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AN EXACT FORMULA FOR LIGHT TRAPPING FRACTION IN OPTICAL FIBERS FLORIN COTOROBAY

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Julian Budagov
SSC Laboratory

AN EXACT FORMULA FOR LIGHT TRAPPING FRACTION IN OPTICAL FIBERS

FLORIN COTOROBAY

(Presented by Julian Budagov)

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ABSTRACT: Julian Budagov's Dubna group is now participating in SDC's calorimetry. Here we present exact formulas which were derived for trapping fraction of light emitted from an interior point in scintillating or WLS-fibres, and also for trapping fraction averaged over distance to fibre axis of the emitting point. These formulas contain explicit dependence on the ratio n_{21} = (cladding refr. index)/(core refr. index).

Papers dealing with scintillating or WLS-fibres use expression $f=0.5*(1-n_{21})$ for internally emitted light trapping fraction. This expression is exact only for light emitted by points lying on fibre axis.

We will consider a point situated at a distance r from fibre axis, fibre core radius being noted with R. The solid angle spanned by directions for which emitted light encounters core-cladding interface at incidence angles greater than critical angle ($\sin\varphi=n_{21}$) can be calculated analytically. So we obtain expression:

Formula (1)

$$\Omega = 4\left[\frac{\pi}{2} - \int_0^{\phi_0} d\phi \sqrt{\frac{n_{21}^2 R^2 - \sin^2\phi}{\frac{R^2}{r^2} - \sin^2\phi}}\right] \quad ; \phi_0 = \begin{cases} \frac{\pi}{2} \text{ for } \frac{r}{R} < n_{21} \\ \arcsin\left(n_{21}\frac{R}{r}\right) \text{ for } \frac{r}{R} > n_{21} \end{cases}$$

For r = 0 this leads to known relation

Formula (2)

$$\Omega = 2\pi(1-n_{21})$$

For r = R the integral can be evaluated analytically, resulting expression

Formula (3)

$$\Omega = 2\pi\sqrt{1-n_{21}^2}$$

In practically interesting case (n_{21} = .94) this is roughly 6-times greater than corresponding value for points placed on fibre axis (r = 0). So, extrapolating trapping fraction expression deduced in axial case for any fibre points, irrespective to their position, leads to considerable inaccuracies.

Instead, we have to average Formula (1) over all points in a transversal section of fibre. Averaged trapping solid angle for internal points of fibre is:

Formula (4)

$$\langle\Omega\rangle = \frac{\int_0^R rdr\,\Omega(r)}{\int_0^R rdr}$$

which results in expression:

Formula (5)

$$\langle\Omega\rangle = -4\left(1-n_{21}^2\right)\left\{\int_0^{n_{21}} \frac{dt}{t^2\sqrt{1-t^2}} \ln\left(\frac{\sqrt{1-t^2}+\sqrt{n_{21}^2-t^2}}{1+n_{21}}\right) + \frac{\sqrt{1-n_{21}^2}}{n_{21}} \ln\sqrt{\frac{1-n_{21}}{1+n_{21}}}\right\}$$

For practical uses (n_{21} close to 1) this expression can be simplified, behaving like

Formula (6)

$$\langle\Omega\rangle \sim 2\pi\left(1-n_{21}^2\right)$$

which is numerically 2-times greater than corresponding value of (2).

This confirms the assertion made earlier about general use of axial case Formula (2). Figures 1 and 2 show respectively, radius dependence of Ω and n_{21} dependence of $\langle\Omega\rangle$, compared with usage of relation (2).

Concluding, we state that in cases of practical interest (n_{21} above 0.8) a much more accurate formula for mean trapping fraction in scintillating or WLS plastic fibres is:

Formula (7)

$$f = 0.5\,(1-n_{21}^2)$$

Fig. 2

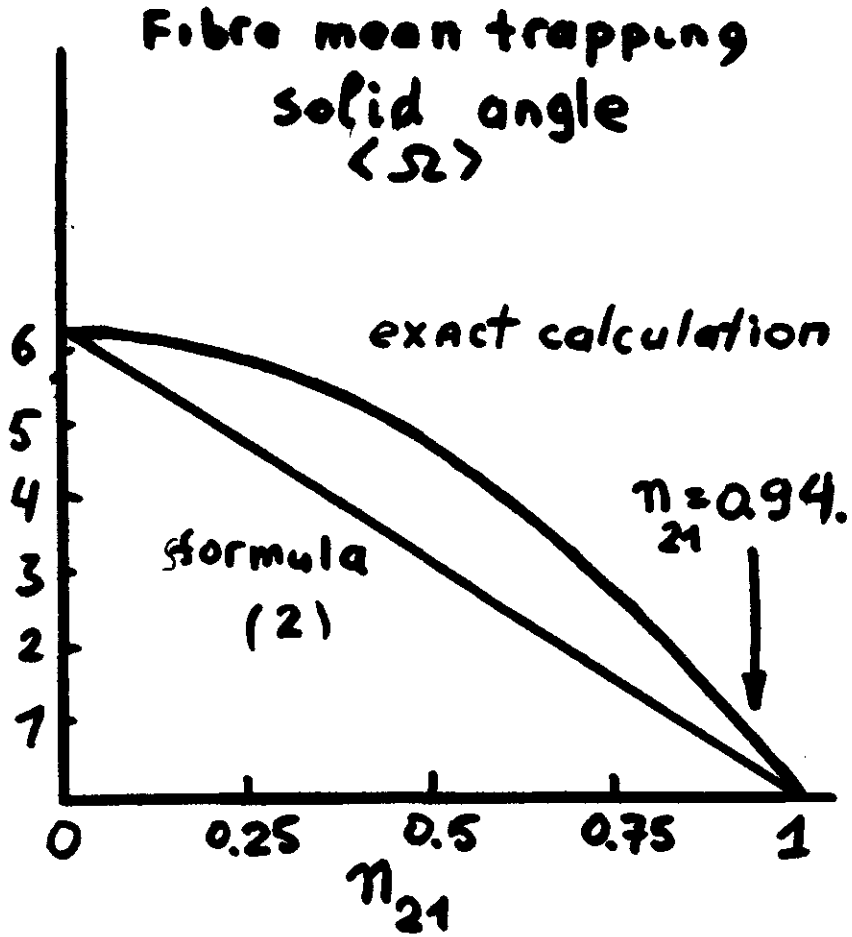
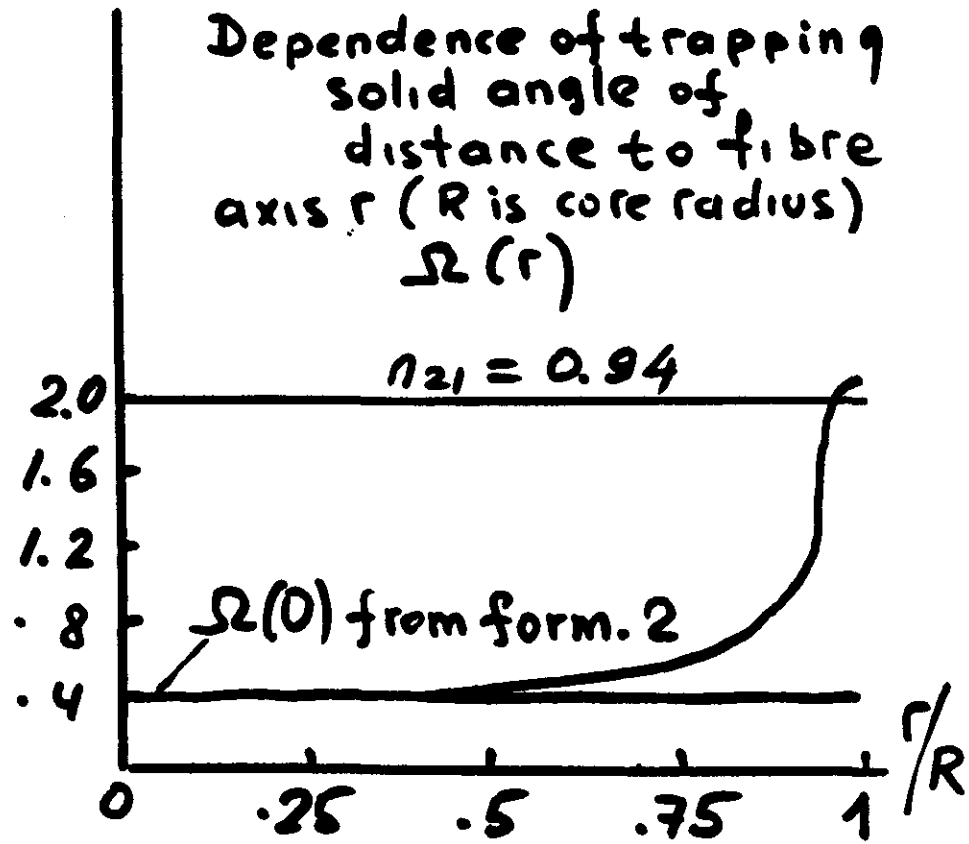

Fibre mean trapping
solid angle
⟨Ω⟩
exact calculation
formula
(2)
n_{21}=0.94.
6
5
4
3
2
1
0
0.25
0.5
0.75
1
n_{21}

Fig. 1


Dependence of trapping
solid angle of
distance to fibre
axis r (R is core radius)
Ω(r)
n_{21} = 0.94
Ω(0) from form. 2
2.0
1.6
1.2
.8
.4
0
.25
.5
.75
1
r/R