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LARGE–SCALE VELOCITY FIELDS IN NON–GAUSSIAN SEEDED MODELS

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Abstract

The large–scale velocity field is analyzed in the framework of non–Gaussian seeded models. In particular, we calculate the skewness of the parallel component of the velocity difference distribution $p[v_{\|}(\mathbf{r}_1) - v_{\|}(\mathbf{r}_2)]$ for global textures in an $\Omega = 1$ Cold Dark Matter (CDM) model. We demonstrate that the analysis of the distribution of velocity differences is a good method for detecting intrinsic non–Gaussian behaviour.

1 Introduction

We analyze the large–scale velocity field in a class of intrinsically non–Gaussian models, in which the initial inhomogeneities are stimulated by a primordial population of *seeds*, e.g. global textures [5] or generic seeds [7]. Our aim is to find possible indicators of the non–Gaussian behaviour of the seeded mass distribution in the large–scale peculiar velocity field. The problem has been addressed in [3]. A common feature of the *seeded models* is that the linear density field $\delta(\mathbf{r})$ may be written as a convolution of a density profile with a distribution of point–like seeds $n(m,\mathbf{x}) = \sum_h \delta_D(m - m_h)\,\delta_D(\mathbf{x} - \mathbf{x}_h)$, namely $\delta(\mathbf{r}) = \int dm\, d\mathbf{x}\; n(m,\mathbf{x})\, A(m, \mathbf{r} - \mathbf{x})$, where $\langle n(m,\mathbf{x})\rangle = n(m)$ is the mean number density of seeds characterized by the parameter m (usually the gravitational mass), and $\mathbf{x}$ are the (comoving) positions of the seeds. The function $A(m, \mathbf{r} - \mathbf{x})$ is the *accretion function* in $\mathbf{r}$ due to the presence of a seed with mass m in $\mathbf{x}$: it describes the accretion pattern density around each seed ([7], [6], [4]). In particular, we consider only models in which the seeds are *randomly* distributed, as in the seeded Hot Dark Matter model [7] or in the textures+CDM model [2]. Moreover, we assume the accretion pattern to be spherically symmetric i.e. $A(m, \mathbf{r} - \mathbf{x}) = A(m, |\mathbf{r} - \mathbf{x}|)$, which is surely a good approximation in any model involving gravitational accretion around *static* seed masses, but it

can be applied to the texture model, where the interaction is not purely gravitational [2]. The general statistical properties of the seeded density perturbation field $\delta(\mathbf{r})$ have been clarified by [4]. In the hypotheses, the seed mean number density $n(m)$ and the accretion function A *completely* determine the statistics of the seeded density field δ.

2 Seeded Velocity Field

The seeded perturbation $\delta(\mathbf{r})$ induces the peculiar velocity $\mathbf{v}(\mathbf{r})$ according to the Poisson equation or, in other words, $\mathbf{v}(\mathbf{r}) = \int dm\, d\mathbf{x}\; n(m,\mathbf{x})\, \mathbf{v}_{m,\mathbf{x}}(\mathbf{r})$, where $\mathbf{v}_{m,\mathbf{x}}(\mathbf{r})$ is the contribution to $\mathbf{v}$, measured at the point $\mathbf{r}$, due to a single m–seed located at the point $\mathbf{x}$ i.e., in comoving coordinates,

$$\nabla \cdot \mathbf{v}_{m,\mathbf{x}}(\mathbf{r}) = -H\,\Omega^{4/7}\, a\, A(m, |\mathbf{r}-\mathbf{x}|)\,; \tag{1}$$

here H is the Hubble constant, Ω is the density parameter; a is the scale factor. In practice, $A(m, |\mathbf{r}-\mathbf{x}|)$ acts exactly as a density perturbation source for the single seed–velocity contribution $\mathbf{v}_{m,\mathbf{x}}(\mathbf{r})$, and the total velocity $\mathbf{v}(\mathbf{r})$ is the superposition, for the entire population of seeds, of any single seed–velocity. As shown in [3], the one-point velocity distribution $p(\mathbf{v})$ is a rather poor indicator of non–Gaussian behaviour for models with randomly distributed seeds. In particular, for the seeded Hot Dark Matter model, a non–Gaussian velocity field requires an extremely low seed density, while the textures+CDM model has a velocity field which is nearly indistinguishable from a Gaussian one. Smoothing tends to make the velocity field even more Gaussian. Here we test for the non–Gaussian character of the large–scale matter distribution, by looking at the two–point velocity distribution and related statistics. In particular, the probability distribution of the *velocity difference* may be written as the Fourier transform of the moment generating function $\mathcal{Z}$, namely

$$p(\mathbf{v}_1 - \mathbf{v}_2) = \frac{1}{(2\pi)^3} \int d\mathbf{S}\; \mathrm{e}^{-i\mathbf{S}\cdot(\mathbf{v}(\mathbf{r}_1)-\mathbf{v}(\mathbf{r}_2))}\; \mathcal{Z}[\mathbf{S}]\,, \tag{2}$$

where

$$\mathcal{Z}[\mathbf{S}] \;\equiv\; \exp \int n(m)\, dm\, d\mathbf{x} \left(\mathrm{e}^{i\mathbf{S}\cdot(\mathbf{v}_{m,\mathbf{x}}(\mathbf{r}_1)-\mathbf{v}_{m,\mathbf{x}}(\mathbf{r}_2))} - 1 \right)\,. \tag{3}$$

Starting from this, we have numerically evaluated the *skewness* (i.e. the third central moment) of the distribution of the component of the velocity difference along the separation $\mathbf{r}_1 - \mathbf{r}_2$, $v_{||}(\mathbf{r}_1) - v_{||}(\mathbf{r}_2)$, as a function of separation distance for the $\Omega = 1$ textures+CDM model. The texture accretion function may be found e.g. in [2]. It results that the skewness of the parallel component of velocity difference (normalized to the velocity variance $\sigma_{\mathbf{v}}$) is a decreasing function of the separation distance of the observation points; in particular, it changes from 3.75 at 1 Mpc to 0.75 at 50 Mpc, the statistic being useful up to about a 10–50 Mpc separation (for $h = 1/2$). Thus, we see that, for relatively small separations, the difference of velocities *is* a good tracer of the non–Gaussian behaviour. It is trivial to extend these techniques to the case of a smoothed density field.

Applications and extensions of this formalism and analyses of higher order moments of the velocity distribution in the seeded Hot Dark Matter model and textures+CDM model are in progress [1].

References

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