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NEW FACES OF THE PROBLEM OF ANISOTROPY OF COSMIC BACKGROUND RADIATION

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Abstract The anisotropy properties of Cosmic Microwave Background Radiation (CMB) are considered within the framework of photon beam mixing effect developed earlier. The existence of an observable characteristics of CMB is shown, namely the geometrical shape of anisotropy spots, **their degree of complexity**, which can contain unique information on cosmological parameters and the life history of the Universe. If future experiments (COBE and others) indicate such features of anisotropy maps than one can have serious indication of the negative curvature of the Universe.

1 Introduction.

The mystery of perfect isotropy of the Cosmic Microwave Background Radiation is a long time a central one both for theoretical and experimental cosmological investigations.

Hence the outstanding interest to the Cosmic Background Explorer (COBE) data on the possible discovery of anisotropy of CMB [1] is quite natural, though the necessity of further confirmation of those results is evident. In our opinion, however, extreme cautious approach to theoretical interpretation [2] of observational data should be revealed as well.

Particularly, recognizing completely the great heuristic content of the inflation paradigm, one nevertheless cannot neglect the giant approximation to be done from the simplest considerations of that idea to measurable effects. In this situation, special importance should have thorough study of all features of the observational data, and not of some chosen parameter.

The aim of this paper is to show that an observable characteristic does exist, namely the geometrical shape of anisotropy spots, their **degree of complexity**, which can be a carrier of a rather important cosmological information.

Our approach proceeds from the effect of the isotropization of CMB developed in our papers [3,4] and based on the idea, that in Friedmannian Universe with negative curvature, $k < 0$, initial perturbations of the beam of photons decrease due to deviation of close null geodesics.

Among the predictions of that effect are those differing from adopted viewpoints:

a. Perturbations of photon gas can decrease also in post–scattering epoch due to geometrical effects;

b. Photon beams arriving from regions with no causal connection can have similar features.

The second conclusion, equivalent to the fact that *two ink drops mix in a cup of water in a same manner without having information on each other*, removes the mystery of horizon problem, one of the main motivations of the nice idea of inflation.

Before inquiring into the question whether COBE results can be used to confirm or to reject this alternative viewpoint let us briefly consider the free motion in the Universe and the effect of photon beam mixing; more detailed account of the subject can be found in the monograph [5].

2 Free Motion in the Universe.

Let W be a 4–dimensional manifold with Lorentzian metric ${}^4\mathbf{g}$ which is oriented and time–oriented. Let M be a 3–dimensional manifold and let

$$\imath : M \to W,$$

be an embedding of M such that the embedded manifold $\imath(M) = U$ is spacelike, i.e. the $\imath^*({}^4\mathbf{g}) = \mathbf{g}$ is a Riemannian metric on M. Let $C^\infty_{space}(M; W, {}^4\mathbf{g})$ denote the set of all such spacelike embeddings which is a smooth manifold.

Consider a curve in $C^\infty_{space}(M; W, {}^4\mathbf{g})$, i.e. a curve $\imath_t$ of spacelike embeddings of M into $(W, {}^4\mathbf{g})$. The derivative of this curve defines one-parameter family of vector fields $\mathbf{Y}_{U_t}$ on the embedded hypersurfaces $U_t \equiv \imath_t(M)$ by equation

$$\frac{d\imath_t}{dt} \equiv \mathbf{Y}_{\imath_t} = \mathbf{Y}_{U_t} \circ \imath_t : M \to TW.$$

Normal and tangential projections of Y_{U_t} define a curve of functions

$$N_t = \mathbf{Y}_\perp : M \to R$$

and vector fields

$$\mathbf{X}_t = \mathbf{Y}_{||} : M \to TM$$

on M by the equation

$$\mathbf{Y}_{U_t} \circ \imath_t(x) = \mathbf{Y}_\perp(x,t)\mathbf{Z}_{U_t} \circ \imath_t(x) + (\imath_t)_*|_x \mathbf{Y}_{||}(x,t)$$

where $\mathbf{Z}_{U_t}$ is the forward pointing unit timelike normal to U_t.

Assume that the map

$$\jmath : M \times R \to W : (x,t) \mapsto \imath_t(x)$$

is a diffeomorphism. Using $\jmath$ as a coordinate system for W the metric ${}^4\mathbf{g}$ of W can be written in a form

$$\jmath^{*4}\mathbf{g} = (\jmath^{*4}\mathbf{g})_{\alpha\beta}\mathbf{dx}^\alpha \otimes \mathbf{dx}^\beta = -N^2\mathbf{dt}\otimes\mathbf{dt} + g_{ab}(\mathbf{dx}^a + X^a\mathbf{dt})\otimes(\mathbf{dx}^b + X^b\mathbf{dt}).$$

where $g_{ab} = (g_t)_{ab}$ and $\mathbf{g}_t = \imath_t^{*4}\mathbf{g}$.

By means of the map $\jmath$ one can define uniquely the projection π

$$\begin{array}{ccc} W & \overset{\jmath^{-1}}{\rightarrow} & M \times R \\ & \searrow & \downarrow \pi_M \\ & \pi & M \end{array} \qquad \pi = \pi_M \circ \jmath^{-1},$$

where

$$\pi_M : M \times R \rightarrow M : (x,t) \mapsto x.$$

which depends on N and $\mathbf{X}$.

As a result any curve γ in W can be projected on the curve c determined on U, as represented below:

$$\begin{array}{ccc} W & \overset{\pi}{\rightarrow} & M \\ & \searrow & \\ \gamma \uparrow & \phi & \uparrow c \\ R & \overset{\rightarrow}{\sigma} & R \end{array} \qquad \begin{array}{l} \sigma = \phi \circ \gamma : R \rightarrow R \text{ is a diffeomorphism} \\ c = \pi \circ \gamma \circ \sigma^{-1} : R \rightarrow M : \lambda \mapsto c(\lambda). \end{array}$$

One can now inquire into the following question:what conditions should satisfy ${}^4\mathbf{g}, N, \mathbf{X}, \phi$ in order the projection of any null geodesic on W to be a geodesic on M .

Sufficient conditions can be shown to be [6]

$$N = 1, \tag{1}$$

$$\mathbf{X} = 0, \tag{2}$$

$$\mathbf{g} = a^2(t)\cdot\mathbf{h}, \tag{3}$$

$$\phi(x,t) = \phi_0 + \int_{t_0}^{t} a^{-1}(s)ds, \tag{4}$$

$$\lambda(t) = \int_{t_0}^{t} a^{-1}(s)ds = \eta(t) - \eta(t_0), \tag{5}$$

where $\mathbf{h}$ is one of the metrics of maximal symmetric 3–dimensional manifolds.

3 Photon Beam Mixing in Closed Friedmannian Universe

Now we shall consider the behavior of the beam of null geodesics in closed homogeneous Universe by means of the described transformation of geodesics while projection from 4 to 3–dimensional spaces.

To do it let us remind some properties of correlation functions of dynamical systems [7]. Particularly it is shown that geodesic flow T on closed manifold with negative curvature on all two–dimensional directions is Anosov system [8], for which the correlation function decrease by exponential law, i.e. $\exists C_{f,g}, \beta_{f,g}, t_c > 0$ such that

$$b_{f,g}(t) \leq C_{f,g} \cdot \exp(-\beta_{f,g} h(T) \cdot t), \tag{6}$$

where $h(T)$ is a KS–entropy of the flow T and

$$t_c = (\beta h(T))^{-1}, \tag{7}$$

t_c is the characteristic time of decrease of perturbations. Thus such flow at $t \gg t_c$ loses completely an initial information.

Now let us apply this mechanism of decay of correlations to the problem of evolution of perturbations of photon gas in the compact Friedmannian Universe with negative curvature: $k = -1$.

Note that the homogeneous metric does not specify uniquely the global topology of the Universe but only local one.

Let $W = U \times R$ be $(3+1)$–dimensional manifold with Robertson–Walker metric.

Geodesic flow on this space, thus can be reduced to a geodesic flow on three–dimensional closed manifold $M = U$ with metric $a_0^2\mathbf{h}$ and affine parameter λ.

The deviation of two geodesics in 3–manifold implies that

$$l(\lambda) = l(0) \exp(\chi\lambda), \tag{8}$$

where Lyapunov exponent $\chi = a_0^{-1}$, ($\chi = 0$ when $k = 0$ or $k = +1$), and fulfills also for non–compact spaces. Hence

$$\lambda_c^l = \frac{1}{\chi}.$$

Correlation function decreases as follows:

$$b(\lambda) \leq C \exp(-\beta h \lambda), \tag{9}$$

where KS–entropy $h = 2\chi$, and

$$\lambda_c^b = \frac{1}{\beta h}.$$

This property in W reads as

$$L(t) = L(t_0)\frac{a(t)}{a(t_0)}\exp(\chi\lambda(t)). \tag{10}$$

In the case of Friedmannian World with negative spatial curvature and expansion rate

$$a(t) = a(t_0)\left(\frac{t}{t_0}\right)^{\alpha},$$

the initial moment of free photon motion is determined by the recombination time (the last scattering):

$$t_0 = t(z = 1000) \simeq 10^5 yrs.$$

Therefore one has

$$\lambda(t) = \frac{1}{(1-\alpha)}\frac{t_0}{a(t_0)}\left(\left(\frac{t}{t_0}\right)^{1-\alpha} - 1\right) = f_\alpha(z)\sqrt{1-\Omega(t)},$$

$$L(t) = L(t_0)(1+z)\exp(f_\alpha(z)\sqrt{1-\Omega(t)}),$$

where

$$f_\alpha(z) = \begin{cases} \frac{\alpha}{1-\alpha}\left(1-(1+z)^{1-1/\alpha}\right) & 0 < \alpha < 1 \\ \ln(1+z) & \alpha = 1. \end{cases}$$

The information on any small perturbation on the geodesic flow at $1 + z = 1000$ should be lost up to $z = 0$ by a factor

$$\exp(-m) = \exp(-2\beta f_\alpha(999)\sqrt{1-\Omega}). \tag{11}$$

In the case of $k = -1$ α can be considered as effectively varying from $2/3$ to 1 and, hence, $2f_\alpha(999)$ from 4 to $6\ln 10 \simeq 13.8$.

Let us numerically evaluate the role of the considered mechanism of decrease of fluctuations of in the case of CMB.

Particularly, if one has initial perturbations of a size r in phase space X on average distance ρ, then the time during which all initially perturbed regions will mix is λ_m, i.e.

$$\rho \doteq r\exp(h(T)\cdot\lambda_m), \tag{12}$$

$$\lambda_m = \lambda_c^b \ln(\rho/r) = \lambda_c^b \ln(c). \tag{13}$$

The sizes of initially perturbed regions can be estimated using the typical present distances between clusters of galaxies and ρ and their sizes r: $\ln(c) = \ln(\rho/r) = \ln(25Mpc/5Mpc) \simeq 1.6$.

For this quantities we have

$$m = 2\beta f_\alpha(z)\sqrt{1-\Omega}.$$

For example, the decrease of perturbations by now when $\Omega \ll 1$ and $\beta \sim 1$ we have [3]

$$\delta(z=0) \simeq \delta(z=1000)10^{-3}. \tag{14}$$

We see that due to geometry of Friedmannian World, small perturbations of CMB existing at the epoch of last scattering, by now might be decreased sufficiently.

Similar analysis can be performed for arbitrary evolution laws of the Universe with $k = -1$, $\Omega < 1$. Particularly, in the case of inflationary Universe one has $\Omega = 1$ and $m = 0$ and therefore this effect is absent.

4 Degree of Complexity of Anisotropy Spots

In the previous sections we considered the behavior of photon beam on a space with negative curvature. The difference of the situation in the case of other geometry of the space is clearly seen from the Jacobi equation

$$\frac{d^2\mathbf{n}}{d\lambda^2} + k\mathbf{n} = 0, \tag{15}$$

where $\mathbf{n}$ is the vector of deviation determining the behavior of close geodesics. Eq.(15) has the following solutions (cf. (8))

$$\mathbf{n}(\lambda) = \begin{cases} \mathbf{n}(0) + \dot{\mathbf{n}}(0)\lambda & k = 0, \\ \mathbf{n}(0)\cos\lambda + \dot{\mathbf{n}}(0)\sin\lambda & k = +1, \\ \mathbf{n}(0)\cosh\lambda + \dot{\mathbf{n}}(0)\sinh\lambda & k = -1. \end{cases} \tag{16}$$

To illustrate the meaning of these solutions let us look for the evolution of photons with isotropic (random) distributions of velocity vectors within a ball with unit radius.

Consider also the decomposition of the initial ball to smaller ones with nearly same directions of velocity vectors of photons.

According to Eq.(16) the behavior of small balls should be quite different depending on k and as a result the evolution of the initial unit–radius balls should crucially be determined by k as shown in Figure 1.

Particularly, in the case of $k = -1$ the geodesic flow represents an Anosov system (locally, if U is not compact) being a dynamical system with maximally strong statistical properties. Therefore, if the initial distribution of velocity vectors of photons within the ball is random, the evolution of a perturbed ball in U should appear as a multitail complex structure being a unification of elongated ellipsoids, while at $k = 0$ and $k = +1$ one should have unification of balls. This effect arises due to following reasons. The initial ball has a regular form (ball) in coordinate space and rather complicated form (multitail structure) in velocity one. During the evolution at $k = -1$ when one has an Anosov system with exponential rate of mixing properties (homogeneous mixing), the form in coordinate space will become similar to that of the velocity space (multitail structure). At $k = 0$ and $k = +1$ no such effect exists due to absence of exponential instability.

Let us also remind that the final structure in W should be a result of increase of a scale by a factor

$$1 + z \simeq 1000,$$

being the only effect usually taken into account while considering the evolution of perturbations in post–scattering epoch (Figure2). Meanwhile the present (at $z = 0$) amplitude and form of perturbations depend on the curvature and the age of the Universe.

Let us stress again that the described effect of complex geometry of perturbations occurs only in a Universe with negative curvature $k = -1$ and is absent when $k = 0$, i.e. when the Universe is flat or is so large that can be considered as flat as well. The last case is what follows from inflationary scenario.

We see that a unique parameter, **degree of complexity**, appears which can be rather informative not only concerning the mechanism of mixing of photon beams but also the main parameters of the Universe.

Recalling the COBE observations of the anisotropy of CMB one hardly can have any unique indication of the degree of complexity due to domination of noise. However future observations by COBE and other facilities can give serious information on this point, though it seems to be more difficult than simply the evaluation of anisotropy level.

Dependence of anisotropy structures on the wavelength should only indicate that the isotropy level of initial photons distributions was different

for different energies.

Thus, if the detection of CMB anisotropy and their complex geometrical form could be indicated in future observations, then one can come to the following conclusions:

1. The Universe has not passed an inflationary phase of evolution and, particularly its today curvature is not zero as it is predicted by that model;

2. Curvature of the Universe is negative, i.e. $\Omega < 1$.

More accurate observations of anisotropy spots of CMB should enable one proceeding from calculations of the effect of photon beam mixing [3, 4], to do definite quantitative estimations of main cosmological parameters.

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Figure captions.

Figure 1 *The evolution of photon beam anisotropy due to mixing effect in the Universe with different curvatures.*

Figure 2 *Evolution of perturbations of photon gas only due to the expansion of the Universe.*

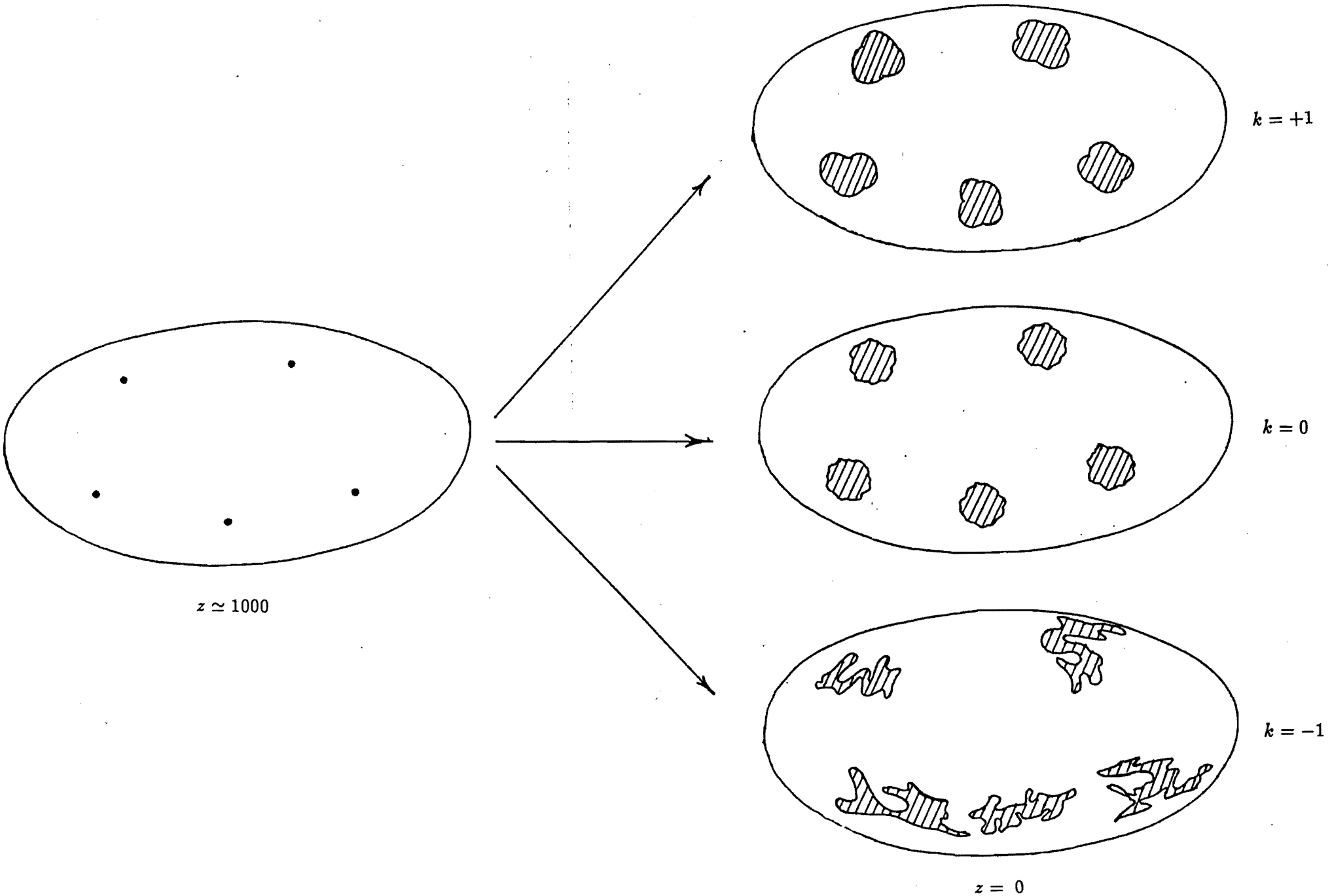

FIGURE 1.

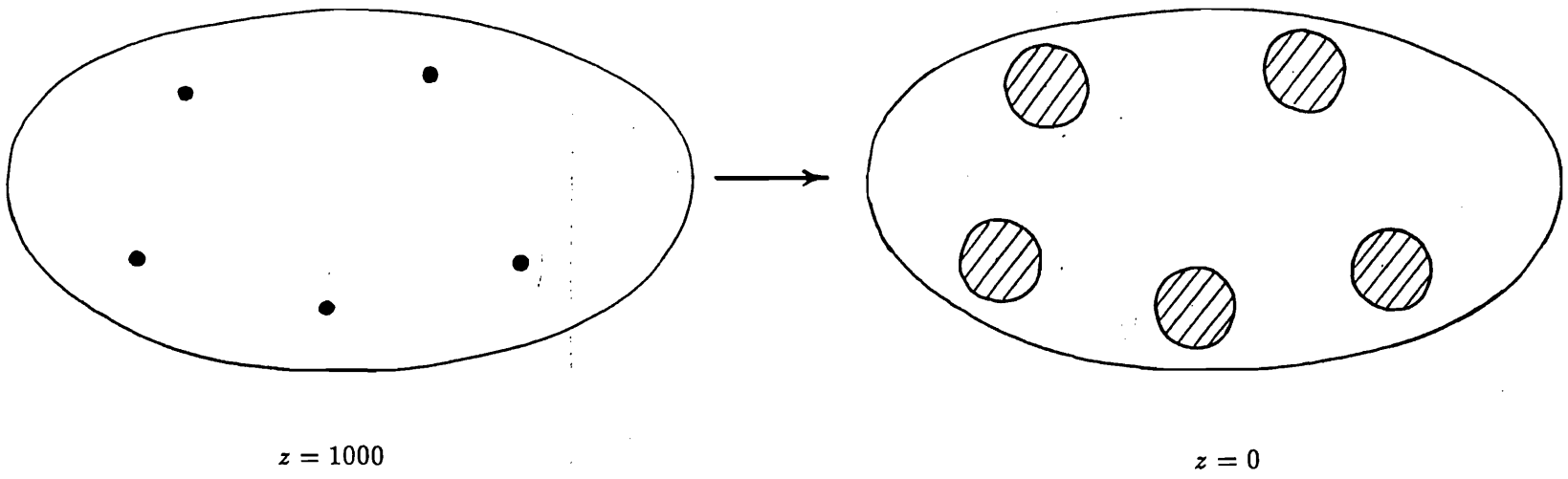

FIGURE 2.