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SISSA-147-95-EP

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Acceleration and Cauchy horizons in 2d black holes

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Abstract

In this paper we investigate classical and quantum properties of the solutions of a one-parameter class of exactly solvable models of two dimensional dilaton gravity. It is shown that the particular structure of these geometries, where the natural frame in which the metric is static is accelerated with respect to a distant inertial observer, may have interesting consequences.

An important feature of these spacetimes is the appearance, in some cases, of an acceleration horizon besides the usual black hole horizon associated to the black hole and the maximal extension of the geometry beyond it is not trivial.

Furthemore the redshift and blueshift factors at the black hole horizons are no more exponential as in general relativity, but power law. In particular, as a consequence, classical charged black hole solutions possessing Cauchy horizons can be stable under generic gravitational perturbations generated in the external region and the mass inflation behaviour doesn't show up. Despite this fact, however, quantum back-reaction seems to produce a scalar curvature singularity there.

Moreover, the presence of a 'natural' accelerated frame implies that at the quantum level an additional flux, the acceleration heat bath, can be present. Some possible implications of it are considered.

December 1995

1. Introduction

Two-dimensional theories of gravity have proven to be very useful and simple toy models to understand the issues related to the processes of black hole formation and evaporation. In particular the quantum corrected CGHS action [1], in the form advocated by RST [2], turned out to be an exactly solvable model describing the semiclassical physics of an evaporating black hole.

In [3] we considered a one-parameter class of exactly solvable simple models, which contain RST as a special case. We showed that the associated quantum physics is similar to the usual one, but that the classical black hole solutions present interesting differences.

The most striking feature of our models is that the natural frame in which the metric is static is not asymptotically minkowskian; this means that an inertial observer 'sufficiently far away' sees the black hole to be accelerated.

Depending on the value of the parameter characterizing our geometries the 'asymptotic region' where the spacetime becomes flat corresponds either to 'true' infinity, in the sense that it takes an infinite amount of proper time to reach it, or is just the location of another horizon, the acceleration horizon, and inertial observers there can cross it in a finite time.

Another characteristic of our spacetimes is that the redshift factor at the event horizon, which is exponential in the asymptotic inertial time coordinate t in both general relativity and CGHS theory, has now a power-law behaviour. This simple fact turns out to have interesting implications.

In fact let us consider the Reissner-Nordström solution of General Relativity. It describes charged black hole solutions and possesses two horizons, one of which is the event horizon and the other is a Cauchy horizon. Besides, since in this geometry the singularity is timelike, it seems possible that an observer travels through the black hole tunnel and then emerges into another universe [4]. However, as many authors have shown [5], the Cauchy horizon is unstable because generic small perturbations in the external universe are seen infinitely (exponentially) blueshifted by free falling observers. The backreaction on the geometry of such perturbations drastically changes the form of the spacetime, resulting in a scalar curvature singularity at the Cauchy horizon. This is the mass inflation phenomenon [6].

Our models turn out to possess classical charged black hole solutions, whose causal structure is similar to the Reissner-Nordström one, but now at the Cauchy horizon the blueshift factor is power law. And in the space of the parameters characterizing the geometry it is possible to find a region of nonzero measure for which both instability and mass inflation at the Cauchy horizon are avoided. However quantum backreaction seems to alter these conclusions. Such an analysis has already been carried out in the RST model in [7] and here similar results are obtained, suggesting that a scalar curvature singularity always forms at the Cauchy horizon, thus forbidding any further extension of the geometry above it.

2. The Classical theory

Our models are described by the classical action $S_{cl} = S_0 + S_M$, where

$$S_0 = \frac{1}{2\pi}\int d^2x\sqrt{-g}[e^{-\frac{2}{n}\phi}(R + \frac{4}{n}(\nabla\phi)^2) + 4\lambda^2 e^{-2\phi}] \tag{2.1}$$

and

$$S_M = -\frac{1}{4\pi}\sum_{i=1}^{N}\int d^2x\sqrt{-g}(\nabla f_i)^2 \ . \tag{2.2}$$

In the case $n = 1$ this reduces to the classical CGHS [1] action.

The equations of motion derived from this action are

$$g_{\mu\nu}[\frac{4}{n}(-\frac{1}{2} + \frac{1}{n})(\nabla\phi)^2 - \frac{2}{n}(\nabla^2\phi) - 2\lambda^2 e^{\frac{2-2n}{n}\phi}] + \frac{4}{n}(1 - \frac{1}{n})\partial_\mu\phi\partial_\nu\phi$$

$$+\frac{2}{n}\nabla_\mu\partial_\nu\phi + e^{\frac{2}{n}\phi}T^M_{\mu\nu} = 0 \ , \tag{2.3}$$

$$\frac{R}{n} - \frac{4}{n^2}(\nabla\phi)^2 + \frac{4}{n}\nabla^2\phi + 4\lambda^2 e^{\frac{2-2n}{n}\phi} = 0 \ , \tag{2.4}$$

$$\nabla^2 f_i = 0 \ . \tag{2.5}$$

We have already derived in [3] the solutions to the above equations. Now we just outline that we can express the general static eternal black hole solutions in the 'Schwarzschild' frame (σ, t), where

$$ds^2 = e^{2(1-n)\lambda\sigma}[-fdt^2 + \frac{1}{f}d\sigma^2], \quad \phi = -n\lambda\sigma, \tag{2.6}$$

and f is defined by $f = 1 - \frac{2m}{\lambda}e^{\frac{2}{n}\phi}$. m can be regarded as the mass of the black hole. Charged black hole solutions generalizing the definition of f to

$$f = 1 - \frac{2m}{\lambda}e^{\frac{2}{n}\phi} + \frac{Q^2}{\lambda^2}e^{\frac{4}{n}\phi} \tag{2.7}$$

have been found in the case $n = 1$ [8]. For $n \neq 1$ such solutions exist if we add to the action the electromagnetic term

$$S_{EM} = \frac{1}{2\pi}\int d^2x\sqrt{-g}e^{-2a\phi}(-2F_{\mu\nu}F^{\mu\nu}), \tag{2.8}$$

where $F_{\mu\nu}$ is the e.m. field tensor and a is a constant to be fixed later in terms of n (for the $n = 1$ theory $a = 1$).

The equations of motion of $F_{\mu\nu}$ are easily derived from S_{EM}:

$$\nabla_\nu(e^{-2a\phi}F^{\mu\nu}) = 0 \ . \tag{2.9}$$

The e.m. field tensor is totally antisymmetric and so can be written as $F_{\mu\nu} = Fe_{\mu\nu}$, where $e_{\mu\nu} = e_{[\mu\nu]}$ and $e_{01} = \sqrt{-g}$. Inserting this into (2.9) we get the solution $F_{\mu\nu} = Qe^{2a\phi}e_{\mu\nu}$, where Q is a constant representing the charge on the black hole.

Since in the following we will be interested also in solutions arising from purely incoming massless matter it is suitable to work in the light-cone gauge and look for metrics of the form

$$ds^2 = e^{2(1-n)\lambda\sigma}(-fdv^2 + 2dvd\sigma), \tag{2.10}$$

where v is an advanced time coordinate defined by $dv = dt + \frac{d\sigma}{f}$. Besides in analogy with the $n = 1$ theory we require the dilaton to be linear in the σ coordinate $\phi = -n\lambda\sigma$. The addition of S_{EM} modifies (2.4) to

$$\frac{R}{n} - \frac{4}{n^2}(\nabla\phi)^2 + \frac{4}{n}\nabla^2\phi + 4\lambda^2 e^{\frac{2-2n}{n}\phi} + 4aQ^2 e^{\frac{2an+2}{n}\phi} = 0 \ . \tag{2.11}$$

It is easy to show that

$$R = e^{-2\lambda(1-n)\sigma}[-f_{,\sigma\sigma} - 2\lambda(1-n)f_{,\sigma}] \tag{2.12}$$

and

$$\nabla^2\phi = e^{-2\lambda(1-n)\sigma}(-n\lambda)f_{,\sigma}, \quad (\nabla\phi)^2 = e^{-2\lambda(1-n)\sigma}fn^2\lambda^2 \ . \tag{2.13}$$

We fix the value of a requiring that all the Q^2 terms in (2.11) are coupled in the same way to the dilaton. This gives $a = \frac{2-n}{n}$.[1] Moreover (2.3) now reads

$$g_{\mu\nu}[\frac{4}{n}(-\frac{1}{2} + \frac{1}{n})(\nabla\phi)^2 - \frac{2}{n}\nabla^2\phi + F^{\mu\nu}F_{\mu\nu} - 2\lambda^2 e^{\frac{(2-2n)}{n}\phi}] + \frac{4}{n}(1 - \frac{1}{n})\partial_\mu\phi\partial_\nu\phi -$$

$$-4F_{\mu\alpha}F^\alpha_\nu + \frac{2}{n}\nabla_\mu\partial_\nu\phi + e^{\frac{2}{n}\phi}T^M_{\mu\nu} = 0 \ . \tag{2.14}$$

Allowing for purely ingoing f_i-wave $f_i = f_i(v)$ the vv component of (2.14) gives the additional condition

$$\frac{dm}{dv} = \frac{1}{4}\sum(\partial_v f_i)^2. \tag{2.15}$$

For $f_i \neq 0$ the solution resembles the Vaidya solution of General Relativity, apart from the conformal factor $e^{2(1-n)\lambda\sigma}$. In general m is a function of v and we will refer to the scalar m in

$$(\nabla\phi)^2 = n^2\lambda^2 e^{-2(1-n)\lambda\sigma}(1 - \frac{2m}{\lambda}e^{-2\lambda\sigma} + \frac{Q^2}{\lambda^2}e^{-4\lambda\sigma}) \tag{2.16}$$

as the mass function of the black hole.

[1] Note that for $n = 2$ the e.m. field is completely decoupled from the dilaton and the insertion of S_{EM}, once (2.9) are taken into account, is then equivalent to adding a cosmological constant to the action.

2.1. Causal structure

Consider the case $f_i = 0$, that is from (2.15) $m = m_0 = \text{const.}$, and let us discuss the causal structure of these spacetimes separately for the cases $Q = 0$ and $Q \neq 0$.

The singularity, as can be seen from a direct computation of R, is located at $\sigma = -\infty$. The horizons, as usual, are defined as the surfaces where the Killing vector $\xi^\alpha = \frac{\partial x^\alpha}{\partial t}$ has zero norm. It is easy to see, from (2.6), that

$$\xi^\alpha \xi_\alpha = -e^{2(1-n)\lambda\sigma} f \ . \tag{2.17}$$

For $n \leq 1$ the horizons are located where $f = 0$. This equation, as $Q = 0$, defines the black hole horizon $\sigma = \sigma_{bh}$, where

$$e^{2\lambda\sigma_{bh}} = e^{-\frac{2}{n}\phi_{bh}} = \frac{2m_0}{\lambda} \ . \tag{2.18}$$

When $Q \neq 0$, along with $m_0 > |Q|$, we have two horizons located at $\sigma = \sigma_\pm$, where

$$e^{2\lambda\sigma_\pm} = e^{-\frac{2}{n}\phi_\pm} = \frac{(m_0 \pm \sqrt{m_0^2 - Q^2})}{\lambda} \ . \tag{2.19}$$

σ^+ is the location of the outer, or black hole, horizon and σ^-, the inner horizon, is also a Cauchy horizon. $m_0 = |Q|$ represents the extremal case, where the two horizon coalesce, and for $m_0 < |Q|$ the equation $f = 0$ has no real solutions and the spacetime is that of a naked singularity. The Penrose diagram is identical to that of the Schwarzschild geometry for $Q = 0$ and to Reissner-Nordström as $Q \neq 0$ (see for example [9]).

More interesting is the case $n > 1$. As can be seen from (2.17), besides (2.18) or (2.19) there's one more horizon, namely that defined by $\sigma = \infty$. While for $n \leq 1$ the region $\sigma = \infty$ is truly the asymptotic region, in the sense that it is infinitely distant, in this case inertial observers can cross it in a finite time despite the fact that the accelerated (σ, t) observers never reach it. This is, in fact, just an acceleration horizon and the geometry can therefore be further extended beyond it.

Explicitly, let us consider the line element (2.6) in the double null form

$$ds^2 = -e^{2(1-n)\lambda\sigma} f du dv \ . \tag{2.20}$$

The relation between σ, t and u, v is given by

$$v = t + \int \frac{d\sigma}{f}, \tag{2.21}$$

$$u = t - \int \frac{d\sigma}{f} \ . \tag{2.22}$$

For $Q = 0$ it is

$$\int \frac{d\sigma}{f} = \sigma + \frac{1}{2\lambda} \ln |f|$$

and as $Q \neq 0$ (together with $m_0 > |Q|$)

$$\int \frac{d\sigma}{f} = \sigma + \frac{1}{2\lambda} \frac{e^{2\lambda\sigma_+}}{(e^{2\lambda\sigma_+} - e^{2\lambda\sigma_-})} \ln |1 - e^{2\lambda(\sigma_+ - \sigma)}| - \frac{1}{2\lambda} \frac{e^{2\lambda\sigma_-}}{(e^{2\lambda\sigma_+} - e^{2\lambda\sigma_-})} \ln |1 - e^{2\lambda(\sigma_- - \sigma)}|.$$

In the region $\sigma \to \infty$ it is $v \to t + \sigma$ and $u \to t - \sigma$, so we can introduce the 'asymptotic' minkowskian coordinates

$$\lambda y^+ = \frac{e^{\lambda(1-n)v}}{(1-n)}, \tag{2.23}$$

$$-\lambda y^- = \frac{e^{-\lambda(1-n)u}}{(1-n)} \tag{2.24}$$

for which the metric in region I of figs 1 and 2 ($y^+ < 0$, $y^- > 0$) takes the form (for simplicity we consider the case $Q = 0$)

$$ds^2 = -f^n(y^+, y^-) dy^+ dy^- = -\frac{dy^+ dy^-}{\left[1 + \frac{2m}{\lambda}\left(-\lambda^2(1-n)^2 y^+ y^-\right)^{\frac{1}{n-1}}\right]^n} . \tag{2.25}$$

The boundary of region I ($y^+ = 0$ or $y^- = 0$) is the acceleration horizon. There the scalar curvature vanishes. The metric of eq. (2.25) can be easily extended in region III ($y^+ > 0$, $y^- < 0$) just noting that f depends only on the product $y^+ y^-$.

In order to extend the metric in the two remaining regions II ($y^+ > 0$, $y^- > 0$) and IV ($y^+ < 0$, $y^- < 0$) one has to distinguish two different cases:

i) odd extension $(-1)^{\frac{1}{n-1}} = -1$;

ii) even extension $(-1)^{\frac{1}{n-1}} = 1$.

When neither i) nor ii) are verified then $(-1)^{\frac{1}{n-1}}$ is complex and we don't know how to give a physical meaning to a complex geometry.

New coordinates (u, v) can be defined in II, III and IV. In III the metric takes exactly the same form as in I, while in II and IV the line element becomes

$$ds^2 = e^{2(1-n)\lambda\sigma} f du dv , \tag{2.26}$$

where

$$f = 1 - (-1)^{\frac{1}{n-1}} \frac{2m}{\lambda} e^{-2\lambda\sigma} . \tag{2.27}$$

Let's consider first the odd extension. It is easy to see that both to the future of region II and to the past of region IV there is a spacelike singularity located at $\sigma = -\infty$, where the scalar curvature diverges. The proper time needed to reach the singularity is finite. The

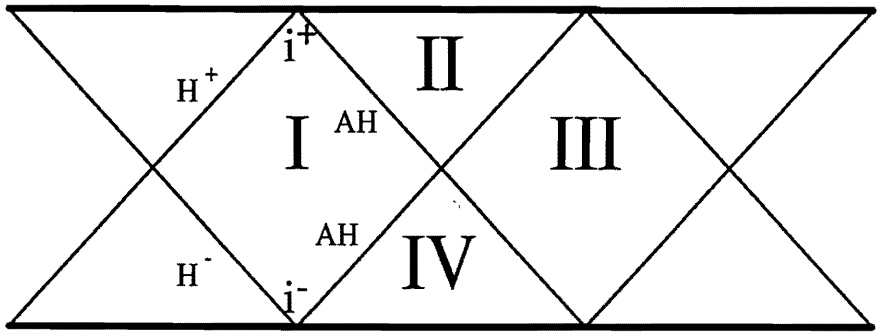

Fig. 1: Causal structure in the odd extension. The bold lines represent the singularity. $H^{\pm}$ denote future and past black hole horizons and AH is the acceleration horizon.

resulting Penrose diagram is given in fig. 1, which closely resembles that of a Schwarzschild black hole immersed in a closed Friedmann universe.

In the case of the even extension region II is free of singularities. The only difference with respect to (2.20) is the signature of the metric. The boundaries $y^+ = +\infty$ and $y^- = +\infty$ of this region are Killing horizons. By further extending the metric across these horizons we find a region where the curvature diverges on timelike singularities (see fig. 2). As can be seen from fig. 2, these horizons are Cauchy horizons for initial data given on a spacelike hypersurface extending from region I to III. Again we can repeat the procedure for further extension in the future and in the past and the result is again shown in fig. 2.

Special care is needed in order to understand the meaning of the points $i^{\pm}$ of figs 1 and 2. These are the only points in region I which are located at infinity for the accelerated observers. Every geodesic observer either cross the black hole horizon or the acceleration horizon.

As $Q \neq 0$ the procedure is analogous, but the structure much more complicated, due to the splitting of the black hole horizon in an outer and an inner one. The details will be given elsewhere. Here we just consider the analytic extension inside the black hole horizon, which leads to a structure similar to Reissner-Nordström as in the case $n \leq 1$, where the asymptotic region ($I^{\pm}$) is now replaced by the acceleration horizon.

A quantity that will be useful in the following is the local surface gravity k, defined by (see for example [10])

$$\kappa = \frac{1}{2}\left|\frac{\partial_\sigma g_{tt}}{\sqrt{-g_{\sigma\sigma}g_{tt}}}\right| = \left|\lambda(1-n)f + \frac{1}{2}f_{,\sigma}\right| \ . \tag{2.28}$$

One finds that at $\sigma_\pm$ (or, for $Q = 0$, at σ_{bh}) it is $\kappa_\pm = \frac{1}{2}|f_{,\sigma_\pm}| = \lambda e^{-2\lambda\sigma_\pm}(e^{2\lambda\sigma_+} - e^{2\lambda\sigma_-})$ (or $\kappa_{bh} = \frac{1}{2}|f_{,\sigma_{bh}} = \lambda|$). Note that, provided $m_0 > |Q|$, it is always $k_- > k_+$. In the case $n > 1$ at the acceleration horizon $\kappa_{ah} \equiv \kappa(\sigma = \infty) = \lambda(n-1)$.

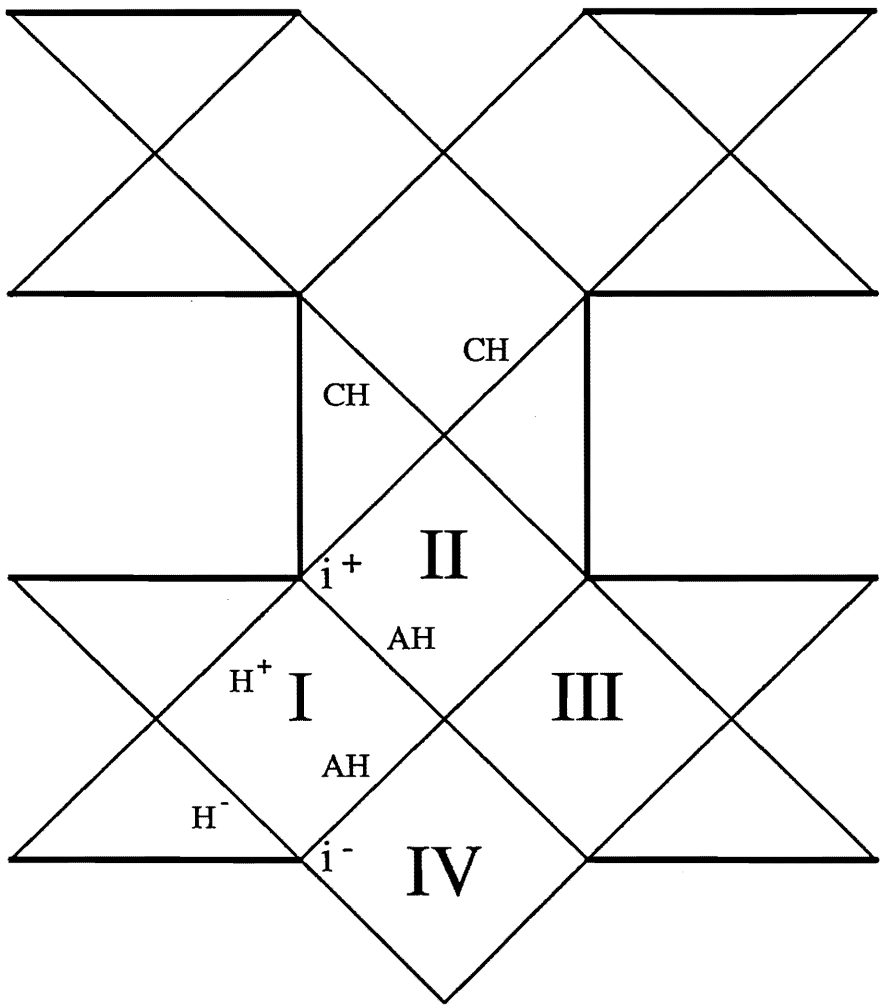

Fig. 2: Penrose diagram of the even extension. The meaning of bold lines, $H^{\pm}$ and AH are as in fig 1. CH is the Cauchy horizon for initial data in regions I and III.

3. Stability of the Cauchy horizons

Cauchy horizons in our geometries arise in several cases. As $n \leq 1$ they are present only for $Q \neq 0$ at $\sigma = \sigma_-$. When $n > 1$ we will consider two different possibilties. The first is in the charged case at $\sigma = \sigma_-$ and the second is $Q = 0$ in the case of fig. 2. The considerations that follow concern only the first such surface; we will comment on the stability of the second one at the end of the paragraph.

It has been shown [5] that the Cauchy horizon of the Reissner-Nordström spacetime is unstable and that perturbations on the background geometry induced by the gravitational collapse grow and become infinitely large in its vicinity, so that a free falling observer crossing this surface measures a divergent energy flux. A similar analysis has been carried out in the Reissner-Nordström type solution of $n = 1$ dilaton gravity [7] and the conclusions are the same.

Now we will check if the same behaviour is present for $n \neq 1$. We will see that this is not true in general and that there remains a region of nonzero measure in parameter space $(n,\, k_-)$ where the Cauchy horizon is stable to the same type of perturbations.

Such perturbations close to the Cauchy horizon follow ingoing null geodesics and decay with an inverse power law in the advanced time of the asymptotic inertial observer.

y^+, defined in (2.23), is the 'asymptotic' inertial ingoing null coordinate. In the case $n = 1$ it is $y^+ = v$, which is ∞ both on future null infinity and on the Cauchy horizon, while it is not in general for $n \neq 1$ because of the conformal factor present in the metric (see for example (2.6)). In general when expressed in 'asymptotically' minkowskian coordinates the metric is non static and the black hole is itself seen to be moving, so we will just write down the form of T_{++} and then transform to T_{vv}. Note that for $n < 1$ $y^+ \in\ [0, +\infty[$ and $y^- \in\ ]-\infty, 0]$, while as $n > 1$ $y^+ \in\ ]-\infty, 0]$ and $y^- \in\ [0, +\infty[$.
Now we can construct the perturbation appropriate for the problem we are studying and write, following [11], $T_{++} = \frac{1}{2}(\frac{df}{dy^+})^2 = \gamma(\lambda y^+)^p$, where $p > 0$ for $n > 1$ and $p < 0$ when $n < 1$, with $|p| > 12$. γ is just a constant. Transforming to v coordinates we get

$$T_{vv} = \frac{\gamma e^{\lambda(1-n)(p+2)v}}{(1-n)^p}, \tag{3.1}$$

so that the mass function from (2.15) is given by

$$m(v) = m_0 - \frac{\gamma e^{\lambda(1-n)(p+2)v}}{2\lambda(1-n)^p(n-1)(p+2)} \ . \tag{3.2}$$

This formula clarifies the fact that the black hole settles down to a final mass m_0. Let us consider a free falling observer close to the Cauchy horizon defined by $v = \infty$. Solving the geodesic equation we get $\dot{v} \sim e^{k_- v}$, where a dot means derivative with respect to the proper time. Thus the energy density he measures is

$$\rho_{obs} = T_{vv}\dot{v}^2 \sim e^{\lambda(1-n)(p+2)v + 2k_- v} \ . \tag{3.3}$$

This quantity remains finite whenever

$$(1-n)(p+2)\lambda + 2k_- \leq 0 \ . \tag{3.4}$$

There is a region, in particular close to extremality as $k_- \sim 0$, where this inequality can be satisfied by properly choosing the value of n. [2] In the case $n > 1$ a similar analysis has already been made in [12]. There the role of the acceleration horizon was played by the cosmological horizon of Reissner-Nordström-De Sitter spacetime. In particular they

[2] Again, the key feature is that while for $n = 1$ the blueshift is exponential in the null inertial time coordinate v, it has a power law behaviour in y^+ as $n \neq 1$.

considered the effect of finite perturbations on the cosmological horizon and noted that the Cauchy horizon is stable if $(k_- - k_c) \leq 0$, where k_c is the surface gravity associated with the cosmological horizon. If we now replace k_c with k_{ah} and set $p = 0$ (3.4) reproduces the same result.

This simple calculation shows that in our spacetime the Cauchy horizon is not necessarily a pathological surface and suggests that a further extension of the geometry above it could in principle be meaningful. To check the validity of the above affirmation we now consider a more realistic situation where also an outgoing flux is present and see how the geometry gets modified. Remember that both in general relativity and $n = 1$ dilaton gravity this results in a drastic change of the geometry and a scalar curvature singularity appears at the Cauchy horizon, where the mass function is seen to diverge. This is the mass inflation phenomenon.

A final remark concerns the stability of the Cauchy horizon for the case $Q = 0$ depicted in fig 2. What we need to check is essentially whether regular perturbations on the black hole horizon of region I or III are seen divergent or not by our free falling observer at the Cauchy horizon. In principle we should repeat all the previous analysis, neverthless it is enough to note, as pointed out in the discussion following (3.4), that only the comparison between the two surface gravities is relevant. And since in this case the two surface gravities coincide, we conclude that this Cauchy horizon is always stable.[3]

4. Mass Inflation

We want to construct an approximate solution to the equations of motion in the presence both of incoming and outgoing fluxes of null radiation close to the Cauchy horizon. It is useful to write down (2.11) and (2.14) in the conformal gauge and to introduce the quantity $F = e^{2(\rho-\phi)}$. It is

$$\partial_+\partial_-(\ln F) = -2Q^2 F e^{\frac{6}{n}\phi} \ , \tag{4.1}$$

$$\partial_+\partial_-(e^{-\frac{2}{n}\phi}) = (-\lambda^2 + Q^2 e^{\frac{4}{n}\phi})F \ , \tag{4.2}$$

while the constraints are

$$\partial_\pm^2(e^{-\frac{2}{n}\phi}) - \partial_\pm(e^{-\frac{2}{n}\phi})\partial_\pm(\ln F) = -\frac{1}{2}\sum_{i=1}^{N}(\partial_\pm f)^2 \ . \tag{4.3}$$

We imagine for simplicity that the inflow is turned on at a finite advanced time $x^+ = x_0^+$ and the outflow, crossing the Cauchy horizon, at $x^- = x_0^-$. The spacetime is then divided

[3] The stability of the black hole horizon to perturbations in the external regions I and III is currently under investigation.

in four different parts: a pure inflow and a pure outflow regions whose geometry is correctly described by the Vaidya type metric (2.10) and m satisfies (2.15) (in the outflow case v is replaced by u and (2.10) remains the same apart from a minus sign in $dud\sigma$), static 'Reissner-Nordström' and finally the most interesting one where both fluxes are present.

We concentrate our discussion to this last region where $x^- \sim x_0^-$. Note that for $x^- < x_0^-$ and close to the Cauchy horizon the solution is almost static (see (2.15) in the limit $v \to \infty$), with the dilaton reaching a finite value ϕ_-. We fix the x^+ coordinate in such a way that $F(x_0^-, x^+) = F(x^-, x_0^+) = e^{-2\phi_-}$ [7]. This ensures that x^+ is regularly related to the kruskal advanced time coordinate associated with the inner horizon of the Reissner-Nordström spacetime, i.e. $\sim -e^{-k_- v}$. The form of T^M_{++} then follows from (3.1) after a coordinate change, that is

$$T^M_{++} \sim (x^+)^{-\frac{\lambda(1-n)(p+2)}{k_-}-2}, \tag{4.4}$$

while the exact relation between u and x^- along with the shape of T^M_{--} ($\neq 0$) will not concern us here, since the conclusions that we'll be able to draw will result to be independent of them.

An approximate solution to F at first order in $(x^- - x_0^-)$ and close to the Cauchy horizon following from (4.1) and with the above initial conditions is

$$F \simeq e^{-2\phi_-} exp\left(-2Q^2 e^{(\frac{6}{n}-2)\phi_-}(x^- - x_0^-)(x^+ - x_0^+) \right) . \tag{4.5}$$

This permits to solve, in the same limit, the constraints (4.3) along with (4.2) giving

$$e^{-\frac{2}{n}\phi} \simeq e^{-\frac{2}{n}\phi}(x_0^+, x_0^-) - \int_{x_0^+}^{x^+} d\xi F(\xi) \int_{x_0^+}^{\xi} \frac{d\xi'}{F(\xi')} T^M_{++}(\xi') -$$

$$- \int_{x_0^-}^{x^-} d\xi F(\xi) \int_{x_0^-}^{\xi} \frac{d\xi'}{F(\xi')} T^M_{--}(\xi') + x^+(-\lambda^2 e^{-2\phi_-} + Q^2 e^{(\frac{4}{n}-2)\phi_-})(x^- - x_0^-) . \tag{4.6}$$

Note that both F and the metric stay bounded as $x^+ \to 0$. However when one calculates the Ricci scalar $R = 4F^{-1}e^{-2\phi}\partial_+\partial_-(2\phi + \ln F)$ we have a sum of different terms of which only one is potentially divergent, namely that proportional to

$$\int_{x_0^+}^{x^+} \frac{d\xi}{F(\xi)} T^M_{++} \int_{x_0^-}^{x^-} \frac{d\xi'}{F(\xi')} T^M_{--}(\xi'). \tag{4.7}$$

The result we get is that if

$$\lambda(1-n)(p+2) + k_- \leq 0 \tag{4.8}$$

then R remains finite, while when this inequality is not satisfied the Ricci scalar is seen to diverge. We note that also at this level the possibility of having a regular Cauchy horizon is not completely ruled out. Again there's a region in (n, k_-) space where the last inequality is true, in particular close to extremality, and the mass inflation doesn't occur. For $p = 0$ and $n > 1$ the same kind of result, in the context of the Reissner-Nordström-De Sitter spacetime, has been found in [12].

The next step is to see whether or not quantum effects can modify this picture.

5. Quantum effects in fixed background

In two dimensions conservation equations and conformal anomaly determine the v.e.v. of the energy-momentum tensor of a conformally coupled scalar field ϕ up to two arbitrary functions. These functions, in turn, select the vacuum state of the quantum field.
It is convenient to work in the conformal gauge where

$$ds^2 \equiv -e^{2\rho} du dv = -F f du dv. \tag{5.1}$$

Here $F = e^{2(1-n)\lambda\sigma}$ and f has been defined previously in (2.7) (here we consider only the case $m = m_0 =$ const. and $Q \neq 0$). Therefore $\rho = \lambda(1-n)\sigma + \frac{1}{2}\ln f$. A straightforward calculation gives

$$\langle T_{uu} \rangle = -\frac{N}{12}(\partial_u \rho \partial_u \rho - \partial_u^2 \rho + \frac{t_u}{4}) =$$

$$= -\frac{N}{48}[f^2((1-n)\lambda + \frac{1}{2}\frac{f_{,\sigma}}{f})^2 - f f_{,\sigma}((1-n)\lambda + \frac{1}{2}\frac{f_{,\sigma}}{f}) - \frac{f^2}{2}(\frac{f_{,\sigma\sigma}}{f} - \frac{f_{,\sigma}^2}{f^2}) + t_u] \tag{5.2}$$

and similarly for $\langle T_{vv} \rangle$. Let's consider static configurations, for which $t_u = t_v \equiv C =$ const.. We can require that as $\sigma \to \infty$ $\langle T_{uu} \rangle = \langle T_{vv} \rangle = 0$. Since in this limit the Ricci scalar vanishes this choice is the analog to select the Boulware vacuum of the $n = 1$ theory and thus defines a zero temperature ground state (for which also the inertial observers see that 'asymptotically' the energy-momentum tensor vanishes). This fixes $C = -\lambda^2(1-n)^2$. With this choice at the black hole horizon

$$\langle T_{uu} \rangle \to \frac{N}{48}[\lambda^2(n-1)^2 - k_+^2] \ , \tag{5.3}$$

where k_+ is the associated surface gravity, defined in section 2. (5.3) describes a black hole of Hawking temperature $T_H = \frac{\kappa_+}{2\pi}$ immersed in a thermal bath at the temperature $\frac{\lambda|1-n|}{2\pi}$. In general k_+ depends on both Q and m_0 and vanishes in the extremal case $m_0 = |Q|$. That the two terms in (5.3) indeed represent thermal fluxes of radiation at those temperatures can be checked for example using euclidean continuation techniques.

The energy density measured by an infalling observer, for which as $f \to 0$ $\dot{u} \sim \frac{1}{f}$ and $\dot{v}$ is finite, diverges as $\frac{1}{f^2}$. The same divergence is present, once we replace k_+ with k_- and u with v, at the Cauchy horizon.
One interesting possibility is when

$$k_+ = \lambda|1-n|, \quad n \neq 1 \ . \tag{5.4}$$

In this case ρ_{obs} is finite at the event horizon $\sigma = \sigma^+$. This is possible because the relation between the Kruskal retarded time coordinate regular at the event horizon $x^- = -\frac{1}{k_+}e^{-k_+ u}$ and the inertial coordinate at $\sigma = \infty$ $y^- = -\frac{1}{\lambda(1-n)}e^{-\lambda(1-n)u}$ (see (2.24)) becomes very simple and the Bogolubov transformation between the modes of the field ϕ defined in terms

of these two coordinates is trivial. In particular, as $n > 1$ it is $x^- = -\frac{1}{y^-}$, exactly the same relation among the corresponding coordinates of an extremal Reissner-Nordström black hole, which is known to have vanishing Hawking temperature. The new feature here is that while in the extremal Reissner-Nordström case the horizon is quantum mechanically unstable [13], now the black hole horizon is not as a result of our choice of C.

The picture which arises is now quite clear and is very similar to that of [14]. There, as here, the preferred observer is accelerating with respect to the 'asymptotic' inertial one and is comoving with the black hole. As it is well known, due to the Unruh effect [15] he feels immersed in a bath of thermal radiation (just as an accelerated observer in Minkowski spacetime). And with the fine-tuning $k_+ = \lambda|1-n|$ as $\sigma \to \infty$ this heat bath exactly balances the Hawking radiation of the black hole and no thermal effects are detected. A similar cancellation is obtained in [16] in the Reissner-Nordström-De Sitter spacetime and $y^\pm$ for $n > 1$ are the analog of the Kruskal coordinates regular at the cosmological horizon.

However, also in this case the Cauchy horizon is unstable and free falling observers keep on measuring divergent energy densities there.

6. Backreaction

The semiclassical theory we consider that keeps into account the effects of the Hawking radiation is described by the action (see [3])

$$S = S_0 + S_M + S_{EM} + \frac{\kappa}{2\pi}\int d^2x\sqrt{-g}[\frac{1-2n}{2n}\phi R + \frac{n-1}{n}(\nabla\phi)^2 - \frac{1}{4}R(\nabla^2)^{-1}R] \,. \quad (6.1)$$

where $\kappa = \frac{N}{12}$ and S_0, S_M, S_{EM} are considered in (2.1), (2.2) and (2.8).
Let's perform the field redefinitions

$$\chi = \kappa\rho + (\frac{1}{2n} - 1)\kappa\phi + e^{-\frac{2}{n}\phi}, \quad (6.2)$$

$$\Omega = \frac{\kappa}{2n}\phi + e^{-\frac{2}{n}\phi} \quad (6.3)$$

and work in the conformal gauge. If $Q = 0$ (6.1) can be cast in the 'free field' form [17]

$$S = \frac{1}{\pi}\int d^2x[\frac{1}{\kappa}(-\partial_+\chi\partial_-\chi + \partial_+\Omega\partial_-\Omega) + \lambda^2 e^{\frac{2}{\kappa}(\chi-\Omega)}] + \frac{1}{2}\sum_{i=1}^{N}\partial_+ f_i\partial_- f_i] \quad (6.4)$$

and the models become exactly solvable. For $Q \neq 0$ it is no more possible to write down explicitly the action, 'cause we don't know the inverse transformation $\phi = \phi(\chi, \Omega)$. However we can still derive the equations of motion, i.e.

$$\partial_+\partial_-\chi = (-\lambda^2 + Q^2 e^{\frac{4}{n}\phi})e^{\frac{2}{\kappa}(\chi-\Omega)} \,, \quad (6.5)$$

$$\partial_+\partial_-(\chi-\Omega)=\frac{2}{n}\kappa Q^2\frac{e^{\frac{4}{n}\phi}}{\Omega'}e^{\frac{2}{\kappa}(\chi-\Omega)}\ , \tag{6.6}$$

and the constraint equations

$$\kappa t_\pm=\frac{1}{\kappa}(-\partial_\pm\chi\partial_\pm\chi+\partial_\pm\Omega\partial_\pm\Omega)+\partial_\pm^2\chi+\frac{1}{2}\sum_{i=1}^{N}\partial_\pm f_i\partial_\pm f_i. \tag{6.7}$$

Here the choice of t_+ becomes crucial. It measures how the physical vacuum state is related to the vacuum defined in terms of the modes $x^\pm$, regular at the Cauchy horizon, and transforms with the Schwarzian derivative. We could require, as in [7], physical conditions such that the black hole is in thermal equilibrium with a heat bath in order to isolate effects on the interior from those of the evaporation or, as we did in [3], just demand that $t_+(v)=0$. (6.1) was determined by this last requirement in the case $Q=0$. In general the form of the quantum action in terms of the fields ρ and ϕ will depend on the physical conditions assumed, that is on the type of vacuum state of the scalar fields, but what matters here is that (6.5), (6.6) and (6.7) remain the same. And when we go to the Kruskal x^+ coordinate ($=0$ on the Cauchy horizon) we find that the leading behaviour is $t_+(x^+)\sim\frac{\alpha}{x^{+2}}$, where α is a positive constant (this is easily seen by performing the Schwarzian derivative of $e^{-k_+v}=(x^+)^{\frac{k_+}{k_-}}$ or $v=-\frac{1}{k_-}\ln(-k_-x^+)$ with respect to x^+). The form of both $t_-(x^-)$ and T^M_{--} is again not important for our discussion as long as they remain $\neq 0$.

Much care is needed in order to determine whether or not the r.h.s. of (6.6) diverges before the inner horizon is encountered. Classically, there's a range of values of m_0, Q and λ such that it is always $e^{-\frac{2}{n}\phi}>e^{-\frac{2}{n}\phi_{cr}}=\frac{\kappa}{4}$ as $\sigma\geq\sigma_-$. Moreover at the quantum level the t_+ considered is equivalent to introduce a negative energy flux along the Cauchy horizon, thus providing a 'defocussing' effect that is likely to guarantee that $\Omega'\neq 0$ during the entire evolution up to $x^+\to 0$. In fact it is easy to show (see [7]) that, as $x^+\to 0$, we are again in a weak coupling region and (only finite terms are written)

$$\chi\sim\Omega\sim-\alpha\kappa\ln(-\lambda x^+)+\kappa\int^{x^-}d\xi\int^{\xi}d\xi'[t_-(\xi')-T^M_{--}(\xi')]. \tag{6.8}$$

The Ricci scalar in the same limit behaves as

$$R\sim ne^{(\frac{4}{n}-2)\phi}\frac{1}{x^+}\int^{x^-}d\xi[T^M_{--}(\xi)-t_-(\xi)]\ . \tag{6.9}$$

Provided the term inside the square bracket is positive, R diverges to $-\infty$ as $n>0$ and to $+\infty$ for $n<0$.

Therefore while at the classical level we could always choose our parameters n and k_- such that the spacetime is regular at the inner horizon, quantum-mechanically this is no more possible and a singularity always forms.

Acknowledgements: It is a pleasure to thank D. Amati, R. Balbinot, R. Brout and J.G. Russo for help and very useful discussions.

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