the most appropriate for massive AGB stars. An interesting aspect of these new calculations is that they show that the yields are strongly influenced by the initial stellar metallicity acting on the amount of dredged-up material, which is larger in initially low-Z AGB. This fact implies that C and O are higher at low Z whereas N increases slightly with Z.

2.5. Summary of the element production

After having discussed the most important ingredients in the stellar evolution models and the sites of production of the different chemical species and before summarizing what we said in the previous sections we would like to mention the general uncertainties still present in nucleosynthesis and stellar evolution. They are: 1) the rate of the reaction $^{12}C(\alpha,\gamma)^{16}O$, 2) the treatment of convection, 3) the nature of the explosion mechanism in massive stars (prompt or delayed), 4) the range of stellar masses which do indeed explode (which is the limiting mass M_{bh} for forming a black hole?), 5) the velocity of the deflagration front.

Where and how are the elements produced?

- ^{12}C: quiescent He-burning in high and intermediate mass stars
- ^{13}C: quiescent and explosive H-burning (cold CNO cycle $T \leq 5 \cdot 10^7$ K, warm CNO cycle $T \sim 1.2 \cdot 10^8$ K) both in single and binary intermediate mass stars (novae). ^{13}C can be a partially primary element as a result of the third dredge-up and hot-bottom burning (RV)
- ^{14}N: quiescent H-burning (cold CNO) in low and intermediate mass stars. ^{14}N can also be a primary element like ^{13}C
- ^{15}N: explosive H-burning occurring in SNe and novae ($^{15}N/^{14}N$ in cold CNO is 100 times lower than $^{15}N/^{14}N$ in the solar system)
- ^{16}O: He-burning in massive stars
- ^{17}O: cold CNO in single low and intermediate mass stars ($M > 4\ M_\odot$) (also in explosive burning)
- ^{18}O: from destruction of ^{14}N by $^{14}N(\alpha,\gamma)^{18}F(\beta^+)^{18}O$ occurring in stellar regions suffering He-burning. Restored into the ISM from SNe II . Also in quiescent and explosive H-burning
- ^{20}Ne, ^{24}Mg: C-burning in massive stars,
  C-deflagration in C-O white dwarfs,
  Mg is produced also in Ne-burning quiescent and explosive
- ^{28}Si, ^{32}S: O-burning in massive stars and (quiescent and explosive)
  C-deflagration in C-O WDs
- ^{40}Ca: explosive O,Si-burnings in massive stars and C-deflagration in C-O WDs
- ^{56}Fe: Si-burning (quiescent and explosive) in massive stars and C-deflagration in C-O WDs (substantial contribution)

- s-process elements: He-burning in massive stars $^{22}Ne(\alpha,n)^{25}Mg$ (A< 90) and He-shell flashes $^{13}C(\alpha,n)^{16}O$ (A> 90)

- r-process elements: explosive He, C, O or Si burning in type II SNe

3. The birthrate function

The number of stars formed in the mass interval $m, m+dm$ and in the time interval $t, t+dt$ is the birthrate function:

$$B(m,t) = \varphi(m)\psi(t)dmdt \tag{2}$$

where $\varphi(m)$ is the IMF and $\psi(t)$ is the SFR. In fact, usually the dependence from m and t of the birthrate function is separated for the sake of simplicity. As we will see in the following, what we can be able to do is to derive the IMF after having assumed the SFR or viceversa.

How to derive the IMF in the solar vicinity?

Assume that the actual mass distribution of stars of Main Sequence (MS) per unit area, namely the present day mass function (PDMF), is $n(m)$, then:

for $\tau_m \geq t_G$ $(0.1 < m/m_\odot \leq 1)$

$$n(m) = \int_0^{t_G} \varphi(m)\psi(t)dt \tag{3}$$

where t_G is the galactic lifetime. If $\varphi(m)$ is constant in time, as it is usually assumed, then:

$$n(m) = \varphi(m) < \psi > t_G \tag{4}$$

where $< \psi >$ is the average SFR in the past.

For $\tau_m << t_G$ $(m \geq 2\ M_\odot)$

$$n(m) = \int_{t_G-\tau_m}^{t_G} \varphi(m)\psi(t)dt \tag{5}$$

Again if $\varphi(m)$ is constant in time it implies:

$$n(m) = \varphi(m)\psi(t_G)\tau_m \tag{6}$$

where $\psi(t_G)$ is the SFR at the present time and τ_m is the lifetime of a star of mass m.

We can not apply the same approximations used before to the stars in the interval $1-2\ M_\odot$, therefore $\varphi(m)$ in this mass interval depends on the following quantity, which has the dimension of a time and is the timescale for star formation in the solar neighbourhood:

$$T = \frac{< \psi_G > t_G}{\psi(t_G)} \tag{7}$$

A good fit between the two parts of the $\varphi(m)$, namely below 1 $M_\odot$ and above 2 $M_\odot$, requires:

$$6 \cdot 10^9 \leq T \leq 50 \cdot 10^9 \tag{8}$$

as discussed in the extensive review by Scalo (1986).

For $m < 0.1\ M_\odot$ we do not know $n(m)$ (see D'Antona 1995).

Before showing how to derive $\psi(t)$ after assuming an IMF, we would like to briefly recall how is derived the PDMF, $n(m)$.

3.1. How to derive the PDMF

Stars are counted as functions of their absolute magnitude. These counts are then transformed into the actual distribution of MS stars $n(m)$; first of all one corrects the number of stars in each interval of absolute magnitude for the presence of post-MS stars. This implies the knowledge of color and spectral type of the stars. Then, star counts are transformed into the average number of stars per pc^2 at a given galactocentric distance since stars of different mass are differently distributed perpendicularly to the galactic plane (the scale height decreases with increasing mass). At this point the mass-luminosity relation for stars in the MS phase allows us to pass from absolute magnitude to mass. Once $n(m)$ is known and the $\psi(t)$ function for the solar neighbourhood is assumed, we can derive $\varphi(m)$.

3.2. How to derive the $\psi(t_G)$

By integrating over the mass the equation which gives $n(m)$, after assuming an IMF, one can derive $\psi(t_G)(M_\odot pc^{-2} Gyr^{-1})$. For reasonable values of the $\varphi(m)$ Miller and Scalo (1979) suggested the following values for the SFR in the solar vicinity:

$$3 < \psi(t_G) < 7\ M_\odot pc^{-2} Gyr^{-1} \tag{9}$$

3.3. Parametrizations of the SFR

Here we will remind the most common parametrizations of the SFR adopted in galactic evolution models.

- a) a time-independent SFR (Twarog and Wheeler, 1982)
- b) an exponentially decreasing SFR (Tosi, 1988a)
- c) a SFR proportional to some power, k (between 1 and 2) of the total surface gas density. This formulation is similar to Schmidt's(1959) original suggestion that the SFR should vary as a power of the volume density of gas. In principle, Schmidt's formulation of the SFR and (c) are equivalent when $k = 1$, whatever the scale height of gas, but they are also equivalent for any value of the exponent as long as the scale height of gas is constant

with galactocentric radii. The advantage of the formulation in terms of the surface gas density lies in the fact that it does not require the computation of the scale height of the gas layer. Kennicutt (1989) tried to assess the dependence of massive star formation on the surface gas density in disk galaxies and found that in dense regions the SFR can be well represented by a law with $k = 1.3 \pm 0.3$. This law breaks down at densities below a critical threshold value although this value is found to vary by an order of magnitude among galaxies. The adoption of a threshold in the SFR in chemical evolution models can produce very different results from those without such a threshold (see Chiappini et al., 1997).

- d) A more complicated formula for the SFR can be obtained by introducing a dependence on the total surface mass density (Talbot and Arnett 1975, Chiosi 1980, Dopita 1985, Matteucci and Greggio 1986, Matteucci and François 1989, Tosi 1988a, Dopita and Ryder 1994). This kind of formulation ($\psi(t) \propto \sigma_g^m \sigma_T^n$) seems to reproduce better the main features of the Galactic disk, as shown by Tosi (1988a). In particular, Dopita and Ryder (1994) have suggested that a satisfactory agreement with the observed properties of disk galaxies is obtained for $1.5 < m + n < 2.5$.

- e) A SFR proportional only to the surface mass density of molecular hydrogen but with a dependence on the metallicity was proposed by Rana and Wilkinson (1986). Tosi and Diaz (1990) also studied a similar SFR but concluded that such a formulation of the SFR produces results in poorer agreement with the properties of disk galaxies than models assuming an exponentially decreasing SFR.

- f) A bimodal star formation rate, with a "high" mode of star formation exponentially decreasing with time and a "low" mode of star formation independent of time (Larson 1986, Wyse and Silk 1987) and other possible variations (Vangioni-Flam and Audouze 1988, Elbaz et al. 1995). However, when applied to specific cases this kind of formulation gives problems (Tosi 1988a, François et al 1990, Ferrini et al. 1990, Gibson 1996).

- g) An in principle more physical approach based on the analysis of induced and spontaneous star formation has been proposed by Franco and Shore 1984, Shore et al. 1987, Matteucci et al. 1989, Firmani and Tutukov 1992, Shore and Ferrini 1995. This approach, however, does not give particular advantages nor substantially different results relative to the previous formulations.

- h) Another more realistic approach is the Schmidt law of star formation which depends only on the cold gas proposed by Burkert and Hensler (1988) and Theis et al. (1992) in their chemo-dynamical models. The assumed star formation rate is $\psi(t) = \alpha_{SF}\eta_{SF}(T)\rho_{CM}^2$, where the coefficient $\eta_{SF}(T)$ is a temperature dependent factor and ρ_{CM} is the density of the cloud medium. The properties of such star formation rate have been further studied in Köppen et al. (1995), who suggested that feed-back between gas and stars naturally leads to a quadratic dependence of the star formation rate on the gas density, independent of the other assumptions on the stellar birthrate function.

However, in spite of the more physical approach, also the formulations g) and h) contain a number of free parameters, and this is unavoidable given the lack of knowledge of the physical processes which influence the formation of stars.

3.4. Parametrizations of the IMF

The IMF is usually approximated either by a one-slope power law (Salpeter, 1955) or by a multi-slope power law (Tinsley 1980, Scalo 1986, Kroupa et al. 1993). The most common normalization of the IMF in galactic chemical evolution models is :

$$\int_{m_L}^{m_U} m\varphi(m)dm = 1 \tag{10}$$

where m_U and m_L are usually 0.1 and 100 $M_\odot$, respectively. Besides the already mentioned parametrizations which assume a constant IMF in time and space, there have been various suggestions about possible variations of the IMF in the Galaxy in recent years:

- Güsten and Mezger (1983) suggested the existence of a lower mass limit in the IMF in spiral arms of $2-3$ $M_\odot$ and in the interarms of ~ 0.1 $M_\odot$. The proportion of stars in arms and interarms should increase with radius.

- Scalo (1986)found an IMF two-peaked (at ~ 0.25 $M_\odot$ and ~ 1 $M_\odot$). It is not clear what happens at low masses ($M < 0.25$ $M_\odot$).

- Larson (1986) and Wyse and Silk (1987) proposed a bimodal SFR/IMF implying a high mass mode strongly decreasing in time and a low mass mode constant. This formulation overestimates the solar oxygen abundance and produces many stellar remnants that we probably do not need (Kuijken and Gilmore, 1989).

- Rana (1987) suggested the existence of a multimodal IMF with a number of humps, due to the fact that a star forming cloud undergoes mini bursts each creating stars of some specific mass range.

- Ferrini et al. (1989) showed that fragmentation theories support multimodality but not bimodality in agreement with Rana's results.

- Terlevich and Melnick (1983) suggested a variation of the IMF with metallicity in extragalactic HII regions but their suggested increase of the IMF slope, x, with Z leads to problems in reproducing the abundance gradients along the Galactic disk. A metallicity dependence of the IMF was also recently proposed Chuvenkov and Glukhov (1995).

- Mc Clure et al. (1986) compared the IMF of globular clusters and suggested a dependence of x from Z: metal poor globulars seem to have the higher fraction of low mass stars.

- A more physical recent approach is from Padoan et al. (1997) who studied the physics of molecular clouds in order to derive the IMF. They suggested that the IMF is universal in the sense that its functional form arises as a consequence of the statistics of turbulence which are universal properties of the flows. They found that the IMF is a function of temperature, density and velocity dispersion of the gas. This approach produces a Miller and Scalo (1979) IMF for the typical conditions of molecular clouds in the Galaxy at the present time. The application of this IMF to the chemical evolution of the Galaxy can be found in Chiappini et al. (this conference).

4. Supplementary parameters

4.1. Infall

The existence of infall of gas onto the Galactic disk is inferred from high and very high velocity clouds (Mirabel, 1989) and is claimed as a natural consequence of galaxy formation out of gas lost from halos and/or of primordial extragalactic gas (see Chiappini et al. 1997 for example). The infall solves the G-dwarf metallicity problem and is desirable to prevent gas consumption in spirals (see Tinsley 1980, Larson 1991). The total rate of infall of gas estimated for our Galaxy is 0.2-0.5 $M_{\odot} yr^{-1}$ (Mirabel 1989).

The infall rate in galactic models is usually parametrized as:

-constant in time (Twarog and Wheeler, 1982)

-exponentially decreasing in time and constant in space (Tosi 1988a)

-exponentially decreasing in time and also variable in space (Chiosi 1980, Clayton 1987, Matteucci and François 1989)

The chemical composition of the infalling gas is generally assumed to be primordial. Models with enriched infall (Tosi 1988b, Matteucci and François 1989) all conclude that no difference is produced in the results if $Z_{inf} \leq 0.4\ Z_{\odot}$.

4.2. Galactic winds

Supernovae and stellar winds can trigger galactic winds when the thermal content of the gas in galaxies exceeds its potential energy. Galactic winds can explain the mass-metallicity relation in ellipticals and account for the presence of metals in the ICM (Larson 1976, Arimoto and Yoshii 1987, Matteucci and Tornambè 1987, Gibson 1997). Galactic winds in chemical evolution models can be normal or metal enriched (preferential loss of metals), continuous or occurring only during the bursts of star formation (Matteucci and Tosi 1985, Marconi et al. 1994).

4.3. Radial flows

Radial flows, under specific conditions, can help in building up abundance gradients in galactic disks but they are not the unique cause for the formation of gradients. If the flow velocity is not larger than 2 $kmsec^{-1}$ and the SFR depends only linearly from the gas density, then radial flows steepen the gradients (Lacey and Fall, 1985). More recently, Edmunds and Greenhow (1995) showed that there is no "simple one-way" effect of flows on gradients although in the

linear SFR case accelerating flows tend to flatten the gradients and decelerating flows tend to steepen them.

5. Models of chemical evolution of galaxies

5.1. Analytical models

We will first discuss the so-called **Simple Model** for the chemical evolution of the solar neighbourhood, which is defined as a region centered in the sun and extending roughly 1 Kpc in all directions.

The basic assumptions of the Simple Model are:

- - the system is one-zone and closed, namely there are no inflows or outflows
- - the initial gas is primordial (no metals)
- - $\varphi(m)$ is constant in time
- - the gas is well mixed at any time

Following the formalism of Tinsley (1980) we define:

$$\mu = \frac{M_{gas}}{M_{tot}} \tag{11}$$

as the fractional mass of gas, with:

$$M_{tot} = M_* + M_{gas} \tag{12}$$

where M_* is the mass in stars (dead and alive). Possible non-baryonic dark matter is not considered.

¿From eq. (11) and (12) it follows that:

$$M_* = (1 - \mu)M_{tot} \tag{13}$$

The abundance by mass of an element i is defined by:

$$X_i = \frac{M_i}{M_{gas}} \tag{14}$$

where M_i is the mass in the form of the element i. It is well known that the abundances must satisfy the condition, $\sum_i X_i(t) = 1$.

The initial conditions are:

$M_{gas}(0) = M_{tot}$, $X_i(0) = 0$ and the equation for the evolution of the gas in the system is:

$$\frac{dM_{gas}}{dt} = -\psi(t) + E(t) \tag{15}$$

where $E(t)$ is the rate at which dying stars restore both the enriched and unenriched material into the ISM and it contains all the stellar evolution and nucleosynthesis information which we discussed in the previous sections.

$E(t)$ can be written as:

$$E(t) = \int_{m(t)}^{\infty} (m - m_{rem})\psi(t - \tau_m)\varphi(m)dm \tag{16}$$

where $m - m_{rem}$ is the total mass ejected from a star of mass m, and τ_m is the lifetime of a star of mass m.

When eq. (16) is substituted into eq. (15) one obtains an integer-differential equation which can be solved analytically only by assuming instantaneous recycling approximation (**I.R.A., namely the lifetimes of stars above 1 $M_\odot$ are negligible whereas those of stars below 1 $M_\odot$ are larger than the age of the universe**). In this case we can define the constant:

$$R = \int_{1}^{\infty} (m - m_{rem})\varphi(m)dm \tag{17}$$

namely the total amount of mass restored into the ISM by a stellar generation. The other important quantity is the **yield** which has been already defined in section 2:

$$y_i = \frac{1}{1-R}\int_{1}^{\infty} m p_{im}\varphi(m)dm \tag{18}$$

where p_{im} is the fraction (by mass) of the newly produced and ejected element i by a star of mass m.

By means of eq. (17) and eq. (18), eq. (16) can now be written as :

$$E(t) = \psi(t)R \tag{19}$$

and consequently eq.(15) as:

$$\frac{dM_{gas}}{dt} = -\psi(t)(1 - R) \tag{20}$$

The equation for the evolution of the chemical abundances is:

$$\frac{d(X_i M_{gas})}{dt} = -X_i\psi(t) \ + E_i(t) \tag{21}$$

where:

$$E_i(t) = \int_{m(t)}^{\infty} [(m - m_{rem})X_i(t - \tau_m) + m p_{im}]\psi(t - \tau_m)\varphi(m)dm \tag{22}$$

contains both the unprocessed and the newly produced element i, and applies only to elements which are not destroyed inside stars.

Under the assumption of I.R.A. equation (22) becomes:

$$E_i(t) = \psi(t)RX_i(t) + y_i(1-R)[1-X_i(t)]\psi(t) \tag{23}$$

Eq. (23) is then substituted in eq. (21) which is solved analytically. If $X_i << 1$, which is generally true for metals, equation (27) has the following solution:

$$X_i = y_i ln(\frac{1}{\mu}) \tag{24}$$

the famous solution for the **Simple Model**. If X_i is not much lower than the unity, like for example it happens for X_{He} (Maeder 1992), then the solution of eq. (21) is:

$$X_i = 1 - \mu^{y_i} \tag{25}$$

which shows that if $\mu \rightarrow 0$ then $X_i \rightarrow 1$.

The yield which appears in eq. (24) is known as *effective yield*, simply defined as the yield $y_{i_{eff}}$ that would be deduced if the system were assumed to be described by the Simple Model:

$$y_{i_{eff}} = \frac{X_i}{ln(1/\mu)} \tag{26}$$

If $y_{i_{eff}} > y_i$ then the actual system has attained a higher abundance for the element i at a given gas fraction μ.

5.2. Models with gas flows

The equation for the evolution of abundances in presence of gas flows transforms into:

$$\frac{d(X_i M_{gas})}{dt} = -X_i(t)\psi(t) + E_i(t) + X_{A_i}A(t) - X_i(t)W(t) \tag{27}$$

where $A(t)$ is the accretion rate of matter with abundance of the element i X_{A_i} and $W(t)$ is the rate of loss of material from the system.

The case $A(t) = W(t) = 0$ obviously corresponds to the Simple Model.

The case $A(t) = 0$, $W(t) \neq 0$ corresponds to the outflow model.

The easiest way of defining $W(t)$ is to assume:

$$W(t) = \lambda(1-R)\psi(t) \tag{28}$$

where $\lambda \geq 0$ is the wind parameter. The analytical solution for the equation of metals, which can be integrated between 0 e $X_i(t)$ and between $M_{tot} = M_{gas}(0)$ and $M_{gas}(t)$, is:

$$X_i = \frac{y_i}{(1+\lambda)} ln\mu^{-1} \tag{29}$$

For $\lambda = 0$ we recover the solution of the Simple Model. This equation is also valid only if $X_i << 1$. The meaning of eq. (29) is immediately clear, the true yield is lower than the effective yield in presence of outflows.

The case of $A(t) \neq 0$ and $W(t) = 0$ corresponds to the accretion model. The easiest way to choose the accretion rate is:

$$A(t) = \Lambda(1 - R)\psi(t) \tag{30}$$

where Λ is a positive constant different from zero. The solution of the equation of metals for a primordial infalling material ($X_{A_i} = 0$) and $\Lambda \neq 1$ is :

$$X_i = \frac{y_i}{\Lambda}[1 - (\Lambda - (\Lambda - 1)\mu^{-1})^{-\Lambda/(1-\Lambda)}] \tag{31}$$

as shown by Matteucci and Chiosi (1983).

If $\Lambda = 1$ the solution is:

$$X_i = y_i[1 - e^{-(\mu^{-1}-1)}] \tag{32}$$

the well-known case called "extreme infall" in Tinsley (1980). The quantity $\mu^{-1} - 1$ represents the ratio between the accreted mass and the initial mass. Equation (32) shows that when $\mu \rightarrow 0$ $X_i \rightarrow y_i$.

5.3. The equations with type Ia and II SNe

If M_i is the mass of an element i, we can write:

$$\dot{M}_i(t) = -\psi(t)X_i(t) + \int_{M_L}^{M_{Bm}} \psi(t - \tau_m)Q_{mi}(t - \tau_m)\phi(m)dm +$$

$$A\int_{M_{Bm}}^{M_{BM}} \phi(m)[\int_{\mu_{min}}^{0.5} f(\mu)\psi(t - \tau_{m2})Q_{mi}(t - \tau_{m2})d\mu]dm +$$

$$(1 - A)\int_{M_{Bm}}^{M_{BM}} \psi(t - \tau_m)Q_{mi}(t - \tau_m)\phi(m)dm +$$

$$\int_{M_{BM}}^{M_U} \psi(t - \tau_m)Q_{mi}(t - \tau_m)\phi(m)dm + X_{A_i}A(t) - X_iW(t) \tag{33}$$

The total mass $M(t)$ refers to the mass of stars (dead and alive) plus gas. The quantity $W_i(t) = X_iW(t)$ is the galactic wind rate for the element i, whereas the quantity $A_i(t) = X_{A_i}A(t)$ is the accretion rate for the element i. M_{Bm} is the minimum and M_{BM} the maximum mass allowed for binary systems giving rise to type Ia SNe. If we assume the model of a C-O WD plus a red-giant companion for the progenitors of type Ia SNe, then $M_{BM} \leq 8\ M_\odot$. M_{Bm} is more uncertain and often has been taken to be 3 $M_\odot$ to ensure that the primary star (the initially more massive in the binary system) would be massive enough to guarantee that after accretion from the companion the C-O white dwarf eventually reaches the Chandrasekhar mass and ignites carbon. This formulation of the SN Ia rate was

originally proposed by Greggio and Renzini (1983) and included in the complete equations for the chemical evolution by Matteucci and Greggio (1986).

Greggio (1996) presented revised criteria for the choice of M_{Bm}: for the single white dwarf model she adopted as a condition for the explosion of the system that:

$$M_{WD} + \epsilon M_{2,e} \geq M_{Ch} \tag{34}$$

(where $M_{2,e}$ is the envelope mass of the evolving secondary, M_{WD} is the mass of the white dwarf and M_{Ch} is the Chandrasekhar mass). For models involving Sub-Chandrasekhar white dwarf masses she adopted the following criterion:

$$M_{WD} \geq 0.6\ M_{\odot} \tag{35}$$

and

$$\epsilon M_{2,e} \geq 0.15 \tag{36}$$

She also showed that, in order to ensure having SN explosions at late times in the framework of an initial burst of star formation, as required in ellipticals, the value of ϵ has to be larger than 0.5 for Chandrasekhar white dwarfs whereas an efficiency as low as 0.25 is enough for Sub-Chandrasekhar models.

The constant A represents the fraction of binary systems in the IMF having the right characteristics to become type Ia SNe.

The function $\tau_m(m)$ describes the stellar lifetimes. The quantity $Q_{mi}(t-\tau_m)$ contains all the information about stellar nucleosynthesis and is defined as in Talbot and Arnett (1973).

6. Observational constraints in the Galaxy

Here we will discuss the main observational constraints for the Galaxy. A good model of chemical evolution should honour all of them.

- The G-dwarf metallicity distribution as a function of the iron abundance ([Fe/H]). We remind that the G-dwarf problem is the deficit of metal deficient stars in the solar vicinity as predicted by the one-zone model of galactic chemical evolution. The available data in the literature are from: Schimdt (1963), Bond (1970), Pagel and Patchett (1975), Pagel (1989), Rocha-Pinto and Maciel (1996), Wyse and Gilmore (1995). In figure 1 are shown the most recent data compilations compared with the data of Pagel and Patchett (1975). This is the most important constraint that a model of chemical evolution should honour, since the distribution of stars with metallicity contains all of the information on the past history of the star formation and on the details of the formation of the Galactic disk.

- Another very important constraint is represented by the pattern shown in the abundance ratios versus absolute abundances by solar neighbourhood stars, as described in 6.1. The absolute solar abundances (Anders and Grevesse, 1989) are also a necessary constraint although they depend on all model parameters, at variance with abundance ratios which depend mostly on the nucleosynthesis and IMF.

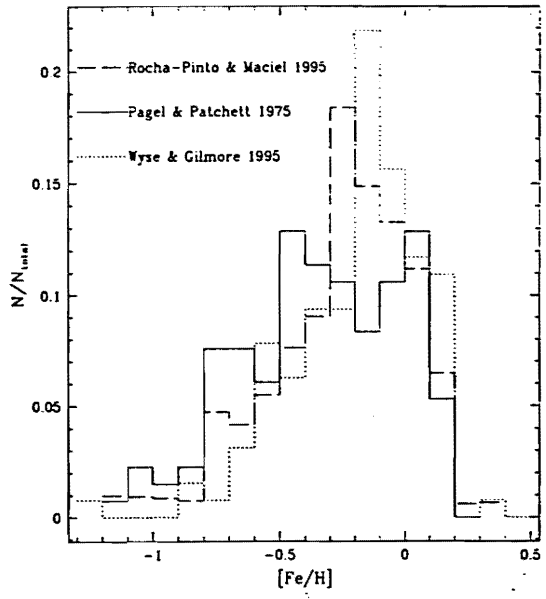

Figure 1. The observed distribution of G-dwarfs versus [Fe/H]. The figure is taken from Chiappini et al. 1997

- On the other hand, the age-iron abundance relation is not a good constraint since it can be usually reproduced by almost any chemical evolution model. The available data are from: Twarog (1980), Nissen et al. (1985), Carlberg et al. (1985), Edvardsson et al. (1993). The more complete and reliable data are those from Edvardsson et al. (1993) which show a very large scatter. This fact implies that is very difficult to fit such a distribution and weakens the concept of a unique age-metallicity relation. The reasons for the scatter can be many, among others inhomogeneous infall and orbital diffusion have been proposed. Orbital diffusion, in particular, would create a situation in which stars born at different galactocentric radii and therefore with different metallicities would be seen in the solar neighbourhood (François and Matteucci 1993 and references therein).

- Present day total surface gas density

$$\Sigma_{gas} = 7 - 14\ M_{\odot} pc^{-2} \tag{37}$$

(Kulkarni and Heiles, 1987)

- Present day star formation rate (very uncertain since it depends on assumptions on the IMF) (Miller and Scalo, 1979):

$$3 - 7\ M_{\odot} pc^{-2} Gyr^{-1} \tag{38}$$

- The relative number of thin disk and metal- poor stars (halo plus part of thick disk stars) (see Chiappini et al. 1997 for references).

- The present time infall rate (see section 4.1).

6.1. Abundaces in stars

Here we will review the current status of the observed abundances in stars in the solar neighbourhood. We remind that the notation $[X/H] = log(X/H)_* - log(X/H)_\odot$.

- **Carbon** is observed to vary in lockstep with iron ($[C/Fe] \sim [Fe/H]$) although a slightly increasing trend with decreasing metallicity may be present (Laird 1985, Carbon et al. 1987, Wheeler et al. 1989. Andersson and Edvardsson 1994, Tomkin et al. 1995).

- **Nitrogen** seems to vary in lockstep with iron although there is a lack of data at low metallicities (Laird 1985,Carbon et al. 1987).

- **α-elements** (are the elements synthetized by adding α-particles such as Mg, Si, Ca, Ti etc..) seem to be overabundant relative to iron in halo stars for metallicities in the range $[Fe/H] = -1.0$ to -4.0. The most recent studies (McWilliam et al. 1995) indicate that the average overabundance is $[\alpha/Fe] = +0.44 \pm 0.02$ dex for Mg, Si and Ca, whereas $[Ti/Fe] = +0.31$ dex. All the α-elements decrease with increasing [Fe/H] down to the solar value ([Fe/H]=0) and below, except that O shows a steeper variation with increasing [Fe/H] than the other α-elements (Edvardsson et al. 1993).

- **Iron-peak elements** namely from scandium to nickel, generally show a roughly solar ratio for most of the [Fe/H] interval, although Cr. Mn and Co seem to decline for [Fe/H]< -2.4 (McWilliam et al. 1995).

- **First nuclei beyond the iron peak elements** such as Cu and Zn. [Cu/Fe] increases almost linearly with [Fe/H] whereas [Zn/Fe] looks solar over the whole range of [Fe/H] (Gratton and Sneden 1988. Sneden and Crocker 1988, Sneden et al. 1991).

6.2. Abundances in the bulge

Stars in the Galactic bulge are distributed in a large metallicity range from metal poor to metal rich. Recent studies (McWilliam and Rich 1994. Sadler et al. 1996) suggest that the average metallicity in bulge stars is roughly solar. Concerning the abundance ratios in these stars, McWilliam and Rich suggest an overabundance of Mg and Ti relative to Fe of $\simeq +0.3$ dex all over the metallicity range. On the other hand, Sadler et al. find Mg enhancements ($\sim +0.2$ dex) only for stars with $[Fe/H] < 0.0$ whereas for stars with $[Fe/H] > 0.0$ they found [Mg/Fe]=0.0. Calcium and Silicon, as observed by McWilliam and Rich, follow the same behaviour as in disk stars, namely they reach solar ratios for $[Fe/H] > -0.2$ dex. Therefore, the situation seems to be still not clear and more abundance measurements in the Bulge and in external ones are needed.

Minniti et al. (1995) found a spatial metallicity gradient in the bulge, although at any given distance from the galactic center there is a large spread in metal abundances. The presence of such a gradient is usually interpreted as evidence of a dissipative collapse, in other words that the formation of the bulge

took a finite time although short if compared with the timescale attributed to the formation of the Galactic disk (see next sections).

6.3. Abundance gradients across the disk

- HII regions (optical, radio and far infrared observations) suggest:

$$\Delta log(O/H)/\Delta R = -0.07 \pm 0.015\ dex Kpc^{-1} \tag{39}$$

$$\Delta log(N/H)/\Delta R = -0.09 \pm 0.015\ dex Kpc^{-1} \tag{40}$$

$$\Delta log(S/H)/\Delta R = -0.07 \pm 0.02\ dex Kpc^{-1} \tag{41}$$

  (Shaver et al. 1983 from optical and radio lines for the galactocentric distance range 4-14 Kpc with the sun at 10 Kpc)

  More recent observations of HII regions (Fich and Silkey 1991, Simpson et al. 1995, Vilchez and Esteban 1996) seem to suggest a two-slope gradient with the slope in the internal part of the disk being steeper than that in the external parts ($R_G > 6$ Kpc), where there seems to be hardly any gradient in agreement with the abundance determinations derived from B stars in the galactocentric distance range 7-18 Kpc (Kaufer et al. 1994).

- Stellar photometry (Friel 1995) suggests:

$$\Delta[Fe/H]/\Delta R = -0.09 \pm 0.014\ dex Kpc^{-1} \tag{42}$$

  for a galactocentric radius interval of $R_G \geq 8$ Kpc

- Planetary Nebulae (PNe) suggest an O gradient very similar to that of Shaver et al. (1983) in a galactocentric distance range of 5-12 Kpc. The gradient for S is similar to that of O whereas that of Ne is slightly lower (Maciel and Köppen 1994, Maciel and Chiappini 1994)

Figure 2, from Tosi (1996), shows the O abundance gradient in the Galactic disk as derived from HII regions and young stars compared with predictions from current models of chemical evolution. In most of these models the abundance gradients arise from assuming a gradient in the star formation rate and/or the timescale of accretion of the Galactic disk. The existence of abundance gradients suggests that the disk might have formed from *inside-out.*

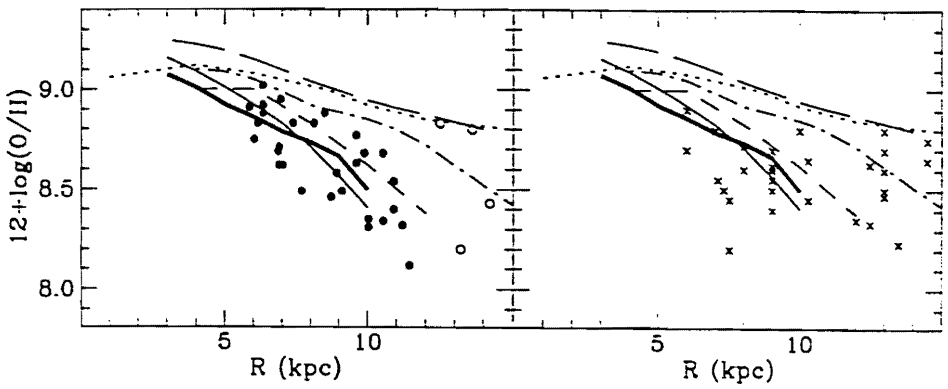

Figure 2. The oxygen abundance gradient along the Galactic disk. The figure is taken from Tosi (1996). Different model predictions are compared to observational data; HII regions are in the left panel: dots are from Shaver et al. (1983), circles are from Fich and Silkey (1991); B stars from Prantzos and Aubert (1995) in the right panel. The models are from Ferrini et al. (1994) (short-dashed lines), Carigi (1994) (dash-dotted lines), Prantzos and Aubert (1995) (dotted lines), Chiappini and Matteucci (1996) (long-dashed lines) and Tosi (1988a) (continuous lines)

6.4. Gas distribution along the disk

- *HI* is deduced from the 21 cm emission. The uncertainties in deriving such a quantity are in the optical depth in the solar vicinity, the amount of HI outside solar vicinity and the adopted rotation curve for the Galaxy. The available studies suggest that the HI distribution along most of the Galactic disk is flat (see Lacey and Fall 1985 for a compilation of data).

- H_2 is measured from z-integrated CO emissivity by assuming a conversion factor $x = N_{H_2}/I_{CO}$, usually considered independent from the position on the disk, except in Bhat et al. (1984) (see Lacey and Fall 1985 for a compilation of data). The resulting distribution for the H_2 along the disk is exponential, namely it follows the distribution of the stellar light. Up to now there have been no models of galactic chemical evolution able to account for the different distribution of HI and H_2 along the Galactic disk.

In figure 3, from Tosi (1996), the gas and SFR distribution along the Galactic disk are shown.

6.5. The SFR along the disk

The variation of the current SFR with galactocentric distance can be inferred by a variety of observations under the assumption of the IMF:

- distribution of pulsars

- distribution of SN remnants

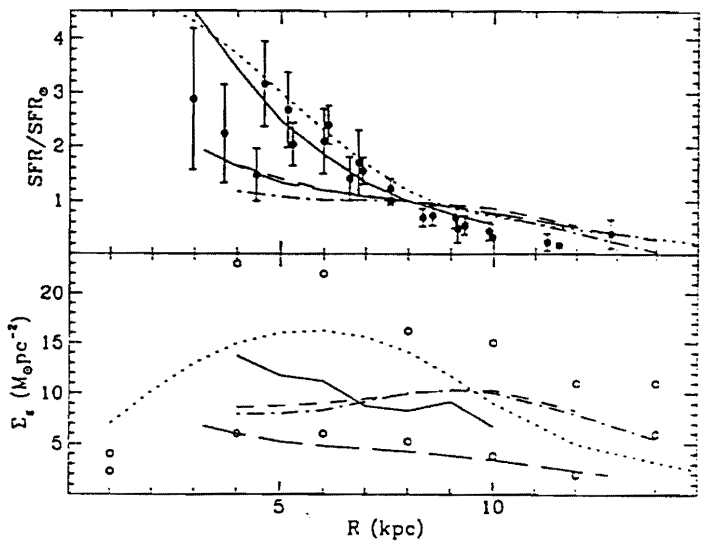

Figure 3. The gas and SFR distribution along the Galactic disk compared to the same models as in figure 2. The figure is taken from Tosi (1996). The upper panel shows the radial distribution of the SFR (normalized to the value at the solar radius). The lower panel shows the radial distribution of the gas density.

- distribution of Lyman continuum photons

These traces all indicate an exponential distribution for the SFR.

The absolute value of the star formation rate is very uncertain since it depends on the IMF, the mass range of stars producing pulsars, SNe, etc.. It is better to use the relative value:

$$\psi(r, t_G)/\psi(r_\odot, t_G) \tag{43}$$

(see Lacey and Fall, 1985 and figure 3 for a compilation of data).

7. Different approaches to the evolution of the Milky Way

The most common approach to the evolution of the Galaxy is to assume a continuity between the halo and disk phases implying that the different phases differ mainly because they formed at different epochs as distincts parts of a unique process. This approach is probably not very realistic although it has allowed us to reach important conclusions on the evolution of α-elements and iron in the halo. The reason for that being the fact that the behaviour of these elements is mainly dependent on the timescale for the formation of α-elements and Fe (Matteucci and Greggio 1986, Matteucci and François 1989, Timmes et al. 1995).

Another approach is to assume that the different phases did not occur one after the other but that they evolved in parallel although at different rates, the rates being the star formation and collapse ones. In this picture the difference in age between the Galactic phases can be very small (for age we intend the last period of star formation) and each component forms out of the gas lost by the previous one (Ferrini et al. 1992, Pardi et al. 1994)

Finally, the more recent approach is to consider the halo and disk phases as almost completely disentangled in the sense that only a negligible fraction of the

gas which formed the halo ended up in the disk. This latter then formed from the accretion of extragalactic gas (Pagel and Tautavaisiene 1997, Chiappini et al. 1997, Gratton et al. 1997, Pilyugin and Edmunds 1996).

7.1. The model of Matteucci and François 1989

The main assumptions of this model are:

- The Galactic disk is approximated by several independent rings, 2 Kpc wide, without excange of matter between them. Continuous infall of gas ensures the temporal increase of the surface mass density σ_T in each ring.

- The instantaneous recycling approximation is relaxed. This is of fundamental importance in treating those elements which are mostly produced by long living stars such as ^{14}N and ^{56}Fe.

- Two different prescriptions for the SFR are considered:

$$i) SFR \propto \sigma_g^k \tag{44}$$

$$ii) SFR \propto \sigma_T^{k_2} \sigma_g^{k_1} \tag{45}$$

  where σ_T is the total surface mass density and σ_g is the surface gas density.

- The prescriptions for the IMF are taken from Scalo (1986) and normalized to unity in the mass range $0.1 - 100\ M_\odot$.

- The nucleosynthesis prescriptions are from:
  Renzini and Voli (1981)- intermediate masses
  Nomoto et al. (1984) - SNe Ia
  Woosley (1987) - massive stars

- The infall rate is:

$$\frac{dG_i(r,t)}{dt} = \frac{A(r)(X_{inf})_i e^{-t/\tau(r)}}{\sigma_T(r,t)} \tag{46}$$

  where $\tau(r)$ represents the time scale for disk formation and is assumed to increase with the galactocentric distance. It is 3 Gyr in the solar vicinity. A(r) is a function of the assumed present time total surface mass density distribution and $(X_{inf})_i$ is the abundance of the element i in the infalling material.

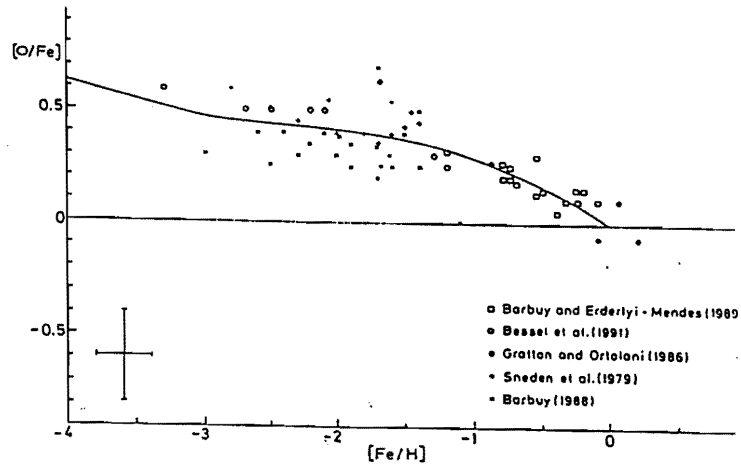

Figure 4. The [O/Fe] versus [Fe/H] relation as predicted by the best model of Matteucci and François (1992) compared with the observations

In figure 4 is shown the [O/Fe] versus [Fe/H] as predicted by the best model of Matteucci and François (1989). The pattern shown by the [O/Fe] ratio is usually interpreted as due to the delayed iron production from type Ia SNe. This delay is responsible for producing the change in slope at [Fe/H] ~ -1.0. The almost constant [O/Fe] for [Fe/H] < -1.0 is due only to the contribution of massive stars. ¿From this model the authors inferred a timescale of the order of 1-1.5 Gyr for the formation of the Galactic halo. This timescale is given by the time at which [Fe/H] becomes greater than -1.0 and which corresponds also to the end of the [O/Fe] plateau. This Fe abundance, in fact, was assumed as the transition metallicity between the halo and disk. This is not entirely correct since roughly 30% of the stars with $[Fe/H] < -1.5$ have recently been identified as thick-disk stars (Beers and Sommer-Larsen 1995). In any case, this is a way of using abundance ratios as cosmic clocks.

7.2. The model of Pardi et al. 1994

The main assumptions of this model are:

- Three main Galactic components are considered: halo, thick-disk, thin-disk. They start forming at the same time but at different rates. They are connected through gas flows in the sense that the thick disk forms out of the gas lost from the halo and the thin disk out of the gas lost from the thick-disk.

- Accretion of extragalactic gas is not considered and the thin disk forms out of the gas lost from the halo and thick disk.

- However, while the difference between the time scale of halo and thick-disk formation is small the difference between thick and thin disk is larger.

- While in other schemes the disk forms out of unenriched material in this case it starts with an initial metallicity different from zero.

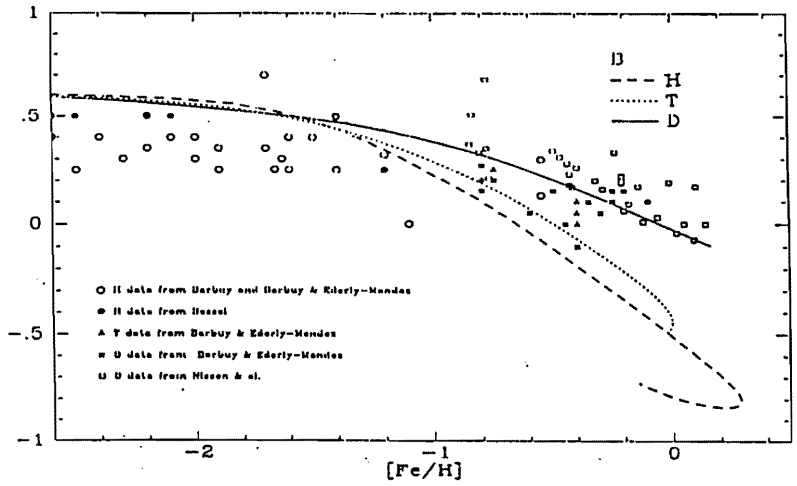

Figure 5. The [O/Fe] versus [Fe/H] relations as predicted by the best model of Pardi et al. 1995 compared with the observations. H=halo; T=thick disk; D=thin disk.

- Nucleosynthesis prescriptions like in Matteucci and François (1989)

In figure 5 is shown the [O/Fe] versus [Fe/H] relations for halo, thick and thin disk as predicted by the Pardi et al. (1994) best model. Here we can see that this model naturally predicts a spread in the [O/Fe] for a given [Fe/H] corresponding to the overlapping of stars belonging to the different Galactic components. An interesting prediction is that there could be halo stars (although very few) with high metallicity.

7.3. The model of Chiappini et al. 1997

The main assumptions of this model are:

- This model is very similar to that of Matteucci and François (1989) except that it assumes that the Galaxy formed by means of two main infall episodes. During the first one the halo and part of the thick disk (some thick disk stars seem to have been accreted, as shown by Gratton et al. 1997) formed, whereas the second one gave rise to the thin disk.

- The timescale for the formation of halo and thick disk is quite short ($\sim$ 0.5 – 1.0 Gyr) whereas the timescale for the formation of the thin disk is quite long ($\sim$ 8.0 Gyr) indicating that the thin disk formed mainly out of extragalactic primordial gas. This timescale ensures a very good fit of the new data on the G-dwarf metallicity distribution shown in figure 1.

- The SFR has the same form as in Matteucci and François (1989) except for the fact that a threshold in the surface gas density ($\sim$ 7 $M_{\odot}pc^{-2}$) is assumed. When the gas density goes under that threshold the star formation stops. Such a threshold has been suggested by star formation studies (Kennicutt 1989).

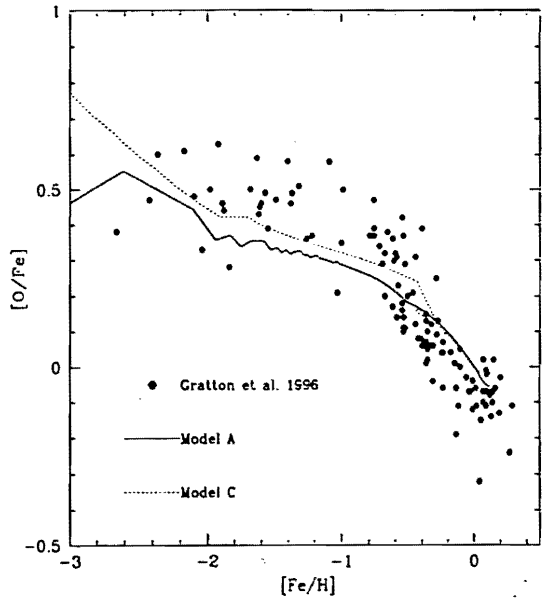

Figure 6. The [O/Fe] versus [Fe/H] relation as predicted by models of Chiappini et al. 1997 compared with the observations (model A is the best model as described in the text, whereas model C is similar but with a shorter timescale for the halo formation). The figure is taken from Chiappini et al. (1997).

- The values of the exponents of the SFR are: $k_1 = 1.5$ and $k_2 = 0.5$, slightly different from those used in Matteucci and François (1989).

- The nucleosynthesis prescriptions are from:

  Renzini and Voli (1981) intermediate masses

  Thielemann et al. (1993) - SNe Ia

  Woosley and Weaver (1995) - massive stars

In figure 6 is shown the [O/Fe] versus [Fe/H] relation as predicted by the best model of Chiappini et al. (1997). The existence of a gas threshold for the occurrence of star formation is shown by the small fluctuations. This kind of model can well reproduce the most recent data compilation by Gratton et al. (1997) showing the presence of a gap in the star formation at the end of the thick-disk phase. The gas threshold, in fact, naturally produces a period of no active star formation between the first and the second infall episode.

8. External galaxies

8.1. Ellipticals

Galaxies of different morphological type suffer different star formation and collapse histories and therefore different chemical evolution. We will only briefly summarize here the main properties of elliptical galaxies.

Elliptical galaxies must have formed on very short timescales (several $\sim 10^8$ years) as suggested by the oversolar [Mg/Fe] ratios deduced for their dominant stellar populations (Worthey et al. 1992, Weiss et al. 1995), -this is another case in which abundance ratios can be used as cosmic clocks and give and independent estimate of the age of galaxies- and by the "tightness" of the fundamental

plane which implies that the scatter of M/L_B, sensitive to the age of stellar populations, is less than 16% (Faber et al. 1987, Renzini 1995). Elliptical galaxies must have suffered early galactic winds as proven by the presence of metals (Fe and some α-elements) in the intracluster medium (Rothenflug and Arnaud 1985, Mushotzky et al. 1996). Supernova-induced galactic wind models are very popular for studying the evolution of ellipticals and explain the chemical enrichment of the intracluster medium (Arimoto and Yoshii 1987, Matteucci and Tornambè, 1987, Ferrini and Poggianti 1993, Gibson 1997). These models can explain the mass-metallicity and velocity dispersion- metallicity relations in ellipticals. In most of the models star formation is assumed to stop after the occurrence of a galactic wind so that the galaxies evolve passively since then. This is a fairly good assumption given the fact that after the early wind the interstellar medium in ellipticals is likely to be made of a low density high temperature gas, unfavorable conditions for the onset of star formation. Moreover, the only supernovae observed in ellipticals are SNe Ia occurring at a rate $R_{SNIa} \sim (0.25 - 0.44)h^2$ SNu (Turatto et al. 1994), thus confirming the absence of recent star formation in these objects.

Elliptical galaxies contain hot X-ray emitting gas. The chemical abundances of this gas are still controversial. ASCA measurements (see Arimoto et al. 1997 for references) indicate $0.1 \rightarrow 0.4Fe_{\odot}$ at variance with the solar abundances deduced for the dominant stellar populations. Several explanations have been proposed to solve this discrepancy among which cooling flows diluting the ISM of ellipticals, uncertainties in the derivation of chemical abundances from X-ray spectra (Arimoto et al., 1997) and the fact that some Fe could be hidden in a cold gas phase (Fujita et al. 1997). In figure 7 is shown the evolution of the Fe abundance in the intracluster medium (ICM) of a poor cluster (i.e. Virgo) as a function of time. The effect of different supernova type on the iron enrichment is also shown. The figure is taken from Matteucci and Gibson (1995) where the interested reader is addressed for more details on the subject.

8.2. Irregulars

The irregular galaxies can be divided in dwarf irregulars and blue compact galaxies. The dwarf irregulars seem to have an almost continuous star formation (gasping) regime (Tosi et al. 1992), whereas the blue compact galaxies (BCG) have probably experienced few short and intense bursts of star formation as indicated by their very blue colors, high content of gas and low metal content (Searle et al. 1973). These galaxies could be important contributors to the population of systems giving rise to QSO-absorption line systems at high redshifts (see for example Matteucci et al., 1997). They are also important since in "bottom-up" scenarios they should be the first gravitating systems to form, a possibility suggested by the Hubble Deep Field recent observations (Mobasher et al. 1997).

Abundances of N, O, C, Ne, S, Ar and Fe are known for some BCG. Most of them show undersolar N/O and C/O abundances and oversolar O/Fe ratios. However, the galaxy IZw18 shows high N/O (Skillman and Kennicutt 1993, Dufour et al. 1988) and particularly high C/O (Garnett et al. 1997). It is interesting to notice that the abundance ratios shown by IZw18 are similar to the abundances observed in two Damped Lyman-α systems (DLA) at high redshift

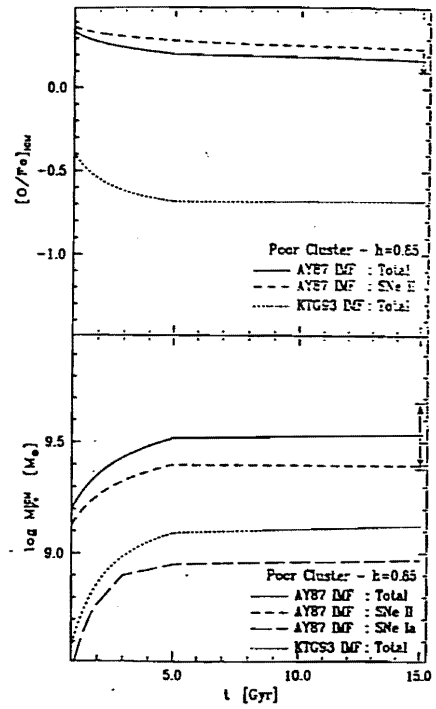

Figure 7. The figure is from Matteucci and Gibson (1995) and shows, in the upper panel, the [O/Fe] vs. time predicted for the gas in the ICM of a poor cluster. The effect of different IMFs and the different roles of SNe are shown. In the lower panel is shown the predicted behaviour of the iron mass in the ICM for the same cluster. The arrow marks the observed range for the iron mass in the ICM of the Virgo cluster (Okazaki et al. 1993) when $H_o = 85 Km/sec/Mpc^{-1}$ is assumed.

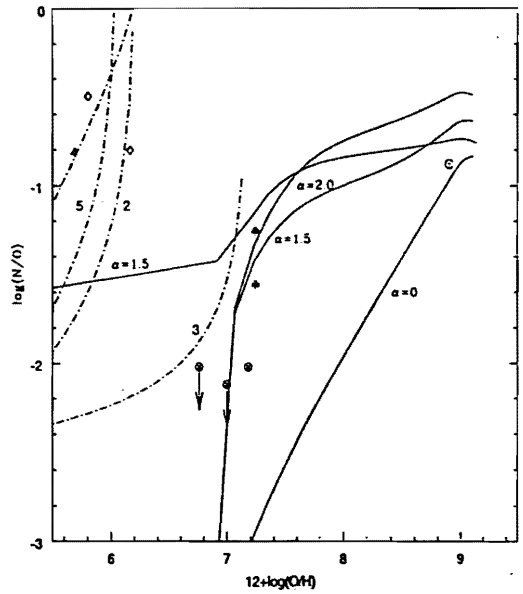

Figure 8. The log(N/O) vs. 12+log(O/H) relation as predicted by different models of chemical evolution. The figure is taken from Matteucci et al. (1997). In particular, the continuous lines refer to a model for the solar neighbourhood for different prescriptions concerning the nucleosynthesis of N. The value of the α parameter in Renzini and Voli (1981) is indicated on each curve; α=0 means only secondary N in low and intermediate mass stars, α=1.5 and 2.0 indicate different amounts of primary N produced during the third dredge-up according to the efficiency of convection. The line with a plateau and labelled α=1.5 represents a model where N is assumed to be primary in massive stars (in all the other models is secondary) plus primary N from intermediate mass stars with α=1.5. The dash-dotted lines are the predictions of starburst models with only one burst, as described in Matteucci et al. (1997). The symbols refer to measurements of N/O and O/H ratios in the sun and in some DLA systems. In particular, the romb with the highest N/O is the system observed by Molaro et al. (1996) whereas the other romb is the DLA system observed by Green et al. (1995). The clovers refer to different measurements for IZw18, the one with the highest N/O is from Skillman and Kennicutt (1993), whereas the other is from Dufour et al. (1988). The crossed circles are the DLA systems discussed in Pettini et al. (1995) and Lipman (1995).

(Molaro et al. 1996, Green et al. 1995) thus suggesting that these DLA systems could be the progenitors or relatives of IZw18. IZw18 itself could be a primordial galaxy in the sense that its nuclear evolution started only few Myears ago. At a first sight these high abundances of elements mostly produced in long living stars (i.e. C, N) would suggest that the present time starburst of IZw18 is not the first. However, the high C and N abundances could be also explained only by one very intense burst of star formation plus nitrogen of primary origin in massive stars and a strong O-enhanced galactic wind, as suggested by Matteucci et al. (1997). In spite of the still uncertain situation in interpreting these data, we like to stress the importance of deriving abundances in DLA systems as a unique tool to understand the nature and the age of primeval galaxies. In figure 8 we show the predictions from models of chemical evolution for galaxies of different morphological type compared to the abundances measured in some DLA systems and in IZw18.

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