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MORE ON MUON MATCHING (LAYER PLACEMENT)

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This note reports on calculations on how readout layer placement affects our ability to match tracks measured in the Central Tracker (CTD) with hits in the muon system without confusion from adjacent candidate tracks found in the CTD. We begin by reviewing the calculational technique. We then give a physically intuitive argument to explain how measurement of both the angle and intercept after the first multiple scattering slab significantly reduces matching confusion. We illustrate this concept for the case of BW1 and turn next to a discussion on how the radial order of readout layers in BW1 affects matching performance in the barrel region. We conclude with a discussion of how additional stereo layers in FW1 will significantly reduce matching confusion in the forward muon system.

1. Review of Calculational Technique

Here is a quick review of the computational matching techniques described in more detail in our November 11,1991 memo, *Muon Matching Studies* (SDC-91-156). We assume that the off line matching of the muon chamber hits with a possible CTD track is performed using a χ^2 test to the extrapolated track trajectory. A track is matched if it has a minimum, acceptable confidence level to the muon chamber hits. Confusion results when an adjacent track has sufficiently similar track parameters that its extrapolation satisfies the muon confidence level cut. In order to quantify matching confusion, one can define a *confusion volume* as the volume of the space of parameter differences which gives a χ^2 change of 1 unit. The confusion volume serves as a sort of N-dimensional Rayleigh criteria as to when two inner detector tracks have track parameters which are too close to resolve within the muon system. The confusion volume is computed as described below.

We begin by describing the expected value of the matching χ^2 of a putative track to the muon system hits due to an adjacent real muon. Let the relevant track parameters (as measured in the CTD) be denoted as $t_\alpha^{(\mu)}$ for the real muon and $t_\alpha^{(p)}$ for the putative track (or candidate muon). We define a *transport matrix* $T_{i\alpha}$ which multiplies the α'th track parameter to predict the i'th hit in the muon system (where we call this hit w_i (or wire number)). In our approach we assume that the i'th hit is due to the muon and follows the muon trajectory $T_{i\alpha}t_\alpha^{(\mu)}$ but suffers fluctuations δw_i due to multiple scattering, measurement, and extrapolation error. The matching χ^2 will be constructed from the inner putative track trajectory and the inverse coordinate covariance matrix $C_{ij}^{(p)\ -1}$ describing hits in the muon system but computed for the putative track. This inverse coordinate covariance matrix is computed for the putative track and includes the measurement errors in extrapolating the putative track into the muon system, smearing due to multiple Coulomb scattering, and measurement error within the muon system.

The χ^2 of the hypothesis that the putative track matches the muon hits is:

$$\chi^2 = \left(T_{i\alpha}t_\alpha^{(p)} - T_{i\alpha}t_\alpha^{(\mu)} - \delta w_i\right)\ C_{ij}^{(p)\ -1}\ \left(T_{j\beta}t_\beta^{(p)} - T_{j\beta}t_\beta^{(\mu)} - \delta w_j\right) \tag{1}$$

Taking the expectation value and autocorrelating we get:

$$< \chi^2 >= \Delta t^t\ H^{(p)}\ \Delta t + Tr\ C^{(\mu)}\ C^{(p)\ -1} + Tr\ \mathcal{E}^{(p)} C^{(p)\ -1} \tag{2}$$

where:

1. Δt^t is the transpose of the vector describing the difference between the muon and putative track parameters.
2. $H^{(p)}$ is the fit matrix for the putative track. The inverse of the fit matrix is the putative track parameter error matrix. The components are $H_{\alpha\beta}^{(p)} = C_{ij}^{(p)\ -1}\ T_{i\alpha}T_{j\beta}$

3. $C^{(\mu)}$ and $C^{(p)}$ are the coordinate covariance matrices for the muon and putative tracks including measurement and multiple scattering effects. $\mathcal{E}^{(p)}$ is the error matrix describing the extrapolation of the putative track from the inner detector to the muon system.

The first term(the H-term) is essentially the inverse of the effective track parameter error matrix of the muon detectors. The $\chi^2 = 1$ boundary will form a hyperspace ellipsoid with a volume given by the reciprocal square root of the $H^{(p)}$ matrix determinant along with possible geometrical factors (*eg* π). If only one track parameter is being compared, the confusion volume is essentially the resolution of the track parameter as measured by the muon system.

The second term involving $Tr\ C^{(\mu)}\ C^{(p)\ -1}$ shows that comparing the pattern of multiple coulomb scattering serves as an additional way of matching muon hits. We think this is interesting but will ignore it for now.

The calculations which follow use a general resolution program which incorporates the myriad of scattering , magnetic, and readout geometries present in the SDC to construct the H matrix and compute confusion volumes.

2. How do BW1 internal lever arms affect muon matching?

In our November 11,1991 memo, *Muon Matching Studies* (SDC-91-156), we noticed a surprisingly large factor of ≈ 2.5 in reduced matching confusion through the use of two ϕ (left-right resolving) doublets (four layers) in BW1 as opposed to one.

Why do two $\hat{\phi}$ doublets in BW1 make such a difference?

BW1 essentially sees the barrel calorimeter as a single scattering slab (of thickness Δ). The presence of two $\hat{\phi}$ doublets allows one to measure both the angle and intercept of the muon track. One can use the measurement of the angle and intercept downstream of the slab to back extrapolate to the center of the slab and compare to the forward extrapolation of the candidate tracks as measured by the CTD. Let us compute the resolution for this procedure for the case of an $\eta = 0$, $\phi = \pi/2$ track. The relevant comparison variable would be the x extrapolation of the track to the slab center or x_c. Many of the effects of multiple coulomb scattering are reduced by this procedure. The fluctuations in this muon chamber extrapolation from a doublet centered at radius R due to multiple scattering can be written as:

$$\Delta x_c = \Delta x - \Delta\phi\,(R - R_c) \tag{3}$$

where the fluctuations between the intercept (Δx) and scattering angle ($\Delta\phi$) are given (as explained in the Reviews of Particle Properties by the PDG):

$$\Delta x = \sigma_o\,\tilde{g}_1\,\frac{\Delta}{\sqrt{12}} + \sigma_o\,\tilde{g}_2\,(R - R_c)\quad,\quad \Delta\phi = \sigma_o\,\tilde{g}_2 \tag{4}$$

Where $\tilde{g}_1$ and $\tilde{g}_2$ are two independent, 0 mean, unit variance, Gaussian distributed random variables and:

$$\sigma_o = \frac{.0141}{P}\sqrt{\frac{\Delta}{\lambda}}$$

Inserting the angle and position fluctuations given by Eqn. (4) into Eqn. (3) essentially cancels out the second term of Eqn. (4) and limits the effective scattering lever arm to a fraction of the slab thickness (Δ). Thus:

$$\sigma(x_c) = \sigma_o\,\frac{\Delta}{\sqrt{12}}$$

Dividing $\sigma(x_c)$ by the distance from the origin to the slab center (R_c) gives the ϕ confusion resolution:

$$\sigma_\phi = \sigma_o\,\frac{\Delta}{R_c\sqrt{12}}$$

The smallest possible resolution that one can get with a single measurement plane occurs at the closest accessible distance ($R = R_c + \Delta/2$) and is:

$$\sigma(x) = \sigma_o\,\frac{\Delta}{\sqrt{3}}$$

which is at least a factor of two greater than the resolution obtainable from two doublets using the variable x_c. If the single $\hat{\phi}$ measuring station is forced to be downstream of the slab, even larger gains will be achieved.

We note that in matching CTD tracks to the hits of BW1, one joins extrapolated tracks in the center of a single scattering slab (the barrel calorimeter). By way of contrast, in matching CTD tracks to the hits in BW2 or BW3 one is smeared from scattering from two displaced slabs (the calorimeter and barrel muon toroid) and the slab center matching technique becomes ineffective. For this reason BW1 is the critical matching chamber rather than BW2 or BW3 which is why the inclusion of $\hat{\phi}$ doublets in BW1 was so critical.

For the case of the SDC muon system we have :

R	R_c	Δ	λ
6.17 m	3.21 m	2.22 m	.0176 m

or:

$$\sigma_\phi = \frac{\sigma_o \, \Delta}{\sqrt{12} R_c} = \frac{32 \text{ mrad} - \text{GeV}}{P_t}$$

This agrees with the results of the earlier memo and is reprised in the low momenta limit of lower curves of Figure 1 which gives confusion volume as a function of P_t for $\hat{\phi}$.

By way of contrast, a single plane located at $R = 6.17\ m$ will give a ϕ resolution (confusion volume) given by:

$$\sigma_\phi = \frac{\sigma_o \sqrt{\frac{\Delta^2}{12} + (R - R_c)^2}}{R} = \frac{78 \text{ mrad} - \text{GeV}}{P_t}$$

Again this is in excellent agreement with the calculations summarized by the upper solid curve of Figure 1.

How do doublet lever arms affect performance?

The lever arm between two $\hat{\phi}$ doublets controls the ability to extrapolate the muon to the center of the barrel calorimeter. When the error due to measurement extrapolation equals the $\sigma(x_c)$ error due to multiple scattering one naively expects a $\sqrt{2}$ increase in $\Delta\phi$ confusion volume. Let the doublet measurement resolution be σ_d, and the doublet lever arm be ℓ. The momenta P_* beyond which measurement effects will dominate MCS effects is then:

$$\frac{\sqrt{2}\,\sigma_d\,(R - R_c)}{\ell} = \frac{.0141\ \Delta}{P_* \sqrt{12}} \sqrt{\frac{\Delta}{\lambda}}$$

For $\sigma_d = 226\mu m$, we have:

$$P_* \approx 110\ GeV\ \ell \tag{5}$$

Our previous barrel matching studies indicate that the relevant momentum scale for high P_t $b\bar{b}$ pair physics is roughly 50 GeV which indicates the desirability of $\ell \approx 0.45\ m$ lever arms between $\hat{\theta}$ or $\hat{\phi}$ doublets in BW1. This is nearly possible within the 85 cm thick BW1 package if one alternates doublets rather than using adjacent $\hat{\theta}$ or adjacent $\hat{\phi}$ doublets.

3. Results

We have considered several scenario's for the placement of readout doublets in BW1 which are described in the below table. Going from the inner radius to the outer radius the doublets are denoted as $d_1 \to d_4$. We think of the layers as appearing in a left-right ambiguity resolving doublet. We always assume that the resolving properties of the doublet require them to be in adjacent layers.

5 BW1 Doublet Scenarios

Scenario	code	d_1	d_2	d_3	d_4
1	solid	$\hat{\theta}$	$\hat{\phi}$		$\hat{\theta}$
2	solid	$\hat{\theta}$	$\hat{\theta}$	$\hat{\phi}$	$\hat{\phi}$
3	dash	$\hat{\theta}$	$\hat{\phi}$	$\hat{\theta}$	$\hat{\phi}$
4	dot	$\hat{\phi}$	$\hat{\theta}$	$\hat{\theta}$	$\hat{\phi}$
5	dash dot	$\hat{\theta}$	$\hat{\phi}$	$\hat{\phi}$	$\hat{\theta}$

The 1st Scenario has only one set of left-right resolving $\hat{\phi}$ doublets in BW1 and is included only to emphasize the importance of having two $\hat{\phi}$ doublets in improving matching. In addition to describing the assumptions of the calculation we "rank" the performance of each possible Scenario n in terms of the size of its confusion volume C_n where the smaller the volume the better the performance of the scenario.

1. Figure 1 gives $P_t \times \Delta\phi$ matching volume for the case where only BW1 is used. $C1 > C2 \approx C5 > C3 > C4$

2. Figure 2 gives $P_t \times \Delta\phi$ matching volume for the case where all barrel muon stations (BW1 $\rightarrow$ BW3) are used. $C1 > C2 \approx C5 > C3 > C4$

3. Figure 3 gives $P_t^2 \times \Delta\phi\Delta\theta$ matching volume for the case where only BW1 is used. Extrapolation errors from the inner CTD are **ignored**.
$C1 > C2 > C5 \approx C4 > C3$

4. Figure 4 gives $P_t^2 \times \Delta\phi\Delta\theta$ matching volume for the case where only BW1 is used. Extrapolation errors from the inner CTD are **included.**
$C1 > C2 > C5 > C4 \approx C3$

5. Figure 5 gives $P_t \times \Delta\phi\Delta\theta\Delta P_t$ matching volume for the case where all muon stations are used. Extrapolation errors from the inner CTD are **neglected.**
$C1 > C2 > C5 > C4 \approx C3$

6. Figure 6 gives $P_t \times \Delta\phi\Delta\theta\Delta P_t$ matching volume for the case where all muon stations are used. Extrapolation errors from the inner CTD are **included.**
$C1 > C2 > C5 > C4 \approx C3$

The calculations described above are fairly easy to understand. In the case of $\hat{\phi}$ matching only, one wishes to maximize the lever arm between the two $\hat{\phi}$ doublets and hence the case where the phi doublets sandwich the two $\hat{\theta}$ doublets (Scenario 4, dotted) is the preferred choice. Using the 93 mm tube diameter we compute $\hat{\phi}$ doublet separations ranging from 18.75 cm in Scenario 2 and 5 (or by Eqn. 5 , an effective momenta of 20 GeV), 37.5 cm for Scenario 3 (an effective momenta of 40 GeV) to 56.25 cm for Scenario 4 (an effective momenta of 60 GeV). Indeed the confusion volume rises close to a factor of $\sqrt{2}$ of its low momenta value at these approximate momenta for Figure 1 where only BW1 is used in the matching.

When matching is done in both $\hat{\theta}$ and $\hat{\phi}$ with no CTD extrapolation error (Figures 3), both $\hat{\theta}$ and $\hat{\phi}$ doublets are equally important and one prefers interleaving the doublets to insure maximal lever arms between the $\hat{\phi}$ and $\hat{\theta}$ doublets. (Scenario 3, dash). In fact the CTD extrapolation errors in $\hat{\theta}$ are about an order of magnitude worse than in $\hat{\phi}$ which breaks the near degeneracy between Scenario 4 and 5 (dot and dash dot) in favor of Scenario 4 (maximum $\hat{\phi}$ lever arm). In this case the dashed curve (interleaved doublets) is preferred at low momenta while the dotted curve ($\hat{\phi}$ doublets sandwiching the $\hat{\theta}$ doublets) is preferred at higher momenta.

Clearly the worse thing one can do to degrade matching is remove one of the two $\hat{\phi}$ doublets from BW1. The next worse thing would be reduce both the $\hat{\phi}$ and $\hat{\theta}$ lever arms unnecessarily by placing both $\hat{\phi}$ doublets adjacent to each other followed by both $\hat{\theta}$ doublets. A bad alternative choice would be to sandwich the $\hat{\phi}$ doublets between two $\hat{\theta}$ doublets.

The two remaining defendable choices are between alternating $\hat{\theta}$ and $\hat{\phi}$ doublets (Scenario 3) and $\hat{\phi}$ doublets which sandwich adjacent theta doublets. On the basis of the foregoing, the choice between scenario 3 and 4 is very close and should probably be made on the basis of other criteria. Only in the case of pure $\hat{\phi}$ matching is Scenario 4 favored strongly of Scenario 3. It is conceivable that the projective trigger might work better if both theta doublets are adjacent. In this case scenario 4 would be preferred. Alternatively spacing the $\hat{\theta}$ doublets out some makes the toroid bend measurement more of an angle-angle measurement and thus less reliant on good alignment between BW1 and BW2/3. This effect is probably small but could be calculated if requested.

4. Stereo View in FW1?

The present design of the forward system has all radial views in FW1 and radial and two small angle stereo doublet views ($\pm 7.5^o$) in FW2. In our model the two stereo views are separated (in Z) by 45 cm. In terms of an effective lever arm for 250 micron chambers this provides a ϕ angular measurement doublet lever arm of only 5.9 cm. When coupled with the small θ covered by the region this implies effective P_t^*'s of only a few GeV. Hence the ability to reduce matching confusion by at least a factor of two by extrapolating to the center of the endcap calorimeter disappears at very low P_t since measurement errors rapidly dominate multiple scattering errors in the present system.

The ϕ measurement lever arm and effective momentum can be dramatically extended through the inclusion of stereo views in FW1 and further extended by increasing the stereo angle from $\pm 7.5^o$ to $\pm 22.5^o$. Figure 7 gives the confusion volume ($P_t \times \Delta\phi\Delta\theta\Delta P_t$) as a function of P_t for an $\eta = 2$ putative track measured with the present forward system design (solid), a design augmented with two $\pm 7.5^o$ stereo doublets in FW1 (dash), and with two stereo doublets in FW1 and all stereo angles set to $\pm 22.5^o$. Extrapolation error has been included in these calculations.

A factor of 2 improvement in reducing matching confusion over the relevant momentum range is indeed possible through the inclusion of stereo doublets in FW1. We suspect that confusion is much worse in forward direction than in barrel since it is so active and hence any help is important help.

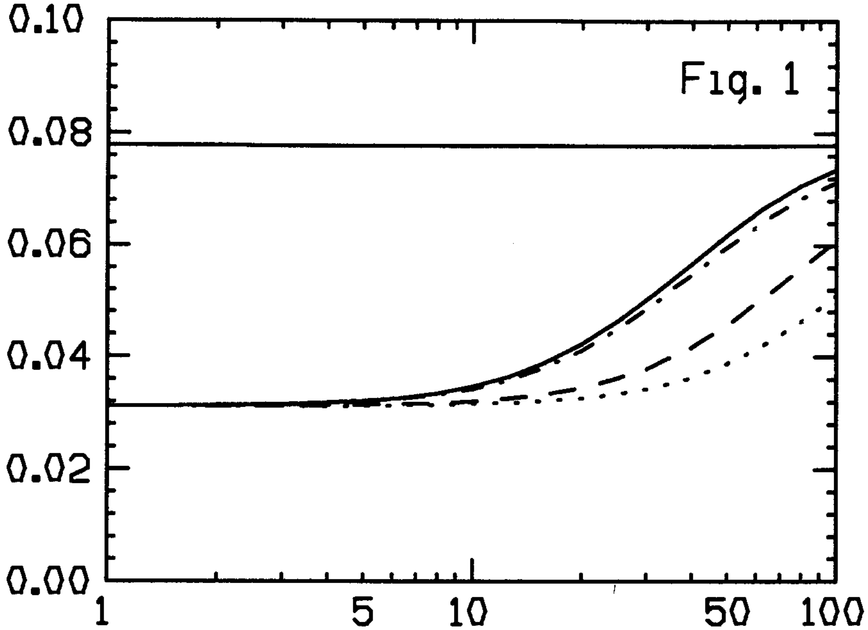

Fig. 1

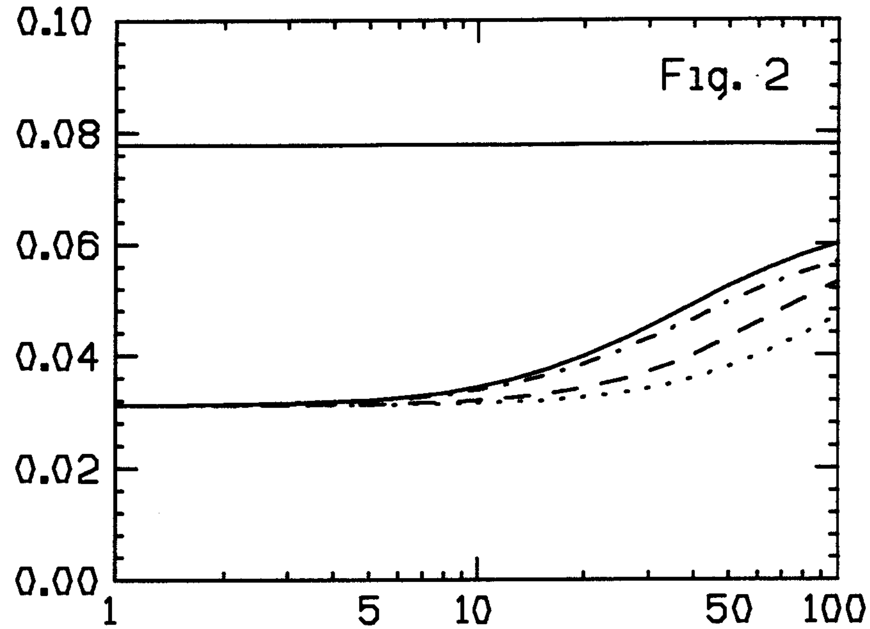

Fig. 2

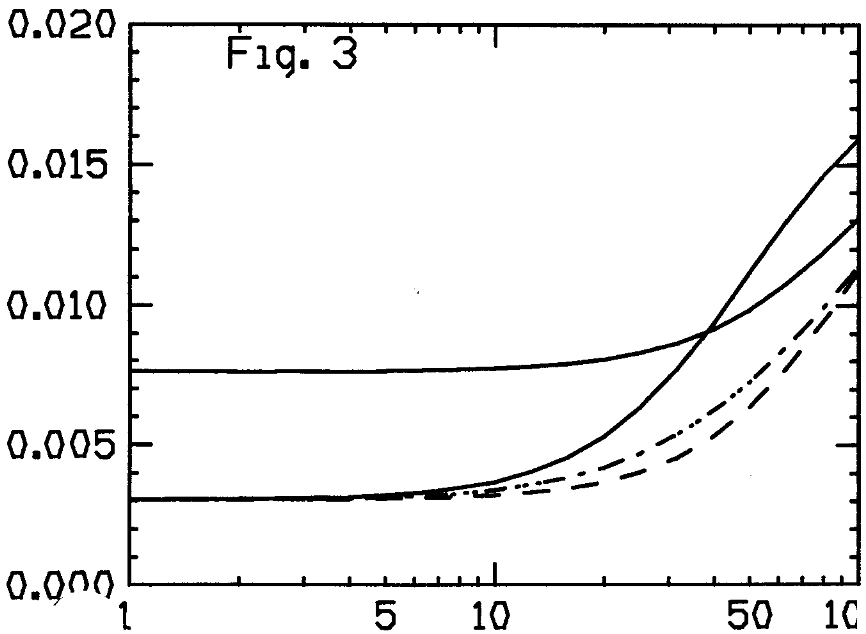

Fig. 3

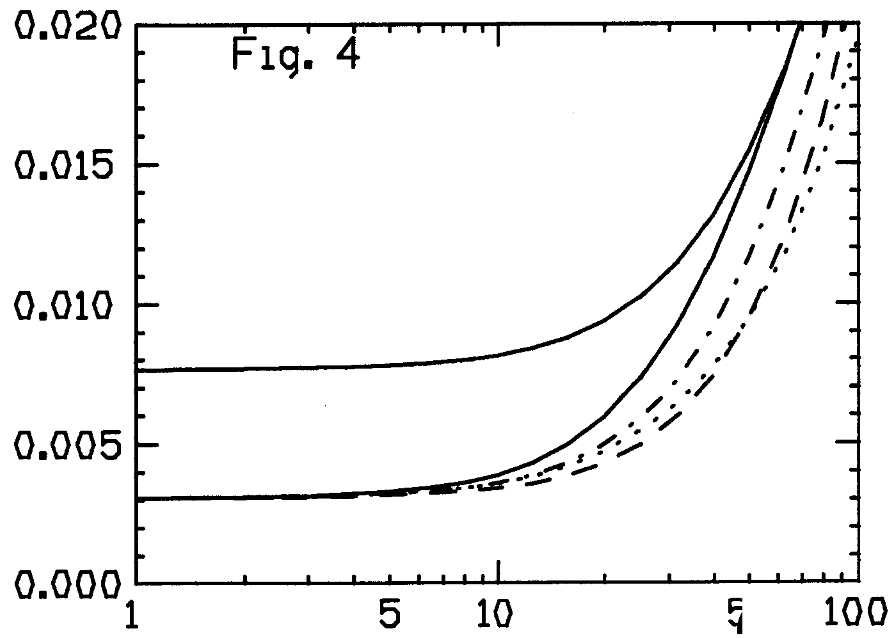

Fig. 4

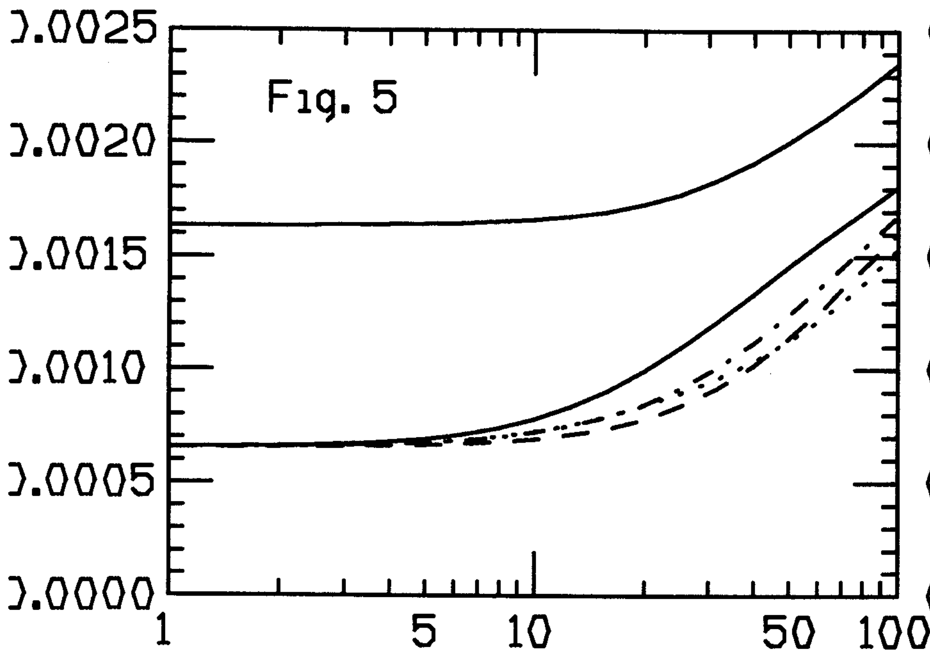
Fig. 5
0.0025
0.0020
0.0015
0.0010
0.0005
0.0000
1
5
10
50
100

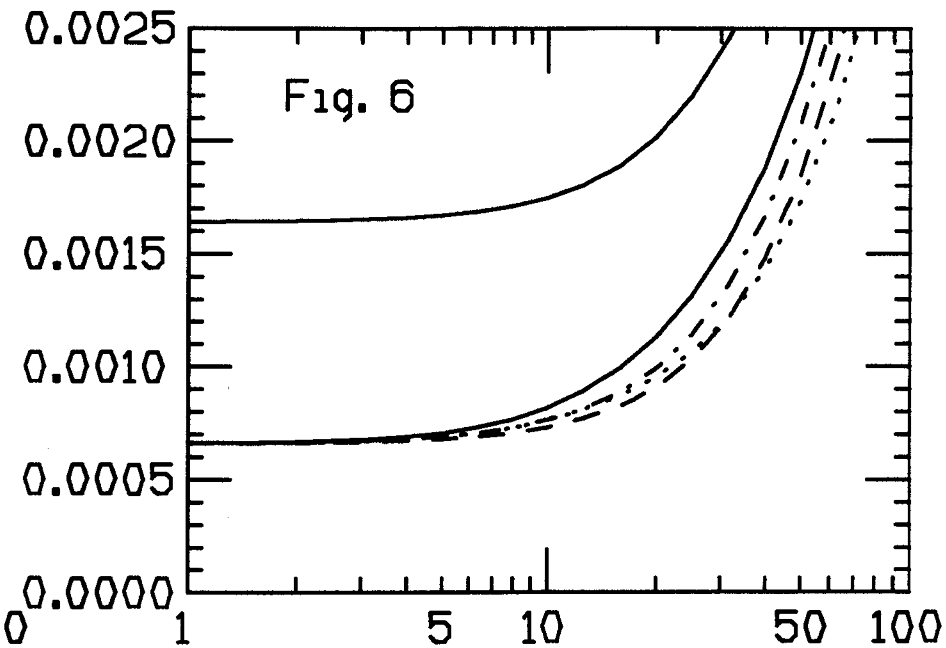
Fig. 6
0.0025
0.0020
0.0015
0.0010
0.0005
0.0000
1
5
10
50
100

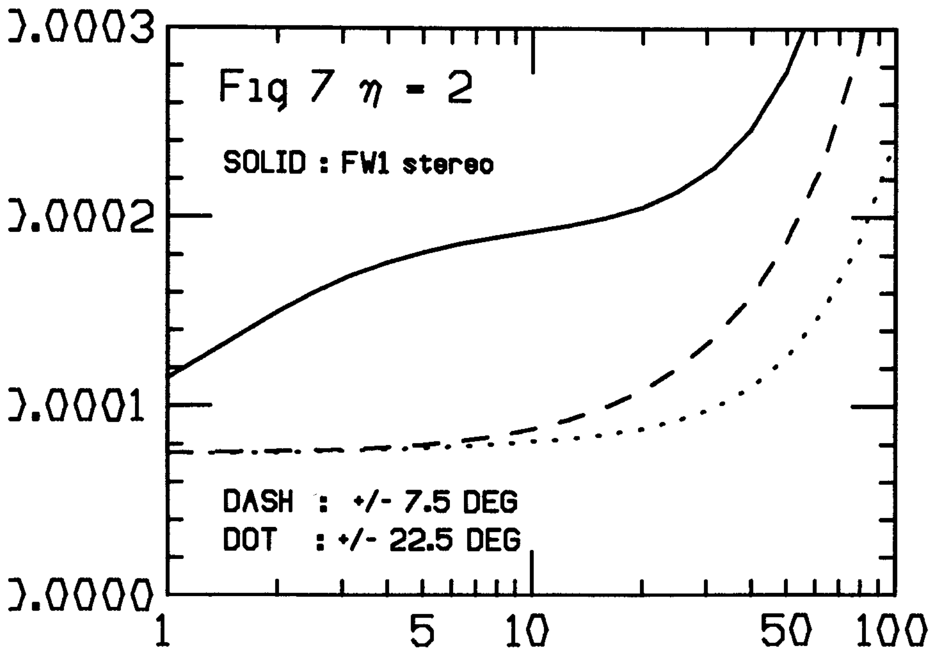
Fig 7 η = 2
SOLID : FW1 stereo
DASH : +/- 7.5 DEG
DOT : +/- 22.5 DEG
0.0003
0.0002
0.0001
0.0000
1
5
10
50
100