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Comment on "Centrifugal force: A gedanken experiment"

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Abstract

In a recent paper [G.Z. Machabeli and A.D. Rogava, Phys. Rev. A **50**, 98 (1994)], a special relativistic gedanken experiment was described exhibiting surprising behavior which was interpreted in terms of reversal in direction of the centrifugal force. It is here pointed out that there is a completely conventional explanation for this behavior and that a description in terms of reversal of centrifugal force does not seem appropriate.

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1. Introduction

In their paper, Machabeli & Rogava [1] described a gedanken experiment which seemed to exhibit the possibility of a reversal in direction of the centrifugal force in special relativity similar to that which has been discussed by a number of authors in a *general* relativistic context. The experimental set-up consisted of a straight, long and narrow pipe, rotating with constant angular velocity ω about an axis normal to the pipe, and a small bead which could move inside the pipe without friction. In the particular cases studied in detail, the bead starts from the rotation axis with initial radial velocity v_0 along the pipe, and is then found to move out along the pipe until it reaches a distance $r = c/\omega$ from the pivot (*i.e.* until it reaches the light cylinder) at which point its motion reverses and it moves back again towards the pivot. A sequence of oscillations about the pivot then follows. Such a behavior is clearly indicative of the presence of some restoring force pulling the bead back towards the pivot and this was identified as being provided by a reversal in direction of the centrifugal force during some or all of the motion (reversal signifying attraction towards the rotation axis rather than the usual repulsion away from it). It is the purpose of the present note to re-investigate this interpretation. We mainly follow the notation adopted there and use units in which $c = 1$.

The motion is considered in the reference frame of the rotating pipe, for which the metric of the two-dimensional spacetime was written as

$$ds^2 \equiv -d\tau^2 = -(1 - \omega^2 r^2)dt^2 + dr^2, \tag{1}$$

where τ is the proper time of the bead.

As correctly emphasized by the previous authors, the gedanken experiment is very highly idealized. It would be impossible to construct such an apparatus in practice since the tube could not remain straight and rigid when the bead velocity became relativistic and an infinite amount of energy would be required. However, the idea is not to give a direct description of some real situation but rather to explore a feature of special relativity dynamics which might then have a detailed application elsewhere. This is a fair point of view and we broadly follow the spirit

of it here but we take a different view of the situation at $r = 1/\omega$. This point is a real singularity of the two-dimensional metric (1) and the relevant Riemann tensor component for this two-dimensional spacetime,

$$R_{trtr} = \frac{\omega^2}{1 - \omega^2 r^2}, \tag{2}$$

is infinite there, so that any particle arriving at $r = 1/\omega$ would be destroyed by an infinite tidal force. We disagree with the previous authors when they say that because the particle crosses the singularity at only one instant of time, the analysis of motion can be continued across the singularity. In our opinion, the possibility of the oscillatory motion described in [1] is only a mathematical curiosity and has no physical significance. However, the question of whether the centrifugal force does or does not reverse its direction remains something which needs to be addressed.

2. Basic equations

To provide the necessary orientation, we here give a brief summary of the basic equations from [1] which we will be using in our discussion. It is straightforward to derive the equations of motion

$$-\left(1 - \omega^2 r^2\right)\left(\frac{dt}{d\tau}\right) = const \equiv -E, \tag{3}$$

$$\frac{d^2 r}{d\tau^2} = \omega^2 r \left(\frac{dt}{d\tau}\right)^2, \tag{4}$$

where the constant E is interpreted as the energy of the bead in the reference frame of the pipe and

$$\left(\frac{dt}{d\tau}\right) = \left(1 - \omega^2 r^2 - v^2\right)^{-1/2} \equiv \gamma, \tag{5}$$

with $v \equiv dr/dt$. Equations (3) – (5) can then be manipulated to give

$$v \equiv \frac{dr}{dt} = \left[\left(1 - \omega^2 r^2\right)\left\{1 - \frac{\left(1 - \omega^2 r^2\right)}{E^2}\right\}\right]^{1/2}, \tag{6}$$

$$\frac{d^2 r}{dt^2} = \frac{\omega^2 r}{1 - \omega^2 r^2}\left[1 - \omega^2 r^2 - 2v^2\right]. \tag{7}$$

On the left hand side of (7), we have the radial component of three-acceleration of the bead as measured in the reference frame of the pipe and Machabeli & Rogava then refer to the right hand side of (7) as an expression for the force f acting on the bead. They describe this as a generalized expression for relativistic centrifugal force, noting that in the non-relativistic limit $f \rightarrow \omega^2 r$ which is the familiar Newtonian expression. A feature of this definition is that the quantity inside the square brackets can become negative in which case f also becomes negative and so is directed towards the rotation axis. An integration of (6) reveals that f always eventually becomes negative as r grows and that if $v_0 > 1/\sqrt{2}$ then it is negative for all of the motion.

3. Explanation for the behavior seen in the gedanken experiment

The standard special relativistic definition of the four-force is

$$\mathbf{F} = \frac{d}{d\tau}(m_0 \mathbf{U}), \tag{8}$$

where m_0 is the rest mass of the particle and $\mathbf{U}$ is the four-velocity. (We follow Rindler [2] and use uppercase symbols for four-vectors and lowercase symbols for three-vectors.) The corresponding relativistic three-force is

$$\mathbf{f} = \frac{d}{dt}(m\mathbf{v}), \tag{9}$$

where m is the relativistic mass ($m = \gamma m_0$) and $\mathbf{v}$ is the three-velocity. Applying formula (9) to the situation of the gedanken experiment, we obtain (using (5) and (7))

$$f = \frac{d}{dt}(mv) = m\omega^2 r, \tag{10}$$

as the expression for the effective relativistic three-force acting along the tube. (Note that this is entirely an apparent force resulting from the rotation of the tube: there is no real applied force acting in the radial direction.) Equation (10) gives the straightforward special relativistic generalization of the centrifugal force and differs from the Newtonian expression only in that the mass involved is

now velocity-dependant. With this definition, the centrifugal force is always and everywhere positive, acting in a direction away from the rotation axis.

In [1], the expression used for the force (there a force per unit mass) was

$$f_{mr} = \frac{d^2r}{dt^2} \equiv \frac{dv}{dt}, \tag{11}$$

and so

$$f_{mr} = \frac{1}{m}\left(f_{cent} - v\frac{dm}{dt}\right), \tag{12}$$

where f_{cent} is the quantity given by (10) above. We then have

$$f_{mr} = \omega^2 r - \left[\frac{2\omega^2 r v^2}{(1-\omega^2 r^2)}\right]. \tag{13}$$

The first term on the right hand side (the standard centrifugal term) is always positive (outward-pointing) while the second term (arising from variation in the relativistic mass) always gives a negative (inward-pointing) contribution. When the second term is sufficiently large, f_{mr} itself becomes negative. The apparent reversal of direction of the centrifugal force noted in [1] is then seen to originate in the unconventional definition which was used there for the force.

4. Conclusion

In this note we have re-examined the gedanken experiment described in [1] and have reached a different conclusion regarding the explanation for the behavior seen. If the standard relativistic definition is used for forces, the relativistic centrifugal force does not reverse its direction as suggested by the previous authors. Such reversal seems to be reserved for strong-field situations in *general* relativity.

Acknowledgement

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References

[1] G.Z. Machabeli and A.D. Rogava, Phys. Rev. A **50**, 98 (1994).

[2] W. Rindler, *Introduction to special relativity* (Oxford University Press, Oxford, UK, 1982) p. 101.