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DIRAC FERMION IN EUCLIDEAN METRIC AND THE STRONG CP PROBLEM

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ABSTRACT

In Euclidean metric the usual description of a Dirac fermion needs two independent Dirac fields for $\psi(x)$ and $\bar{\psi}(x)$. In the instanton sector this leads to the first option for the fermion path integral which vanishes in the chiral limit due to the zero eigenvalues of the Euclidean Dirac operator. We propose here a novel representation of the conjugate field $\bar{\psi}(x)$ which depends on $\psi(x)$, and is consistent with the Osterwalder-Schrader injunction $\bar{\psi}(x) \neq \psi^{\dagger}(x)$ and the chiral properties of the fermion Lagrangian. This alternative scenario yields the second option for FPI whose chiral limit coincides with the product of only the nonzero eigenvalues of the singular Dirac operator. The strong CP problem and 't Hooft's $U(1)$-dilemma in effective Lagrangian models, both are legacies of the first option for FPI. The second option is free from these blemishes.

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1. Introduction

In Euclidean metric Yang-Mills fields reveal interesting mathematical structures. The structure which seems to have had profound effect on the developement of QCD over the years is the instanton configuration characterised by a non-trivial winding number $\nu \neq 0$

$$\nu \equiv \frac{1}{16\pi^2} \int Tr(F_{\mu\nu}\tilde{F}_{\mu\nu})\, d^4x. \tag{1.1}$$

Effects of instanton configurations manifest directly in the QCD action

$$S_{QCD} \equiv \int \bar{\psi}(\not{D} - im)\psi\, d^4x + \frac{1}{4}\int Tr(F_{\mu\nu}F_{\mu\nu})\, d^4x + \Delta S \tag{1.2}$$

through the θ-term

$$\Delta S \equiv \frac{i\theta g^2}{16\pi^2} \int Tr(F_{\mu\nu}\tilde{F}_{\mu\nu})\, d^4x. \tag{1.3}$$

If $\nu = 0$, i.e., in the absence of instantons, the θ-term drops out. The θ-term is odd under CP and T transformations, and, therefore, a potential source of CP violation in strong interactions.

In the fermion sector the signature of a non-trivial winding number of Yang-Mills fields is the emergence of normalisable zero modes of the Euclidean Dirac operator. In the kernel space, i.e., in the space spanned by eigenfunctions corresponding to zero eigenvalues, the Dirac operator $\not{D}$ and the γ_5 matrix can be simultaneously diagonalised. This means that the normalisable zero modes are all of definite chirality. The precise relation between the winding number ν and the zero modes is given by the Atiyah-Singer index theorem

$$\nu = n_+ - n_-, \tag{1.4}$$

where $n_+(n_-)$ is the number of normalisable zero modes of positive (negative) chirality.

The fermionic weight of instanton configuration of gluon fields is represented by the fermion path integral (FPI)

$$Z_f(m) \equiv \int d\mu \, exp\left[-\int \bar{\psi}(\not{D} - im)\psi \, d^4x\right] \tag{1.5}$$

where $d\mu$ denotes the integration measure for Dirac fermions. The first and the popular option is to identify the FPI in chiral limit $m = 0$ with the determinant of the Dirac operator [1-3]

$$Z_f(m = 0) = det\not{D}. \tag{1.6}$$

In non-trivial sector the Euclidean Dirac operator is singular and non-invertible and as a result the fermion propagator is undefined. What is puzzling is that in relativistic Minkowski metric there is no identifiable signature of the zero modes of the Euclidean Dirac operator. The second option proposed sometime ago by Gamboa-Saravi *et al* [4] gets around this problem by prescribing the value of the FPI in chiral limit as the product of only the non-zero eigenvalues of the singular Dirac operator

$$Z_f(m = 0) = \lim_{\epsilon \to 0} \frac{det(\not{D} + \epsilon I)}{\epsilon^n} \equiv det'\not{D}. \tag{1.7}$$

In $\nu = 0$ sector the two options, (1.6) and (1.7) coincide. Differences manifest only when $\nu \neq 0$. A resolution of the ambivalence between the two options, therefore, holds the key to the physics emanating from instanton configurations of gluon fields, and in particular, to problems arising from strong CP violation implied by the QCD θ-term (1.3).

To resolve this ambivalence from first principles the natural starting point is to parametrise the Dirac fields $\psi(x)$ and $\bar{\psi}(x)$ in terms of Grassmann generators [5]. Precisely at this stage, one is confronted with the second ambivalence. Question is whether the Grassmann generators needed to parametrise $\bar{\psi}(x)$ should depend on or be independent of those employed for $\psi(x)$. Stated differently, must we employ

two independent Dirac fields for $\psi(x)$ and its conjugate $\bar{\psi}(x)$ to describe a single Dirac fermion in Euclidean metric, or, as in relativistic field theory, the fields $\psi(x)$ and $\bar{\psi}(x)$ associated with the Dirac fermion must be related to and derivable from each other? The only guideline, thanks to Osterwalder and Schrader [6] (OS), is the injunction that in Euclidean metric

$$\bar{\psi}(x) \neq \psi^{\dagger}(x). \tag{1.8}$$

This is dictated by the hermiticity properties of the Dirac propagator in Euclidean metric. To implement the injunction (1.8), one possible way, advocated by the authors of ref.6, is to assume that $\bar{\psi}(x)$ is a new Dirac field independent of $\psi(x)$. Though this was mooted originally as a suggestion, it propagated in the literature [7] as the only available option. Deviation [8] from this orthodox attitude is a recent phenomenon. An extra Dirac field, in principle, suggests that in the passage from relativistic theory to Euclidean metric the degrees of freedom of a Dirac fermion has doubled [6,8]. This is not only unnatural but unnecessary.

A precise description of a Dirac fermion is fundamental to the study of FPI. A resolution of the first ambivalence, therefore, depends in a large measure on the position one adopts vis-a-vis the second, viz., whether only one or two Dirac fields are employed to describe a Dirac fermion. This, however, is not the whole story. A crucial input in this resolution is how a 'global' chiral rotation is realised in Euclidean metric. We shall see in sec.2 that if either $\bar{\psi}(x)$ is independent of $\psi(x)$, or if chiral rotation is realised as a non-unitary scale transfromation [8]

$$\psi(x) \rightarrow e^{\beta\gamma_5}\psi(x) \qquad , \qquad \bar{\psi}(x) \rightarrow \bar{\psi}(x)e^{\beta\gamma_5}, \tag{1.9}$$

the natural result for FPI is the first option (1.6). On the other hand, if we insist that not only should the fields $\psi(x)$ and $\bar{\psi}(x)$ depend on and be derivable from each other but they must also transform unitarily under chiral rotation

$$\psi(x) \rightarrow e^{i\alpha\gamma_5}\psi(x) \qquad , \qquad \bar{\psi}(x) \rightarrow \bar{\psi}(x)e^{i\alpha\gamma_5} \tag{1.10}$$

then, as demonstrated in sec. 3, the inescapable result for FPI is the second option (1.7).

An artifice to which recourse is taken almost universally to extract the CP violating effects of ΔS, is the Baluni trick [9, 10] of implementing a Global chiral $U(1)$ rotation (1.10) of fermi fields in FPI. We show in sec.4 that the two options respond differently to this innocuous procedure. In the first option the parameter θ in ΔS (1.3) can be changed at will and for the special choice $\alpha = \frac{1}{2}\theta$ for the parameter of chiral rotation, ΔS can be traded for the Baluni term [9, 10]

$$\Delta' S = \theta m \int \bar{\psi}\gamma_5\psi \, d^4x \tag{1.11}$$

In the second option, however, ΔS remains unaffected, with the Baluni term (1.11) appearing in the fermion sector as an extra piece involving the θ-parameter. The anomalous transformation of the θ-term under chiral rotation is the genesis of the $U(1)$ dilemma [11, 12] in effective Lagrangian models. It is, therefore, argued in sec.4 that the $U(1)$ dilemma is a legacy of the first option (1.6) for FPI. Lagrangian models conforming to the second option (1.7) are not afflicted with this dilemma.

The Baluni term is potentially unphysical in the framework of the second option for FPI. What prevents it from being unphysical in the first option, is the presence of the fermion zero modes. We show in sec.5 that neither perturbative treatments nor current algebraic methods are capable of probing this essentially non-perturbative feature [13]. Even the state of the art technology in lattice calculations may not prove equal to the task and unravel the CP violating effects of the Baluni term [14]. Taking cue from this, several authors [15] have recently pointed out ambiguities in the derivation of a large NEDM within the framework of effective Lagrangians [16].Irrespective of this controversy, one can at least conclude that, like the $U(1)$ dilemma, the large NEDM obtained in effective Lagrangian models is a legacy of the first option for FPI.

2. The Euclidean Dirac Fermion and the First Option for FPI

In Euclidean metric the Dirac operator defined as

$$\not{D} \equiv \gamma_\mu(i\partial_\mu - gA_\mu), \tag{2.1}$$

with γ-matrices obeying the algebra $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$, is hermitian with real eigenvalues λ_n and orthonormal eigenfunctions $\phi_n(x)$.

$$\not{D}\phi_n(x) = \lambda_n(x)\phi_n(x), \qquad \int \phi_n^\dagger(x)\phi_m(x)\, d^4x = \delta_{mn}. \tag{2.2}$$

Each non-zero eigenvalue λ_n has its chirally conjugate partner $-\lambda_n$ with eigenfunction ϕ_{-n}

$$\not{D}\phi_{-n} = -\lambda_n\phi_{-n} \quad , \quad \phi_{-n} = \gamma_5\phi_n. \tag{2.3}$$

In non-trivial sector there will be zero eigenvalues. The normalisable eigenfunctions corresponding to these eigenvalues are all of definite chirality. Assume, for definiteness, $\nu = 1$ so that there is only one zero eigenvalue with eigenfunction ϕ_0 of positive chirality

$$\not{D}\phi_0 = 0 \quad , \quad \phi_0 = \frac{1}{2}(1+\gamma_5)\phi_0. \tag{2.4}$$

The set $\{\phi_n(x)\}$ together with $\phi_0(x)$ constitute a complete basis in function space. The Dirac field can be expanded in this basis as

$$\psi(x) = \sum_{\lambda_n > 0} (a_n + a_{-n}\gamma_5)\phi_n(x) + a_0\phi_0(x), \tag{2.5}$$

where the sum is over only positive eigenvalues. The four degrees of freedom corresponding to each mode of a Dirac field is accounted for if we split [2] $a_{\pm n}$ as

$$a_{\pm n} = (\alpha_{\pm n} + i\beta_{\pm n}) \tag{2.6}$$

with $\alpha_{\pm n}$, $\beta_{\pm n}$ real-valued Grassmann generators. The mode ϕ_0, which is essentially a Weyl mode, has only two degrees of freedom

$$a_0 = \alpha_0 + i\beta_0 \tag{2.7}$$

The hermitian conjugates $a^*_{\pm n} = (\alpha_{\pm n} - i\beta_{\pm n})$, $a^*_0 = (\alpha_0 - i\beta_0)$ are the involutions [5] of $a_{\pm n}$ and a_0 respectively. The generators, therefore, define a Grassmann algebra with involution [5].

It is convenient to substitute $\chi^\dagger(x)$ for $\bar{\psi}(x)$ in the fermion action

$$S_E(m) \equiv \int \chi^\dagger(x)(\not{D} - im)\psi(x)\, d^4x$$

$$\chi^\dagger(x) \equiv \bar{\psi}(x). \tag{2.8}$$

The second ambivalence is about whether to make an ansatz of the form

$$\chi(x) = O(x)\psi(x), \tag{2.9}$$

where the only constraint imposed by the OS injunction (1.8) is that the operator $O(x)$ should be non-trivial. $SO(4)$ and gauge invariance are implemented by an $O(x)$ invariant under these transformations. In ref.6, Osterwalder and Schrader recommended the alternative scenario wherein $\chi(x)$ is independent of $\psi(x)$ and a relation like (2.9) does not exist. In the OS scenario [6] the conjugate Dirac field admits of an expansion exactly analogous to (2.5)

$$\chi(x) = \sum_{\lambda_n > 0} (b_n + b_{-n}\gamma_5)\phi_n(x) + b_0\phi_0(x), \tag{2.10}$$

with a new set of complex-valued generators $\{b\}$, unrelated to the set $\{a\}$ employed for $\psi(x)$. The hermitian conjugates $b^*_{\pm n}$, b^*_0 are, as before, the involutions in the Grassmann algebra [5]. The formula (2.8) for the fermion action yields the result

$$S_E(m) = \sum_{\lambda_n > 0} [(\lambda_n - im)b^*_n a_n + (-\lambda_n - im)b^*_{-n}a_{-n}] - imb^*_0 a_0 \tag{2.11}$$

where the subsets $\{b\}$ and $\{a^*\}$ are absent. To integrate over the subsets of generators $\{a\}$ and $\{b^*\}$ which appear in the action one introduces the corresponding truncated measure

$$\begin{aligned} d\mu &= db_0^* da_0 \prod_{\lambda_n > 0} db_n^* da_n db_{-n}^* da_{-n} \\ &\equiv \prod db^* da \end{aligned} \tag{2.12}$$

and obtains

$$\int d\mu \, exp[-S_E(m)] = det(\not{D} - im). \tag{2.13}$$

In chiral limit this agrees with the first option (1.6) for FPI.

In the OS scenario [6] the number of effective degrees of freedom is reduced by a factor half by excluding essentially the operation of hermitian conjugation in the fermion sector. This is done in two steps. First, only the component $S_E(m)$ instead of the full hermitian form $[S_E(m) + S_E^\dagger(m)]$ is considered as the fermion action. The second step consists in integrating with the truncated measure (2.12) instead of the full measure $\prod db^* dbda^* da$. In the absence of these prescriptions of truncating both the action and the measure the result would be $det(\not{D}^2 + m^2)$ instead of (2.13) as expected for the FPI for two independent Dirac fields with requisite number of dynamical degrees of freedom. An obvious fallout of the loss of the operation of hermitian conjugation is that the two point function $\langle \psi^\dagger(y)\chi(x)\rangle$, which is hermitian conjugate to the fermion propagator $\langle \chi^\dagger(x)\psi(y)\rangle$, is incalculable in the OS scenario. The degrees of freedom , i.e., the Grassmann generators, associated with $\chi(x)$ and $\psi^\dagger(y)$ are not dynamical. Recall that the injunction (1.8), which is the genesis of the OS scenario [6], has its origin in the mismatch of the hermiticity properties of the Green function $\langle \psi^\dagger(x)\psi(y)\rangle$, and the perturbative Dirac propagator in Euclidean metric. It is, therefore, rather strange that precisely this hermitian conjugate, $\langle \chi(x)\psi^\dagger(y)\rangle$, of the fermion propagator should be incalculable in the OS scenario.

The unnaturalness of having to describe a single Dirac fermion with two independent Dirac fields and the problem of hermiticity arising from this, prompted Mehta [8] to seek a conjugate field $\chi(x)$ of the form (2.9). He argues for $O(x) = i\gamma_5$, which is the obvious choice if $O(x)$ is space-time independent. This only means that in the expansion (2.10), one should substitute ia_n^* (ia_{-n}^*) for b_{-n}^* (b_n^*) and ia_0^* for b_0^*. Once again one obtains the first option for FPI. The problem with the choice $O(x) = i\gamma_5$ is that under chiral rotation, $\chi(x)$ transforms identically as $\psi(x)$. This means that with the unitary realisation $\psi(x) \rightarrow e^{i\alpha\gamma_5}\psi(x)$ of chiral rotation the mass term in the fermion action (2.8) will be invariant and the 'kinetic energy' term non-invariant. The scenario will, therefore, be exactly opposite to what physics demands. On the other hand, the non-unitary realisation (1.9) of chiral rotation, recommended in ref.8, is inappropriate for the popular $U(3)\,X\,U(3)$ chiral symmetry in standared model. Another obvious fallout of this unconventional realisation is that the Jacobian (see sec.4) for chiral rotation is real and cannot cancel the imaginary θ-term (1.3) in the fermion action. Thus in estimating physical effects of the θ-term, e.g., NEDM, the conventional route via a chiral quark mass term, like the Baluni term (1.11), is not available in this approach.

3. The Euclidean Dirac Fermion and the Second Option for FPI

Both in the OS scenario [6] and in the scenario presented in ref.8, the unsatisfactory features arising from the definition of the conjugate Dirac field manifest already in the trivial $\nu = 0$ sector. They have nothing to do with fermion zero modes. Let us first assure ourselves that there exists a representation for $\chi(x)$ free from these blemishes.

First observe that an infinitesimal unitary chiral rotation

$$\psi(x) \rightarrow (1 + i\theta\gamma_5)\psi(x) \tag{3.1}$$

induces among the generators for $\psi(x)$ a transformation

$$a_n \to a_n + i\theta a_{-n} \quad , \quad a_{-n} \to a_{-n} + i\theta a_n. \tag{3.2}$$

Conversely, the transformation (3.2) of the generators implements a unitary chiral rotation (3.1) for $\psi(x)$. It is now easy to verify from (3.2) that the combination $(a_n - a_{-n}\gamma_5)$ transforms as

$$(a_n - a_{-n}\gamma_5) \to (a_n - a_{-n}\gamma_5)(1 - i\theta\gamma_5).$$

This means that the representation [17]

$$\chi(x) = \sum_{\lambda_n > 0} (a_n - a_{-n}\gamma_5)\phi_n(x) \tag{3.3}$$

realises for the conjugate field the desired transformation $\chi(x) \to (1 - i\theta\gamma_5)\chi(x)$ for infinitesimal chiral rotation.

One can motivate the representation (3.3) also from an analogy with the formula $\chi(x) = \gamma_0\psi(x)$ valid in Minkowski metric. The opposite chiral transformation property of $\chi(x)$ is realised through the γ_0 matrix. In Euclidean metric, where no distinction is to be made between time and space components, the opposite chiral property of the conjugate field is achieved by substituting for γ_0 a matrix of the form $P_\mu\gamma_\mu$ with P_μ a suitably normalised four vector. The twin requirements of $SO(4)$ and gauge invariance lead to the unique choice $P_\mu \propto D_\mu$, the covariant derivative, and hence to the representation [17]

$$\chi(x) = \lim_{\epsilon \to 0} \frac{1}{(\not{D}^2 + \epsilon^2)^{1/2}} \not{D}\psi(x). \tag{3.4}$$

for the conjugate field. The infinitesimal parameter ϵ ensures that the normalisation factor makes sense even in the kernel space of $\not{D}$. One easily verifies the representation (3.3) from (3.4). Substituting (3.3) for $\chi(x)$ in the formula (2.8) for fermion action one obtains

$$S_E(m) = \sum_{\lambda_n > 0} \{\lambda_n(a_n^* a_n + a_{-n}^* a_{-n}) - im(a_n^* a_n - a_{-n}^* a_{-n})\}. \tag{3.5}$$

Integration with the full fermion measure appropriate for the single Dirac field $\psi(x)$

$$d\mu = \prod_{\lambda_n > 0} da_n^* da_n da_{-n}^* da_{-n} \tag{3.6}$$

gives the desired result for FPI in the trivial $\nu = 0$ sector

$$Z_{\nu=0}(m) \equiv \int d\mu \, exp(-S_E(m)) = \prod_{\lambda_n > 0} (\lambda_n^2 + m^2). \tag{3.7a}$$

In non-trivial $\nu \neq 0$ sector the central question is whether the zero modes can appear in the representation for $\chi(x)$. In the present framework, where $\chi(x)$ depends on and is derived from $\psi(x)$, as in eq.(2.9), and, further, has the opposite chiral transformation property there is no way for zero modes to appear in $\chi(x)$. A priori, this should be evident from the formula (3.4). The Dirac operator $\not{D}$ annihilates the zero modes, if any, present in $\psi(x)$. To probe further, note that each zero mode is really a Weyl mode endowed with just two degrees of freedom. For the positive chirality Weyl mode $\phi_0(x)$, for example, these two degrees of freedom are already accounted for by a_0, the complex-valued generator (2.7) in the representation (2.5). Under chiral rotation a_0 acquires a phase $a_0 \rightarrow a_0(1+i\alpha)$. If $\phi_0(x)$ were to appear in $\chi(x)$ then, according to eq. (2.9), we must have

$$(1 - i\alpha)O(x) = O(x)(1 + i\alpha). \tag{3.8}$$

to implement the opposite chiral transformation property for $\chi(x)$. This, obviously, is not possible. In the case of the contribution from a Dirac mode $\phi_n(x)$, however, reversal of chiral property is not a problem. This is because each Dirac mode comprises two Weyl modes of opposite chirality. The Grassmann generators corresponding to the two Weyl modes can be swapped to realise the chiral property appropriate for the contribution from the Dirac mode $\phi_n(x)$ to the conjugate field $\chi(x)$. In the OS scenario [6], this problem is a non-issue, simply because the

conjugate field is independent of $\psi(x)$ and one is free to invent the generators for $\chi(x)$ without any prejudice for the generators employed for $\psi(x)$.

The upshot of all this is that fermion zero modes cannot appear in the conjugate field and the representation (3.3) for $\chi(x)$ remains valid even in the nontrivial $\nu \neq 0$ sector. The significance of this remarkable result is immediately realised if one substitutes the representation (3.3) for $\chi(x)$ in the formula (2.8) for fermion action. All the degrees of freedom associated with the zero modes in the representation for the Dirac field $\psi(x)$, e.g., the Grassmann generators a_0 in the representation (2.5), drop out from the fermion action, and one recovers in the nontrivial sector the same expression for $S_E(m)$ as that given in (3.5). The integration measure appropriate for this action is therefore, the same as in eq. (3.6). The Grassmann generators associated with the zero modes in the Dirac field, are therefore, absent both in the action and the integration measure. One recognises the similarity with the sets of generators $\{b\}$ and $\{a^*\}$ in the OS scenario [6] discussed in sec.2. The degrees of freedom associated with the zero modes in $\psi(x)$, are, therefore, redundant, and in calculating the Green function of the theory the effective Dirac field can be represented by the truncated expansion

$$\psi_E(x) = \sum_{\lambda_n > 0} (a_n + a_{-n}\gamma_5)\phi_n, \tag{3.9}$$

without the zero modes. It is now a simple matter to convince oneself that the zero eigenvalues of the Dirac operator do not contribute to FPI in an instanton background

$$Z_{\nu \neq 0}(m) = \prod_{\lambda_n > 0} (\lambda_n^2 + m^2). \tag{3.7b}$$

In the chiral limit (3.7b) coincides with the second option (1.7) for FPI.

It may be verified that Euclidean Green functions in the present formulation correspond to a unique relativistic field theory. For this, it is necessary that they obey the physical positivity condition [17] which allows to define a positive metric

Hilbert space of physical states. The positivity condition is defined in terms of the antilinear mapping Θ. In the fermion sector Θ maps the Dirac field $\psi(x)$ to its conjugate $\chi^\dagger(x)$ and vice-versa [18]

$$\Theta\psi(x) = \gamma_0\chi^\dagger(\underline{x}) \quad , \quad \Theta\chi^\dagger(x) = \gamma_0\psi(\underline{x}) \tag{3.10}$$

with $\underline{x} = (-x_0, x_1, x_2, x_3)$. The positivity condition

$$\langle \Theta F, F\rangle \geq 0 \tag{3.11}$$

was proved by Osterwalder and Seiler [18] for gauge invariant functions $F(x)$ of the Dirac field $\psi(x)$ and its conjugate $\chi^\dagger(x)$ at positive times $x_0 > 0$. This then allows to define [18] the physical Hilbert space with respect to the norm $\| A \|^2 = \langle \Theta A, A\rangle$. For gauge field theory of Dirac fermions the proof of the positivity condition was carried out by Osterwalder and Seiler [18] on the lattice in the framework of OS scenario [6] where $\psi(x)$ and $\chi^\dagger(x)$ are independent of each other. The same proof seems to follow mutatis mutandis when the Dirac field $\psi_E(x)$ and its conjugate $\chi^\dagger(x)$, as given in eqs. (3.9) and (3.3) respectively, are dependent on each other.

Recently, Kupsch [20] has given a construction of Euclidean Dirac fields as transforms of white noise, which reproduces all Schwinger functions and has the correct behaviour under Euclidean and gauge transformations. It is remarkable that this construction is in complete agreement with the representation (3.4) for conjugate Dirac field which has been motivated [17] essentially by intuitive arguments. In formulating Euclidean field theory for fermions, difficulty really stems from short distance singularities which tend to become non-renormalisable when one tries to express Schwinger functions in an interacting theory in terms of *free* Euclidean fields [21]. It is, therefore, significant that gauge interactions of fermion and gauge invariance enter in a fundamental way through the definitions of the conjugate field (3.4) and of the fermion measure (3.6).

4. Global Chiral Rotation and the $U(1)$ Dilemma

It is remarkable that all estimates in literature for NEDM and strong CP violation have been obtained, not directly from the θ-term (1.3), but either from the Baluni term (1.11) or from its representation in effective Lagrangians. The alibi for this is that the fermion measure $d\mu$ in the vacuum functional in QCD

$$Z_{QCD} \equiv \int d\mu \mathcal{D}A \, exp[S_{QCD}] \tag{4.1}$$

would acquire under global chiral $U(1)$ rotation a non-trivial jacobian $J(\alpha)$ related to the Adler-Bell-Jackiw anomaly. For a suitable value of the parameter α of chiral rotation (1.10), $lnJ(\alpha)$ may cancel the θ-term (1.3). The Baluni term (1.11) arising from the chiral rotation of the quark mass term in S_{QCD} (1.2) will in that case be the only surviving signature of the θ-term in QCD action. The credibility of this alibi, and, more important, whether any real simplification is achieved in this apparently innocuous procedure deserve careful scrutiny.

The Jacobians in the two options for FPI will naturally be different and can be calculated directly from Fujikawa's [22] result

$$ln\, J(\alpha) = -2i\alpha \int \sum \phi_n^\dagger(x)\gamma_5\phi_n(x)\, d^4x \tag{4.2}$$

where the sum is over the dynamical modes dictated by the fermion measure $d\mu$. In the second option, only Dirac modes with non-zero eigenvalues are dynamical. From the orthonormality of $\phi_n(x)$ and $\phi_{-n}(x) = \gamma_5\phi_n(x)$, it follows that $ln\, J(\alpha)$ vanishes [4] in the second option (1.7). In the first option (1.6), however, the dynamical modes encompass the Weyl modes (2.4) corresponding to zero eigenvalues. This makes all the difference, because, according to (4.2) $ln\, J(\alpha)$ is in this case given by $-2i\alpha(n_+ - n_-)$ which coincides with $-2i\alpha\nu$ by the index theorem (1.1). The net result is that the Baluni trick leads to different actions in the two options

for FPI :

$$S^1_{QCD} = \int \left[\bar{\psi}(\not{D} - im)\psi + \frac{1}{4}Tr(FF)\right] d^4x + \Delta' S \tag{4.3}$$

in the first option, and

$$S^2_{QCD} = \int \left[\bar{\psi}(\not{D} - im)\psi + \frac{1}{4}Tr(FF)\right] d^4x + \Delta' S + \Delta S \tag{4.4}$$

in the second, with ΔS and $\Delta' S$, as before, the θ-term (1.3) and the Baluni term (1.11) respectively.

Note that in their respective frameworks the actions (4.3) and (4.4) are equivalent to the same old QCD action (1.2). From this, one thing is clear. In the second option the Baluni term is unphysical and cannot be held responsible for any observable CP violation. This is not so for the Baluni term in the action (4.3) which corresponds to the first option. There, the θ-term ΔS has been traded for the Baluni term $\Delta' S$. The generators for fermion zero modes in the measure $d\mu$ (2.12) constitute the link beween $\Delta' S$ and ΔS. If this link is snapped through inadequacies in approximation schemes the interchangeablity of $\Delta' S$ and ΔS is lost and $\Delta' S$ becomes unphysical. Thus, even in the first option (1.6) for FPI, estimates of NEDM and strong CP violation derived from the Baluni term, if non-trivial, should be regarded as spurious unless the theoretical probes satisfy the litmus test of being sensitive to fermion zero modes. Alas, theoretical probes based on perturbative treatments or on current algebraic methods in the soft meson approximation do not qualify in this test. It is not enough, as is popularly believed [14-16] for the probe to be able to detect the triangle (ABJ) anomaly. Perturbative calculations are quite capable of that, but fermion zero modes and a non-trivial θ-term are essentially non-perturbative, and, therefore, beyond the pale of perturbation theory. Even with the state of the art technology in the lattice framework it will be difficult [14] to unravel the CP violating effect of a non-trivial θ-term.

An interesting feature of the θ-term in the first option, revealed by the Baluni trick, is that the θ- parameter can be changed at will by a global chiral rotation. A priori, the QCD θ-term (1.3) is built out of gluon fields and therefore expected to be chirally invariant. The gluon fields can, however, acquire chirality in a certain sense [11] from the hidden fermionic degrees of freedom when they are integrated out. One may think in this connection of the ABJ anomaly equation in the first option:

$$\langle \partial_\mu J_{\mu 5} \rangle^{(1)} = \langle 2m\bar{\psi}\gamma_5\psi \rangle + \frac{ig^2}{8\pi^2} \langle tr\ F_{\mu\nu}\tilde{F}_{\mu\nu} \rangle \tag{4.5}$$

This does not yet guarantee a non-trivial chirality for $F\tilde{F}$. For instance, the axial current and its divergence, though built out of quark fields, are chirally invariant. The non-trivial chirality of $F\tilde{F}$ manifests only in the instanton sector from Fujikawa's [22] identity

$$\begin{aligned} \frac{g^2}{16\pi^2} \int tr\ F_{\mu\nu}\tilde{F}_{\mu\nu}\ d^4x &= \int \sum_n \phi_n^\dagger(x)\gamma_5\phi_n(x)\ d^4x \\ &= n_+ - n_- \end{aligned} \tag{4.6}$$

Each Dirac mode (recall our discussions in Sec.3) corresponds to two Weyl modes of opposite chirality and their contributions cancel in the sum. It is the unpaired Weyl modes corresponding to zero eigenvalue that survive. This is the reason why for global chiral rotation the fermionic Jacobian in the first option is non-trivial and brings about a change in the θ-parameter of the θ-term ΔS.

In an effective theory there is no fermion measure and hence no Jacobian. The piece representing the θ-term in the effective Lagrangian should incorporate the effects of the missing Jacobian and therefore break chiral invariance explicitly like the mass term in QCD. There is a price to pay for this procedure when one considers the 'anomaly equation' which in the effective Lagrangian framework really means the normal Ward identity for the divergence of the $U(1)$ axial current. The divergence equation picks up contributions from all terms that break chiral

invariance. The θ-parameter therefore will appear explicitly in the 'anomaly equation', as indeed it does in 't Hoooft's [11] model (see his eq. (4.19)). This is in conflict with the anomalous Ward identity (4.5) in the underlying QCD and constitutes the crux of what 't Hooft calls the $U(1)$ dilemma [11,12]. In retrospect, it is the mismatch between the intrinsic and the effective transformation properties of the QCD θ-term under global chiral rotations which is at the heart of the $U(1)$ dilemma. In this sense the $U(1)$ dilemma is a legacy of the first option (1.6) for FPI.

In the second option (1.7), the Weyl modes corresponding to zero eigenvalues of the Dirac operator are not dynamical. Eq. (4.6) now dictates that even in the instanton sector, $\int F\tilde{F}\, d^4x$ should behave as chirally neutral. The mismatch between the intrinsic and the effective chiral transformation properties of the θ-term therefore disappears in the second option. In an effective Lagrangian corresponding to this option, the θ parameter can appear as a coefficient of only a chirally invariant term. To account for the ABJ anomaly in this option [4,23],

$$\langle \partial_\mu^\dagger J_{\mu 5}\rangle^{(2)} = \langle 2m\bar{\psi}\gamma_5\psi\rangle + 2\langle \frac{ig^2}{16\pi^2} tr\ F_{\mu\nu}\tilde{F}_{\mu\nu} - \sum \phi_{0i}^\dagger \gamma_5 \phi_{0i}\rangle \tag{4.7}$$

where ϕ_{0i}'s are zero modes, one has to introduce in the effective Lagrangian a separate term which is invariant under global chiral rotations but not under local ones. This is reminiscent of the 'anomaly term' suggested by Christos [12], viz. $\int_x \left(-\frac{1}{\Box_x}\right)\left(\partial_\mu\ Tr\ log\ (M/M^\dagger)\right)^2$, where M stands for the meson matrix.

5. "CP violation" from the chiral mass term

The Baluni term [9] was used by him and by others [10] to estimate the electric dipole moment of the neutron. The estimate came out to be θ times a rather large number. Since a dipole moment has not been observed in practice, the conclusion seems to be that θ is very small. This is generally regarded as unnatural,

and referred to as the "Strong CP Problem". Attempts to solve this problem by introducing symmetries [24] have led to the prediction of undetected particles [25] and believers have also supplied theories to explain why these particles should be undetectable [26]. It may be be more worthwhile to enquire [13] whether the dipole moment calculations are in fact reliable. This is what we propose to do in this section.

A. Parity in the presence of γ_5 and the chiral anomaly

The usual free Dirac action

$$S_0 = \int d^4x\, \bar{\psi}(i\partial\!\!\!/ - m)\psi \tag{5.1}$$

is known to be invariant under the parity transformation

$$\psi(\vec{x}, x_0) \to \gamma^0\psi(-\vec{x}, x_0), \bar{\psi}(\vec{x}, x_0) \to \bar{\psi}(-\vec{x}, x_0)\gamma^0. \tag{5.2}$$

Under this transformation, $\bar{\psi}\psi$ transforms as a scalar, $\bar{\psi}i\gamma_5\psi$ as a pseudoscalar, $\bar{\psi}\gamma^\mu\psi$ as a (four-)vector and so on.

Let us consider a free Dirac theory where the mass term is not a scalar but involves a scalar piece as well as a pseudoscalar one.

$$S_0' = \int d^4x\, \bar{\psi}'(i\partial\!\!\!/ - m - i\gamma_5 m')\psi' \tag{5.3}$$

What happens to parity in this theory ? Under (5.2), the pseudoscalar mass term picks up a minus sign, while the other terms stay unchanged, indicating that S_0' is not invariant. However, this does not mean that the theory breaks parity. Just as the transformation (5.2) was found in an attempt to keep S_0 invariant, one should attempt to find a transformation that replaces fields at $\vec{x}$ by fields at $-\vec{x}$ and leave (5.3) invariant [13]. After all, symmetry transformations in field theory are determined by the symmetry properties of the Lagrangian in the asymptotic domain and the pseudoscalar mass term, which is quadratic in the fields, must be

taken into account in the *construction* of the symmetry transformation. Let this transformation be

$$\psi'(\vec{x}, x_0) \to V\psi'(-\vec{x}, x_0)\ ,\ \bar{\psi}'(\vec{x}, x_0) \to \bar{\psi}'(-\vec{x}, x_0)\gamma^0 V^\dagger \gamma^0. \tag{5.4}$$

This will leave (5.3) invariant if

$$(\gamma^0 V^\dagger \gamma^0)\gamma^i V = -\gamma^i\ ,\ (\gamma^0 V^\dagger \gamma^0)\gamma^0 V = \gamma^0, \tag{5.5}$$

$$(\gamma^0 V^\dagger \gamma^0)(m + i\gamma_5 m') = m + i\gamma_5 m'. \tag{5.6}$$

Let us try the following ansatz for V :

$$V = \gamma^0 e^{i\beta\gamma_5}, \tag{5.7}$$

where β is a real number. We find that (5.5),(5.6) are satisfied provided

$$\gamma^0 e^{-i\beta\gamma_5}(m + i\gamma_5 m')\gamma^0 e^{i\beta\gamma_5} = m + i\gamma_5 m', \tag{5.8}$$

i.e.,

$$(cos2\beta + isin2\beta\gamma_5)(m - i\gamma_5 m') = (m + i\gamma_5 m') \tag{5.9}$$

i.e.,

$$mcos2\beta + m'sin2\beta = m\ ,\ msin2\beta - m'cos2\beta = m' \tag{5.10}$$

which are simultaneously satisfied if

$$\beta = tan^{-1}\frac{m'}{m} \tag{5.11}$$

Thus the transformation (5.4) with V determined by (5.6) and (5.10) leaves S_0' unchanged, showing that this theory does have reflection invariance. Note that V satisfies

$$V^\dagger = V = V^{-1} \tag{5.12}$$

as usual, and there is no conflict with the fact the square of a parity transformation is the identity. In fact, the transformation (5.4) can be seen to correspond to the *usual* parity transformation in terms of fields ψ which have a real mass term.

It is important to note that along with the definition of parity, the transformation for the fermion bilinears also change. Clearly, $\bar{\psi}'\psi'$ is no longer a scalar and $\bar{\psi}'\gamma_5\psi'$ no longer pseudoscalar :

$$\bar{\psi}'\psi'(\vec{x}, x_0) \rightarrow (cos\ 2\beta)\bar{\psi}'\psi'(-\vec{x}, x_0) + i(sin\ 2\beta)\bar{\psi}'\gamma_5\psi'(-\vec{x}, x_0) \tag{5.13}$$

$$\bar{\psi}'\gamma_5\psi'(\vec{x}, x_0) \rightarrow -(cos\ 2\beta)\bar{\psi}'\gamma_5\psi'(-\vec{x}, x_0) - i(sin\ 2\beta)\bar{\psi}'\psi'(-\vec{x}, x_0) \tag{5.14}$$

The true transformation rules are obeyed by $\bar{\psi}'e^{i\beta\gamma_5}\psi'$ (scalar) and $\bar{\psi}'\gamma_5 e^{i\beta\gamma_5}\psi'$ (pseudoscalar) respectively. Thus the mass term $\bar{\psi}'(m + i\gamma_5 m')\psi'$, which can also be written as $(m^2 + m'^2)^{\frac{1}{2}}\bar{\psi}'e^{i\beta\gamma_5}\psi'$, is actually a scalar under the transformation (5.4). Note that $\bar{\psi}'\gamma^\mu\psi'$ continues to transform like a vector and $\bar{\psi}'\gamma^\mu\gamma_5\psi'$ like an axial vector.

Having found that the presence of γ_5 in the mass term in a free fermionic theory does not spoil reflection invariance, we would like to know what happens when interactions are switched on. There is one further complication in QCD – the fact that the mass matrix M, though diagonal, is not necessarily proportional to the identity in flavour space. This makes β itself a matrix in flavour space. But there is no conceptual difficulty. The matrix V is defined by

$$\begin{aligned} V &= \gamma^0 exp[i\gamma_5 tan^{-1}\{\theta(MtrM^{-1})^{-1}\}] \\ &\simeq \gamma^0[1 + i\gamma_5\theta(MtrM^{-1})^{-1}] \end{aligned} \tag{5.15}$$

Note that this factor may be obtained from γ^0 in the standard parity transformation by using the chiral transformation used by Baluni. The parity transformation defined in this way leaves the pure quark part of the QCD action invariant. The introduction of gluons necessitates the definition of a parity transformation for

them. We adopt the usual definition :

$$A_0(\vec{x}, x_0) \rightarrow A_0(-\vec{x}, x_0), A_i(\vec{x}, x_0) \rightarrow -A_i(-\vec{x}, x_0) \tag{5.16}$$

The integral of the kinetic term for gluons is then invariant as usual. What is more nontrivial is the invariance of the interaction piece. But (5.5) ensures that this occurs even though V differs from the usual γ^0. We conclude then that the action of QCD is invariant under the modified parity transformation in which γ^0 is replaced by the RHS of (5.15).

What implication does this have for the quantum theory corresponding to QCD with the Baluni term? One would be tempted to say that the quantum theory too has a reflection symmetry. However, it is necessary to be more careful. Recall that in a gauge theory of massless fermions, the action is invariant under chiral transformations, so that there is a classical symmetry, but this is broken in the quantum theory by the anomaly. Can our reflecton symmetry be similarly broken by an anomaly ? There is some reason to fear that it *can* be, for the transformation (5.15) does involve a chiral piece, which may lead to some anomaly. Note, however, that we are concerned here with constant chiral transformations rather than spacetime dependent ones : the distinction is important because only the latter have perturbatively nonvanishing anomalies. Thus, for the constant chiral transformation (1.10), the measure of the path integral with the first option changes by an anomalous Jacobian $exp\left(-\frac{ig^2}{8\pi^2}(Tr\alpha)\int d^4x\, Tr(F\tilde{F})\right)$. While expressions like $Tr(F\tilde{F})$ appear in perturbative calculations, it is an exact four - divergence, so that $\int d^4x Tr(F\tilde{F})$ vanishes for gluon configurations that can be handled by perturbative techniques. Hence perturbative calculations should not be able to detect any violation of reflection invariance in the quantized theory. This can be further confirmed as follows. If perturbation theory is used, the vertices are standard and involve γ^μ, and only the quark propagator involves θ.

However, this propagator can be written as

$$[\not{p} - M - i\gamma_5(TrM^{-1})^{-1}\theta]^{-1} = e^{-i\alpha\gamma_5}[\not{p} - M]^{-1}e^{-i\alpha\gamma_5} \tag{5.17}$$

and the wavefunction as

$$u(p, M + i\gamma_5\theta(TrM^{-1})^{-1}) = e^{i\alpha\gamma_5}u(p, M), \tag{5.18}$$

while

$$\gamma^\mu = e^{i\alpha\gamma_5}\gamma^\mu e^{i\alpha\gamma_5} \tag{5.19}$$

with an appropriate α. Thus the θ-dependent factors in the internal quark lines cancel out at the vertices, leaving the propagators and the vertices of the theory without θ. If there are external quark lines, the remaining factors in the propagators cancel out with those in the wavefunctions. Consequently, diagrammatic techniques will never detect the presence of θ.

That still leaves the question whether there can be non-perturbative effects of θ. *We do not know the answer.* But this is precisely the same situation as in the original formulation (1.2) of the theory. The only possible effects of θ in that formulation must be non- perturbative, viz., through gluon configurations where $\int d^4x\, Tr(F\tilde{F})$ does not vanish even though $Tr(F\tilde{F})$ is a four-divergence. The same is true about the present formulation, as could have been expected. But the authors [10] who worked in this formulation apparently thought that effects of θ would be visible even by methods which were blind to non-perturbative gluon configurations. Our conclusion is that they were wrong. (See also [15]). It is worth repeating that it is not enough to have a calculational framework that is sensitive to the perturbative anomaly. The integral of the anomaly, which is nonperturbative, has to be probed. Following our mention of the anomaly in [13], some lattice-based authors [14] stated that lattice calculations, which take *dis*connected diagrams (with quark loops) into account and hence see the perturbative gluon anomaly,

should be able to detect an NEDM, in contrast to calculations which neglect such diagrams. But this is untenable. Lattice methods would need to include instanton-like gluon configurations in their purview. While this is possible in principle, topological calculations on the lattice are in practice beset with such severe ambiguities [27] that there is no hope of any reliable lattice calculation of the NEDM in the near future.

B. Remarks on Current Algebra calculations with the chiral mass term

Effects of the θ-term in the Baluni form have allegedly been seen in current algebra calculations [10]. One might think that these could be genuine effects since, after all, current algebra is not perturbation theory. But current algebra, or to be more precise, methods used in the work [10] under discussion, make no reference to instantons or gluon configurations with non trivial topology which could make $\int d^4x\, Tr(F\tilde{F})$ nonzero. It is even insensitive to the perturbative anomaly. This immediately suggests that the predicted effects are illusions. It is not difficult to see how they arise[13].

The crux of the calculation of the electric dipole moment of the neutron by the current algebraic method [10] is a CP violating "scalar coupling" of pions and nucleons, represented by

$$\bar{g}\bar{\psi}_N(\frac{\lambda^a}{2})\psi_N\pi^a \tag{5.20}$$

Here $\bar{g}(\propto \theta)$ is the scalar coupling invented by the said authors, ψ_N is the nucleon field and π^a the a-th member of the pion (Nambu-Goldstone boson) octet. A term of the form (5.20) has been alleged to occur in the effective theory of pions and nucleons. This was thought to arise from the Baluni term and thus to be related to the nonzero matrix element

$$i(-i\theta)(TrM^{-1})^{-1}\langle\pi^a N \mid \bar{\psi}'\gamma_5\psi' \mid N\rangle \tag{5.21}$$

In the same way, the nonzero matrix element of $\bar{\psi}'\gamma_5\psi'$ between the η and two

pion states suggests [10] an amplitude for the decay proportional to θ. Since the current algebra estimates of the matrix elements have nothing to do with gluon configurations of nontrivial topology, the logic of the previous subsection immediately tells us that something must be wrong here : parity can be broken only through such gluon configurations [13].

Let us try to understand the error [13] in the current algebra calculation [10] of the neutron electric dipole moment. The coupling coming from (5.21) was identified as a scalar coupling because the soft pion theorem yields

$$\begin{aligned}\theta(TrM^{-1})^{-1}\langle\pi^a N \mid \bar{\psi}'\gamma_5\psi' \mid N\rangle &= (\frac{-i\theta}{f_\pi})(TrM^{-1})^{-1}\langle N \mid [Q_5^a, \bar{\psi}'\gamma_5\psi'] \mid N\rangle \\ &= (\frac{2i\theta}{f_\pi})(TrM^{-1})^{-1}\langle N \mid \bar{\psi}'\frac{\lambda^a}{2}\psi' \mid N\rangle\end{aligned} \tag{5.22}$$

But the pion also has a conventional pseudoscalar coupling to the nucleon. This was not treated in any detail by the current algebraists [10], who simply wrote down the phenomenological value for the coupling constant. The pseudoscalar coupling in the conventional theory with $\theta = 0$ is related to the matrix element $m_\pi^2\langle N \mid \pi^a \mid N\rangle$, which, by PCAC, can be written as $(\frac{1}{f_\pi})\langle N \mid \partial_\mu(\bar{\psi}\gamma^\mu\gamma_5\frac{\lambda^a}{2}\psi) \mid N\rangle$. By virtue of the equation of motion, this is proportional to $\langle N \mid \bar{\psi}\gamma_5\{M, \frac{\lambda^a}{2}\}\psi \mid N\rangle$. Now this expression is not invariant under the chiral transformation taking ψ to ψ' :

$$\begin{aligned}\partial_\mu(\bar{\psi}\gamma^\mu\gamma_5\frac{\lambda^a}{2}\psi) &= i\bar{\psi}\gamma_5\{M, \frac{\lambda^a}{2}\}\psi \\ &= i\bar{\psi}'\gamma_5\{M, \frac{\lambda^a}{2}\}\psi' - 2\theta(TrM^{-1})^{-1}\bar{\psi}'\frac{\lambda^a}{2}\psi',\end{aligned} \tag{5.23}$$

where it has been assumed that M differs only infinitesimally from $(\frac{1}{N_f}TrM)\cdot\mathbf{I}$. Thus the pion coupling is related to

$$\begin{aligned}\langle N \mid \partial_\mu(\bar{\psi}\gamma^\mu\gamma_5\frac{\lambda^a}{2}\psi) \mid N\rangle &= i\langle N \mid \bar{\psi}'\gamma_5\{M, \frac{\lambda^a}{2}\}\psi' \mid N\rangle \\ &\quad - 2\theta\langle N \mid (TrM^{-1})^{-1}\bar{\psi}'\frac{\lambda^a}{2}\psi' \mid N\rangle\end{aligned} \tag{5.24}$$

The previous authors [10] identified the first piece on the RHS as a pseudoscalar coupling and obtained the dipole moment from the interference between these two. But this identification is false, for these two terms by themselves do not have well-defined transformation properties. Only the combination has such a property: it is *pseudoscalar*. In other words, there is no scalar coupling, and no dipole moment in the current algebra approximation [13].

6. Concluding Remarks

The question, how to describe a Dirac fermion in Euclidean metric, plays a key role in the present discussions. Everything else flows from the stance one adopts regarding the answer to this question. There is, on the one hand, the traditional and popular OS scenario [6] where each Dirac fermion is associated with two independent Dirac fields. On the other hand, the second scenario [17] presented here, requires only one Dirac field $\psi(x)$ to describe a Dirac fermion in Euclidean metric. The conjugate field $\bar{\psi}(x)$ (or, equivalently $\chi(x)$) is defined in terms of $\psi(x)$. Gauge interactions of fermion and gauge invariance enter in a fundamental way in the definition (3.3) of $\bar{\psi}(x)$ and of the fermion measure (3.6). There is a direct correspondence between the OS scenario and the first option (1.6) for FPI, and between the second scenario and the second option (1.7) for FPI. Thanks to the paper of Osterwalder and Schrader [6] on the one hand and that of Gamboa-Saravi *et al* [4] on the other, insofaras consistency from the mathematical point of view is concerned there is not much to choose between the two options and hence between the two competing scenarios of a Dirac fermion in Euclidean metric. Consideration based on physics and the fallout of the two scenarios on issues in physics emanating from the instanton sector are expected to play the decisive role in this crucial choice.

One recognises strong resemblance between a relativistic Dirac fermion and

the second scenario. This resemblance is not confined merely to the one-to-one correspondence of a Dirac fermion and a Dirac field. It should be observed that the degrees of freedom associated with fermion zero modes, for which there is no identifiable signature in the relativistic world, are redundant in the second scenario.

The ambivalence in representing the QCD θ-term (1.3) in effective Lagrangian models, called the $U(1)$ dilemma by 't Hooft [11], is a legacy of the first option and hence of the OS scenario [6]. The dilemma is absent in the second scenario. The most significant and, perhaps, the most decisive impact of the second scenario is on the strong CP problem which has defied all attempts towards a resolution for the last fifteen years. The strong CP problem, it has to be recognised, is basically a theorists' problem and arises from the large value for NEDM estimated theoretically from the Baluni term (1.11). In the second scenario effects of the Baluni term are unphysical. In a chirally invariant theory like QCD the Baluni term can be trivially transformed away by a chiral rotation. This would not be true in the first option for FPI. There, one can only trade the Baluni term for an additional piece in the QCD θ-term (1.3). Thus, the second scenario, in essence, evades the storng CP problem and states that the Baluni framework for theoretical estimates for NEDM due to the θ-term is untenable.

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