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INFLUENCE OF TILE NONUNIFORMITY ON THE CONSTANT TERM OF E.M. CALORIMETER RESOLUTION

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Abstract

In this paper the influence of tile response nonuniformity on the constant term from the energy resolution of an electromagnetic sandwich tile calorimeter is investigated.

Our treatment is based on the well known parametrization of the mean longitudinal profile of the electromagnetic shower by a Gamma distribution. This permitted us to obtain an estimate of the contribution of tile nonuniformity to the constant term from energy resolution in the form of an explicit dependence on the degree of nonuniformity and the material constants of the calorimeter.

Then, it is shown how a calibration made individualy on each calorimeter tower might improve the situation to meet the requirements imposed by the physics programe at the future accelerators such as the Superconducting Super Collider (SSC) or the Large Hadron Collider (LHC).

Introduction

The calorimeters are the essential components of all major detectors designed to operate at energies in the TeV range at future accelerators: SSC and LHC. The physics programe at these accelerators imposes very strong requirements on the performance characteristics of such detectors. For exemple, to search for Higgs production through decay channels into electromagnetic particles, the resolution of the electromagnetic calorimeter must be at least:

$$\sigma/E = 15\%/\sqrt{E} \oplus 1\% \tag{1}$$

In order to meet such a requirement we need to understand how some factors appearing in real life as: nonuniformity of response from different components, radiation damage, light attenuation etc. will affect the E. M. calorimeter energy resolution.

In this paper the influence of tile-to-tile response nonuniformity on the constant term of the energy resolution of a sandwich tile electromagnetic calorimeter is studied. The best strategy would be to switch off all other effects which contribute to the energy resolution and to obtain the pure effect of tile nonuniformity. Our method to realise this relies on the use of the well known parametrization of the mean longitudinal profile of the electromagnetic shower by a Gamma distribution. In this way, the stochastic fluctuations, which have an inportant contribution to the E. M. calorimeter resolution, are removed. In all numerical computations we considered that the calorimeter is composed from an array of identical towers. Each tower consisted of $N_{tile} = 32$ alternative layers of lead and scintillator tiles of 4mm thickness. These data were chosen according to SDC technical project [1].

Before presenting the results, we will discuss shortly the nature of the different terms which contribute to the energy resolution. As a rule, the resolution of an E. M. calorimeter, when the electronics noise is not taken into account, is expressed as [3]:

$$\sigma_E/E = a/\sqrt{E} \oplus b \tag{2}$$

where:

- a - the scaling factor - corresponds to the sampling fluctuations and the fluctuations connected to photoelectron conversion;

- b - the constant term - corresponds to energy leakage, light attenuation, nonuniformity of tile response etc.

The formula (2) for the energy resolution deserves a more detailed discussion. Both coefficients a and b have a structure, reflecting the contribution of different factors. Lets begin the discussion considering that the formula for the energy resolution in the form as it appears in (2) describes the ideal situation, where the tile response is uniform and in this case b describes the effects of energy leakage. In a recent paper [4] we tried gradually to approach the real life situation, including first the tile response nonuniformity. Then the values of a and b have to be modified and we express this in the form of quadratically addition of new terms:

$$\sigma_E/E = a/\sqrt{E} \oplus a_1/\sqrt{E} \oplus b \oplus b_1 \tag{3}$$

Therefore the nonuniformity will amplify the sampling fluctuations and will increase the value of the constant term.

Until now we considered that we are in the situation of an absolute calibration, i. e. we calibrated each tower at each energy. Now, to approach even more to a realistic situation the calibration is introduced, which is usually made at one energy. We found this introduces a further increase to b term, which can be expressed by an additional term b_2:

$$\sigma_E/E = a/\sqrt{E} \oplus a_1/\sqrt{E} \oplus b \oplus b_1 \oplus b_2 \tag{4}$$

Of course the value of b_2 will depend strongly on type of calibration: global or on each tower individualy. By this treatment we obtained individual contribution of above mentioned terms or combinations of them. The data are yet preliminary, but work is on progress.

In this paper, we present recent results on the efect of nonuniformity and calibration on the b term, obtained analyticaly. This treatment has not the flexibility of choices as in the Monte Carlo treatment of [4], but presents additional advantage of obtaining explicit analytical dependence on degree of nonuniformity or material constants.

The tile nonuniformity was considered as gaussian distributed, the degree of nonuniformity being characterised by the value of the standard deviation σ_{non} of this distribution. Such an approach has a justification in the case when the individual tile characteristics are not measured and the calorimeter towers are assembled without any selection of tiles.

For clarity of further explanation we need to define some basic concepts.
Tile signal. The tile signal is defined as follows:

$$S_i = E_i^{sci} C_i \tag{5}$$

where

- E_i^{sci} is the energy released in i^{th} tile;
- C_i is the response coefficient of ith tile characterizing the tile efficiency for conversion of deposited energy into light signal.

In our case coefficients C_i are defined as:

$$C_i = 1 + \sigma_{non}\, r_i \tag{6}$$

where

- $i(= 1..N_{tile})$ is the tile sequence number;
- r_i is a set of random numbers distributed acording to a Gaussian distribution of mean value zero and the standard deviation equal to one;
- σ_{non} - the response standard deviation characterizing the size of nonuniformity or the quality of the tile set.

Such a definition of C_i corresponds to the normalization of the full calorimeter signal to the full energy deposition in active medium (tiles).
Calorimeter signal. The calorimeter signal is defined as follows:

$$S^{full} = \sum_{i=1}^{N_{tile}} E_i^{sci} C_i \tag{7}$$

Sampling fraction. The sampling fraction (f_{samp}) is defined in an usual way:

$$f_{samp} = \sum_{i=1}^{N_{tile}} E_i^{sci} \;/\; E_{inc} \tag{8}$$

where $\sum_i^{N_{tile}} E_i^{sci}$ is full energy deposition in active medium and E_{inc} is the energy of incident particle.

Influence of tile nonuniformity: Global calibration

At the begining we study what is the contribution of the tile response nonuniformity in the case of a global calibration. By this we mean that a correspondence is established between the averaged calorimeter signal over all towers and the incident energy.

This section was firstly presented in [4], but we repeate it here because in the following section are used the same computing tricks and approximations and there the mathematical formulas are more longer than here and also for completness, because the obtained results are then compared.

In equ. (7) is given the expression for the calorimetric signal in the presence of tile-to-tile nonuniformities for a given tower. Its averaged value over the full surface of the calorimeter is:

$$\overline{S^{full}} = \sum_{i=1}^{N_{tile}} E_i^{sci} \tag{9}$$

because $\overline{C_i} = 1$.

The corresponding reconstructed energy E will be given by the relation:

$$E = K\overline{S^{full}} \tag{10}$$

where the calibration coefficient K is defined as:

$$K = \frac{1}{f_{samp}} \tag{11}$$

The coefficient K, will be the a unique average calibration coefficient, the same for all individual towers.

The variance of the S^{full} is:

$$var(S^{full}) = \sigma_{non}^2 \sum_{i=1}^{N_{tile}} (E_i^{sci})^2 \tag{12}$$

One can notice that as the incident energy will increase, more tiles will contribute to the signal and the variance is expected to have a slow increase.

The contribution of tiles nonuniformity to the energy resolution (i. e. to the b term) will be:

$$\frac{var(S^{full}))^{1/2}}{\overline{S^{full}}} = \sigma_{non} \frac{(\sum_{i=1}^{N_{tile}} (E_i^{sci})^2)^{1/2}}{\sum_{i=1}^{N_{tile}} E_i^{sci}} \tag{13}$$

One can notice the linear dependence of b on σ_{non}, a general result, independent of any shower parametrisation.

The above equation may be used to obtain an analytical expression for tile nonuniformity contribution to the E. M. calorimeter response, if we make use of the existing knowledge about E_i^{dep} in r.h.s. of equ. (13).

It consists from the use of the well known parametrisation by a gamma-distribution [2] for the average longitudinal shower profile:

$$\frac{dE}{dt} = E_{inc}\frac{\beta}{\Gamma(\alpha)}(\beta t)^{\alpha-1}e^{-\beta t} \tag{14}$$

with

$$\frac{\beta}{\Gamma(\alpha)}\int(\beta t)^{\alpha-1}e^{-\beta t}dt = 1 \tag{15}$$

The parameters α, β contain energy dependence of shower profile. For instance, for electron induced showers, one takes:

$$\beta \simeq 0.5; \quad \frac{\alpha-1}{\beta} = ln\frac{E_{inc}}{E_c} - 0.5 \tag{16}$$

where E_c is critical energy for the averaged material of the calorimeter. The depth inside shower is considered relative to the front edge of the calorimeter, and is expressed in radiation lengths X_0 of the averaged material. The use of averaged medium is legitimate for a sufficiently high sampling frequency, of course.

For further calculations a few more assumptions will be made:

- the shower is totally contained in a single tower (i.e. longitudinal and transverse dimensions of tower are sufficient in the considered energy range)
- the tower has a full depth of T rad. lengths, being a stack of N_{tile} identical sections, each consisting of a tile and an absorber plate.

Under these assumptions, in the equ. (15) we can replace the integral by its rectangle-method quadrature approximation over N_{tile} intervals. Multiplying afterwards with E_{inc} we get:

$$E_{inc} = \sum_{i=1}^{N_{tile}} E_i^{dep} \tag{17}$$

$$E_i^{dep} = E_{inc}\frac{\beta\Delta t}{\Gamma(\alpha)}(\beta t_i)^{\alpha-1}e^{-\beta t_i} \tag{18}$$

where E_i^{dep} is total energy deposited in the i-th section of tower, $\Delta t = T/N_{tile}$ and $t_i = (i-0.5)\Delta t$. The energy deposited in the active part of section is $E_i^{sci} = f_{samp}E_i^{dep}$, with f_{samp} as defined in (8). The signal produced in this section is $S_i = C_i E_i^{sci}$, where C_i was defined earlier.

So the reconstructed energy is, according to (10)

$$E = \frac{1}{f_{samp}}\sum_{i=1}^{N_{tile}} E_i^{sci}C_i \tag{19}$$

In other words, the reconstructed energy is a weighted sum of N_{tile} independent random quantities C_i distributed by the same law. This treatment implies that all tiles are manufactured by using the same technology, so C_i obey to a unique distribution law.

Assuming the distribution (6) for C_i, a straightforward calculation gives

$$\sigma_E^2 = [\frac{E_{inc}}{f_{samp}}\frac{\beta\Delta t}{\Gamma(\alpha)}]^2\sigma_{non}^2\sum_{i=1}^{N_{tile}}(\beta t_i)^{2(\alpha-1)}e^{-2\beta t_i} \tag{20}$$

Making use of relation (15) and after some algebra we get:

$$(\frac{\sigma_E}{E})^2 = \frac{\beta\Delta t}{\sqrt{\pi}(2\alpha-1)}\frac{\Gamma(\alpha+0.5)}{\Gamma(\alpha)}\sigma_{non}^2 \tag{21}$$

A further simplification can be made, by using the explicit form for α and β (see (16)) and the Stirling asymptotics for $\Gamma(x)$, valid at energies higher than say, 10 GeV. Finally we get

$$\frac{\sigma_E}{E} = 0.45\sqrt{\frac{\Delta t}{\sqrt{ln(E_{inc}/E_c)}}}.\sigma_{non} \tag{22}$$

that reveals a very slow decrease with energy of σ_E/E, which is proportional with σ_{non} however. The results for five values of the nonuniformity σ_{non}: $5\%, 10\%, 15\%, 20\% and 30\%$ are presented in fig. 1.

Influence of tile nonuniformity: Local calibration

As can be seen from fig.1, tile response nonuniformity induces inacceptable high values for the constant term b. It is expected that calibrating each tower individually the situation might be improved.

Assuming that we calibrate the tower at one energy, the existence of a nonuniformity in the tile response, will made this calibration not to remain exact at another one. Due to the dependence on energy of the mean shower profile, it is obvious that a different number of tiles will contribute to the response of the tower at different energies and also the fractional weight to the tower response of an individual tile will change with energy.

In order to evaluate this effect, let us suppose that the calibration of the tower was made at the energy E_0 of incident electron beam. To simplify the discussion we will consider that there is no energy leakage. Then, because we use a mean profile for the shower, the calibration condition can be obtained in a very simple form:

$$K \sum_{i=1}^{N_{tile}} E_i^{sci} C_i = E_0 \tag{23}$$

At another energy, say $E_0^{'}$, the signal of the tower will be $\sum_{i=1}^{N_{tile}} E_i^{'sci} C_i$ and the reconstructed energy E' will be:

$$E' = K \sum_{i=1}^{N_{tile}} E_i^{'sci} C_i \tag{24}$$

where the value of the calibration factor K is determined from the calibration condition (23). Taking into account the definition of the coefficients C_i from(6), we obtain:

$$E' = E_0 \frac{\sum_{i=1}^{N_{tile}} E_i^{'sci} + \sigma_{non} \sum_{i=1}^{N_{tile}} E_i^{'sci} r_i}{\sum_{i=1}^{N_{tile}} E_i^{sci} + \sigma_{non} \sum_{i=1}^{N_{tile}} E_i^{'sci} r_i} \tag{25}$$

which, after some algebraic manipulations can be transformed into:

$$E' = E_0 \frac{\sum_{i=1}^{N_{tile}} E_i^{'sci}}{\sum_{i=1}^{N_{tile}} E_i^{sci}} \{1 + \sigma_{non} (\frac{\sum_{i=1}^{N_{tile}} E_i^{'sci} r_i}{\sum_{i=1}^{N_{tile}} E_i^{'sci}} - \frac{\sum_{i=1}^{N_{tile}} E_i^{sci} r_i}{\sum_{i=1}^{N_{tile}} E_i^{sci}})\} \tag{26}$$

We will introduce a new variable ξ, defined by:

$$\xi = \frac{E'}{E_0^{'}} \tag{27}$$

This is a random variable which gives a measure of the deviation of the reconstructed energy around the true value E_0' of the incident energy. It can be easily verified that ξ has the mean value one and the standard deviation $\sigma_\xi = \sigma_{E'}/E_0'$. Since the mean value of E' is E_0', σ_ξ will represent the contribution of the nonuniformity to the energy resolution, i. e. the value of the b term in the case of individual tower calibration. Using (26) we obtain the following expression:

$$\sigma_\xi = \sigma_{non}\frac{E_0}{E_0'}\{\sum_{i=1}^{N_{tile}}(\frac{E_i'^{sci}}{\sum_{i=1}^{N_{tile}} E_i'^{sci}} - \frac{E_i^{sci}}{\sum_{i=1}^{N_{tile}} E_i^{sci}})^2\}^{1/2} \tag{28}$$

One can notice that the linear dependence on σ_{non} remained also after tower calibration. This expression can be evaluated using the parametrisation of the mean shower profile [2] and the same computational tricks as in the previous section. Finally we obtain:

$$\sigma_\xi = \sigma_{non}\beta\Delta t_i\{\frac{\Gamma(2\alpha-1}{2^{2\alpha-1}\Gamma^2(\alpha)} + \frac{\Gamma(2\alpha'-1}{2^{2\alpha'-1}\Gamma^2(\alpha')} - 2\frac{\Gamma(\alpha+\alpha'-1)}{2^{\alpha+\alpha'-1}\Gamma(\alpha)\Gamma(\alpha')}\}^{1/2} \tag{29}$$

where α and α' are the values of the parameter α defined in (16), calculated at calibration energy E_0 and respectively E_0'. The values of the b term for different nonuniformities at different energies in the range from 5 to 200 GeV when towers were calibrated at $E_0 = 50GeV$, are plotted in fig 2. One can notice a near linear dependence of the b term on the logarithm of energy.

For a comparison between the global and individual calibration of the towers the the values of the b term for $\sigma_{non} = 10\%$ are plotted on the same picture on the fig. 3. One can observe that its magnitude is reduced substantially in the case of individual tower calibration.

Conclusions

In this paper, the influence of the tile response nonuniformity on constant term from the energy resolution of an E. M. sandwich tile calorimeter was investigated. The main results are the following:

- the linear dependence of the constant term vs. σ_{non};
- significant deterioration of the energy resolution (as compared to ideal case) due to tile response nonuniformity in the case of global calibration. This appears especially as a result of strong increase of b term with σ_{non};
- reasonable small values for b in the case of individual tower calibration even for large nonuniformities in the tile response.

Acknowledgement

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References

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[2] Particle Data Group, Phys. Lett. 239B (1990) pag. III.15

[3] C. Fabjan and R. Wigmans, Rept. Prog. Phys. **52**, 1519 (1989)

[4] I. Chirikov-Zorin et al., Solenoidal Detector Collaboration Note SDC-92-287 (1992).

Figure Captions

Fig.1 The influence of tile nonuniformity on the constant term from energy resolution in the case of global tower calibration;

Fig.2 The influence of tile nonuniformity on the constant term from energy resolution in the case of individual tower calibration;

Fig.3 Comparison between the values of the constant term in the global and individual tower calibration for 10% nonuniformity.

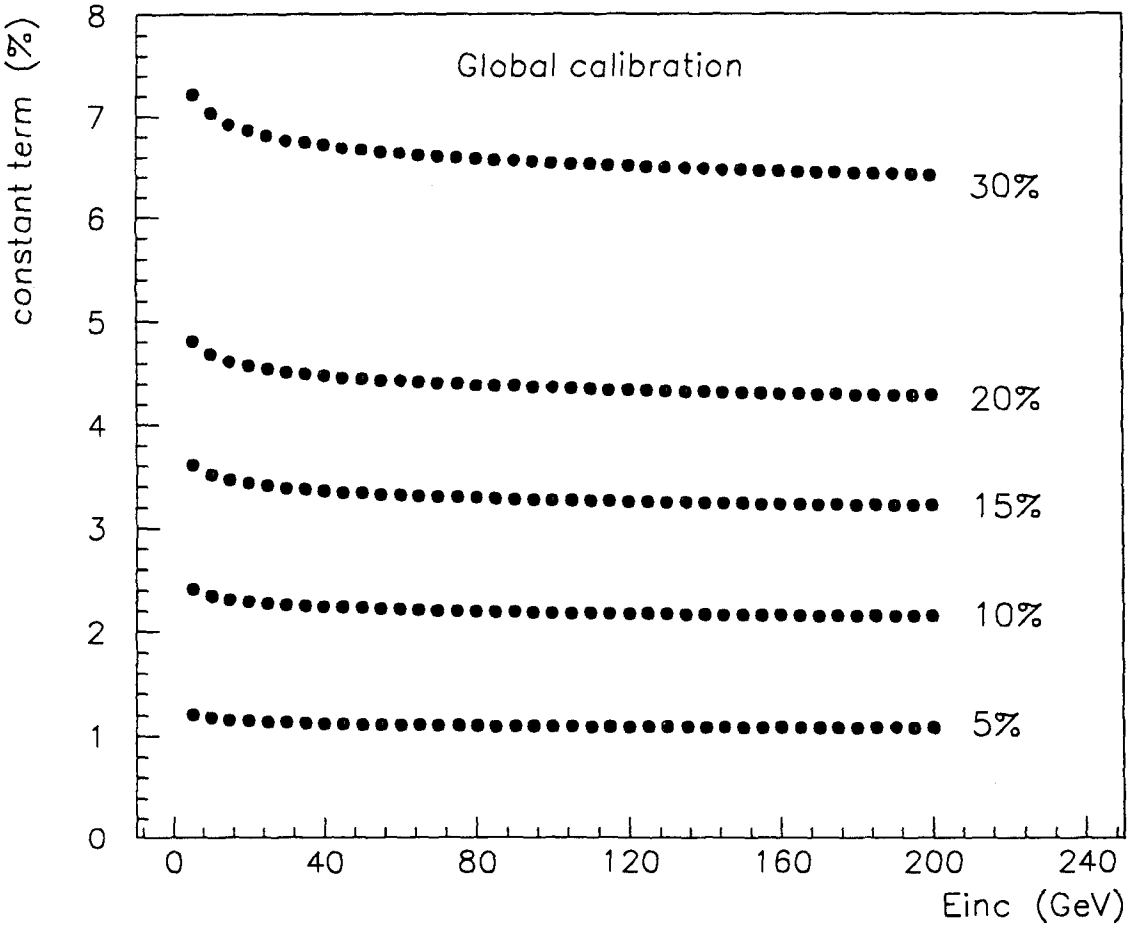
Global calibration
constant term (%)
8
7
6
5
4
3
2
1
0
30%
20%
15%
10%
5%
0
40
80
120
160
200
240
Einc (GeV)

Fig. 1

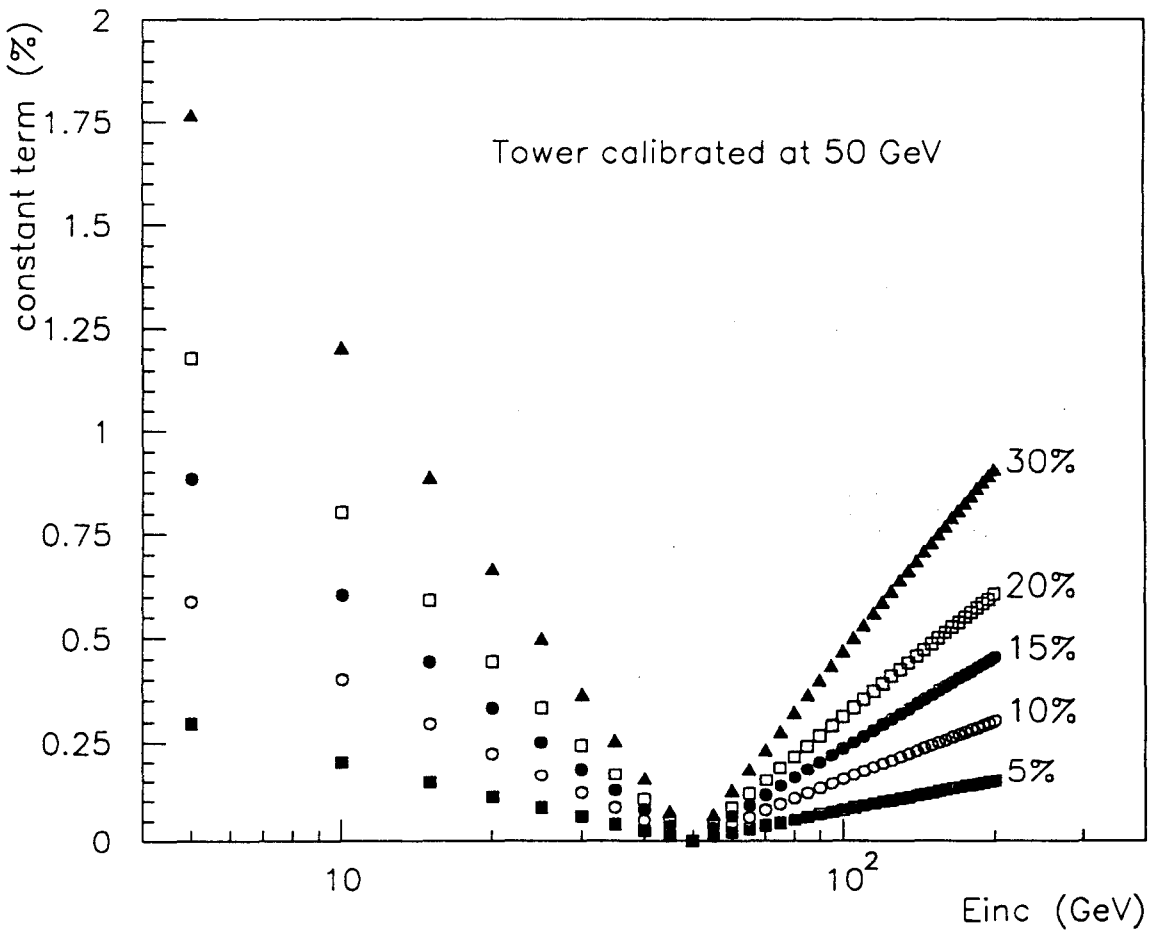
constant term (%)
2
1.75
1.5
1.25
1
0.75
0.5
0.25
0
Tower calibrated at 50 GeV
30%
20%
15%
10%
5%
10
10^2
Einc (GeV)

Fig. 2

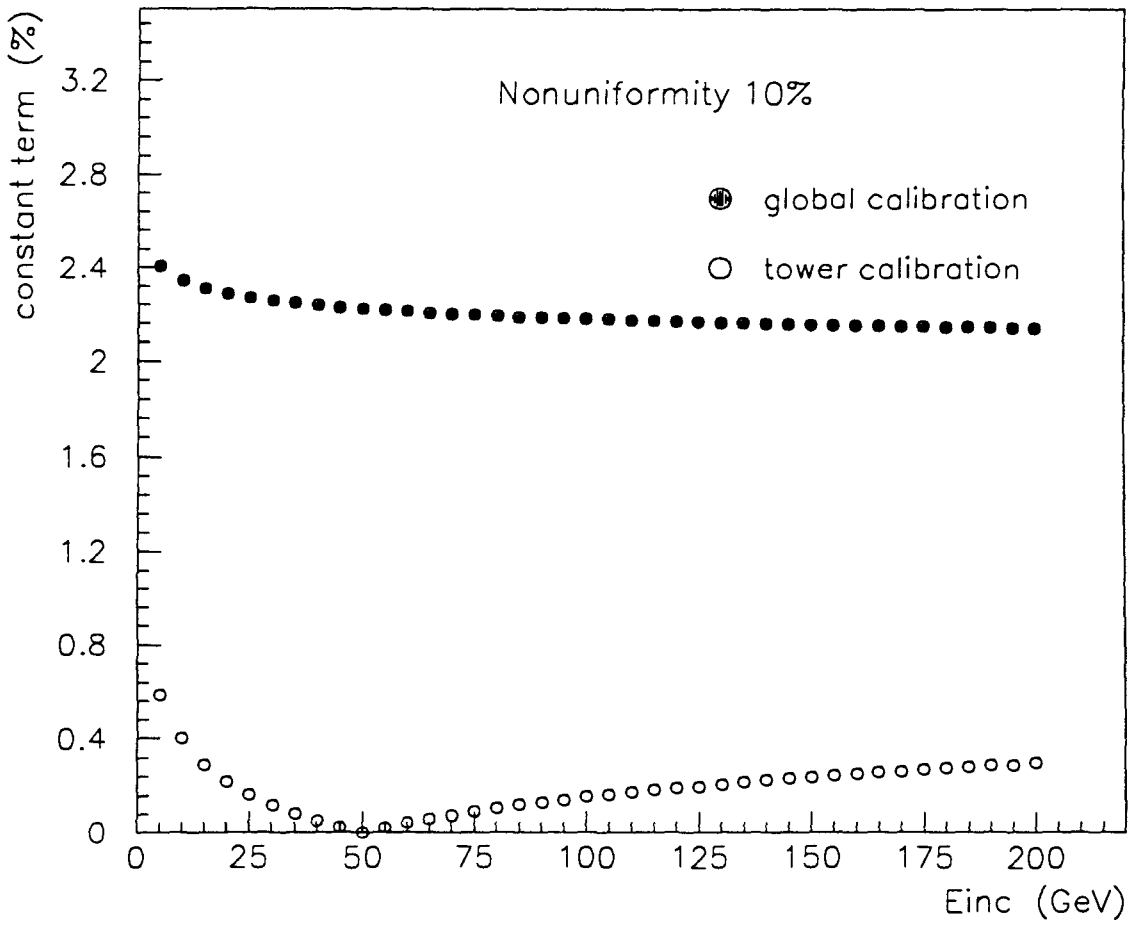
constant term (%)
3.2
2.8
2.4
2
1.6
1.2
0.8
0.4
0
Nonuniformity 10%
global calibration
tower calibration
0
25
50
75
100
125
150
175
200
Einc (GeV)

Fig. 3