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AN EXACT FORMULA FOR LIGHT TRAPPING FRACTION IN OPTICAL FIBRES (Final Version)

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An Exact Formula for Light Trapping Fraction in Optical Fibres
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ABSTRACT

Exact formula are derived for trapping fraction of light emitted isotropically from an interior point in scintillating or WLS-fibres and also for trapping fraction averaged over distance to fibre axis of the emitting point. These formulae contain explicit dependence on the ration n_{21} = (cladding refr. index) / (core refr. index).

Papers dealing with scintillating or WLS-fibres use expression $f = 0.5*(1-n_{21})$ for internally emitted light trapping fraction. This expression is exact only for light emitted by points lying on fibre axis.

We will consider a point inside fibre core situated at a distance r from the fibre axis, fibre core radius being noted with R, as is represented in Figure 1. Such a point is supposed to emit light isotropically.

The solid angle spanned by directions for which emitted light encounters core-cladding interface at incidence angles greater than critical angle ($\sin i_{cr}=n_{21}$) is defined as

$$\Omega(r) = \int_0^{2\pi} d\varphi \int_0^{\theta_m(r,\varphi)} \sin(\theta)\, d\theta \qquad (1)$$

where θ is angle between light ray and fibre axis. For a given φ there is a maximal value $\theta_m(r, \varphi)$ for which this ray will encounter core-cladding interface at angles greater than critical (relative to local normal), to be totally reflected. Geometrical considerations lead for θ_m to expression:

$$\cos(\theta m(r,\varphi)) = \begin{cases} \sqrt{\dfrac{n_{21}^2 R^2 - r^2 \mathrm{Sin}^2\varphi}{R^2 - r^2 \mathrm{Sin}^2\varphi}} & \text{for } 0 \le r \le n_{21}R \\ 0 & \text{for } n_{21}R < r \le R \end{cases} \tag{2}$$

The integration can be performed explicitly in part and following expression is obtained:

$$\Omega(r) = 4\left[\frac{\pi}{2} - \int_0^{\varphi m(r)} d\varphi \sqrt{\frac{n_{21}^2 R^2 - r^2 \mathrm{Sin}^2\varphi}{R^2 - r^2 \mathrm{Sin}^2\varphi}}\right] \tag{3}$$

where

$$\varphi m(r) = \begin{cases} \pi/2 & \text{for } 0 \le r \le n_{21}R \\ \arcsin\left(n_{21}\dfrac{R}{r}\right) & \text{for } n_{21}R \le r \le R \end{cases} \tag{4}$$

Dividing this expression to 4π we obtain the exact formula for trapping fraction. For r=0 this gave the well known expression

$$f(0) = 0.5\,(1 - n_{21}) \tag{5}$$

For points located away from fibre axis f(r) rises monotonically with r, as is seen in Fig. 2. At core surface it attains the value

$$f(R) = 0.5\sqrt{1 - n_{21}^2} \tag{6}$$

In practically interesting case (n_{21} = .94) this is roughly six times greater than corresponding value for points placed on fibre axis (r = 0). So, extrapolating trapping fraction expression deduced in axial case for any

fibre points, irrespective to their position, leads to considerable inaccuracies.

Instead, we have to average formula (3) over all points in a transversal section of fibre. Averaged trapping solid angle for internal points of fiber is:

$$<\Omega> = \left[\int_0^R \Omega(r)\ r dr\right] * \left[\int_0^R r dr\right]^{-1} \qquad (7)$$

For the following calculations, it is more convenient to use another parametrization for light rays inside fibre. As can be seen in Fig. 1, for fixed θ a given class of rays can be defined by different pairs (r,φ) but is uniquely specified by $\rho=r*\sin(\varphi)$. This way we have to do a single integration by ρ, instead of a double one by r and φ, for averaging over possible rays in fibre. The corresponding jacobian for this integration is $\sqrt[4]{R^2-\rho^2}$. Calculations can be done explicitly, leading finally to the exact formula for mean trapping fraction in a fibre:

$$< f > = 0.5\ (1-n_{21}^2) \qquad (8)$$

For practical use (n_{21} close to 1) this expression gives values two times greater than formula (5), currently used in literature. This is a direct consequence of sharp rise of f(r) for points close to core-cladding interface. The difference between these formulae as a function of n_{21} is represented in Fig. 3.

Conclusion

An exact calculation is performed of trapping fraction of light emitted isotropically from points situated inside of a cylindrical scintillating or WLS-fibre. For points located closely to core-cladding interface this fraction is considerably higher than for those lying around fibre axis.

An exact formula is obtained also for trapping fraction averaged over transversal section of fibre. For usual fibres having n_{21}~0.94 this

leads to conclusion that in fact two times more light is trapped than is usually quated (~ 3%). This statement is valid only for a homogeneous distribution of light emitting points in fibre core.

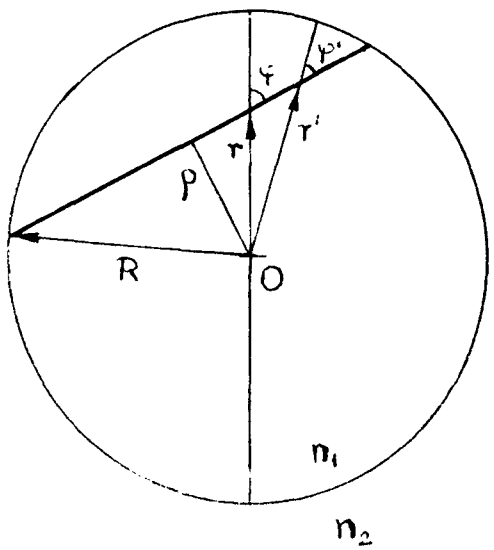
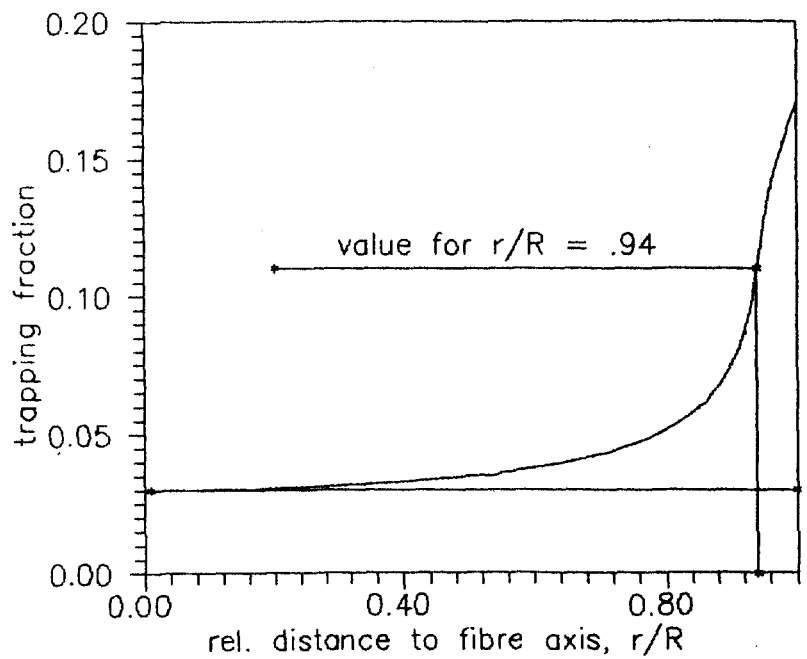


FIG. 1

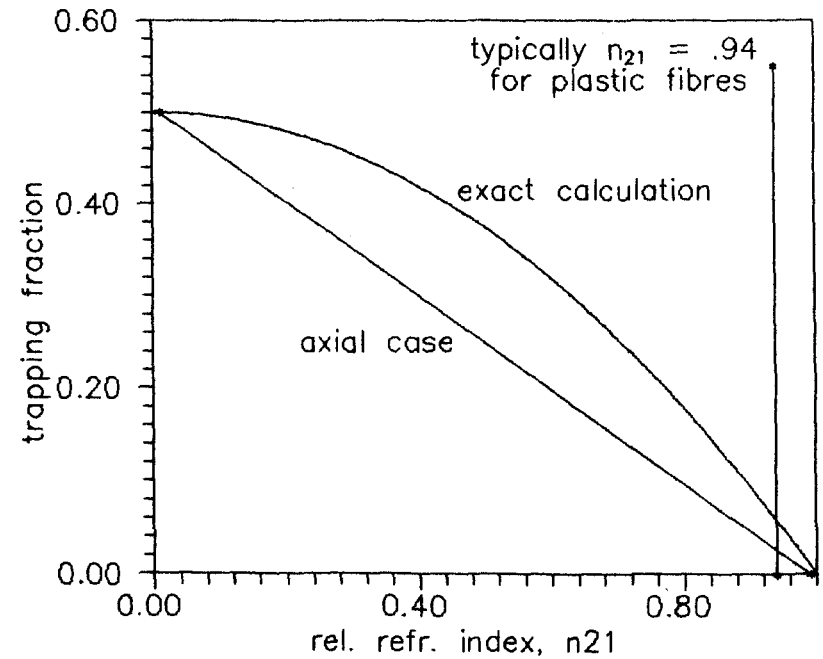
Transverse projection of a ray of light inside fiber with notations used in text

Fig. 2

dependence of trapping fraction on distance from fibre axis



—— the graph is computed for $n_{21}=(n_{clad}/n_{core})=.94$

Fig. 3

n_{21} dependence of mean trapping fraction