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MUON MATCHING STUDIES

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Muon Matching Studies

This note summarizes the results of studies investigating the ability to match muon tracks measured in the Central Tracker (CTD) with hits in the muon system without confusion from adjacent putative (candidate) tracks in the CTD. We began this study to investigate how the placement of ϕ and stereo planes in BW1 $\rightarrow$ BW3 and the thickness of BW1 affects the ability to match CTD tracks to the muon system. After describing the calculational technique, we present some analytic calculations of a quantity we call the confusion volume which serves as a sort of "Rayleigh criterion" as to when a candidate track has track parameters which are too close to those of a muon track to be resolved within the muon system. The (inverse) size of the confusion volume provides figure of merit when comparing possible chamber layouts for the barrel muon system. At low $P_\perp$, where multiple scattering effects predominate, it is better to place ϕ layers in BW1 and thus avoid the multiple scattering of tracks by the toroid. At very high $P_\perp$, where measurement error dominates over multiple scattering, one gets better matching resolution by moving the ϕ layers to BW3 thus allowing the extrapolated tracks to separate more in space relative to the intrinsic chamber resolution. An initially surprising conclusion is that in the low $P_\perp$ limit, layouts with two ϕ layers[1] in BW1 exhibit significantly reduced matching confusion (by a factor of ≈ 2.6) over layouts with no ϕ layers in BW1 but with ϕ layers outside of the toroid. Nearly all of this improvement is made possible by the local ϕ slope measurement provided by the two ϕ layers in BW1. This big BW1 advantage is not present in strawman layouts with only a single ϕ layer, or in arrangements with a short lever arm between the ϕ planes at moderate $P_\perp$

In order to assess the level of confusion, and find out which $P_\perp$ regime is relevant for physics processes we have performed Monte Carlo (ISAJET) simulations

of the high $P_\perp$ $B\overline{B}$ process. These Monte Carlo studies confirm that the low $P_\perp$ (multiple coulomb scattering dominated) limit is appropriate and a factor of ≈ 2 improvement in matching confusion should indeed be realized by placing two ϕ layers in BW1 rather than placing them outside of the muon toroid.

One word of caution... the results discussed here represent a lower bound on the real confusion since we have neglected the effects of δ-rays, bremsstrahlung, etc. which undoubtedly increase confusion[2].

1. Technique

We assume that the off line matching of the muon chamber hits with a possible CTD track is performed using a χ^2 test to the extrapolated track trajectory. A track is matched if it has a minimum, acceptable confidence level to the muon chamber hits where the confidence level is chosen so that the vast majority (*eg* 98 %) of real CTD muons are successfully matched (*eg* , $CL > 0.02$). Confusion results when an adjacent track has sufficiently similar track parameters that its extrapolation satisfies the muon confidence level cut.

It is fairly easy to "simulate" such a matching criteria by calculating the expectation value of the χ^2 for a given putative track to the hits of a given muon track in terms of the difference between the muon track and putative track parameters. We can then approximate the matching criteria by declaring confusion if $< \chi^2 > < \chi^2$ (CL = 0.02) . The matching χ^2 is computed as follows:

Let the relevant track parameters be denoted as $t_\alpha^{(\mu)}$ for the muon and $t_\alpha^{(p)}$ for the putative track. In our studies possible track parameters include the azimuth (ϕ), polar angle (θ), and curvature (Q/P). We define a *transport matrix* $T_{i\alpha}$ which multiplies the α'th track parameter to predict the i'th hit in the muon system (where we call this hit w_i). In our approach we assume that the i'th hit is due to the muon and follows the muon trajectory $T_{i\alpha}t_\alpha^{(\mu)}$ but suffers fluctuations δw_i due to multiple scattering and measurement error. The matching χ^2 will be constructed from the CTD putative track trajectory and the inverse coordinate

covariance matrix $C_{ij}^{(p)\ -1}$ of the putative track. We assume that this inverse coordinate covariance matrix can be computed for the putative track using the momentum as measured by the CTD and the known measurement errors of the muon system. We further assume that the CTD track parameter errors are negligible and ignore issues of left/right ambiguity and δ - ray effects. With these simplifications, the matching χ^2 is given by:

$$\chi^2 = \left(T_{i\alpha} t_\alpha^{(p)} - T_{i\alpha} t_\alpha^{(\mu)} - \delta w_i\right)\ C_{ij}^{(p)\ -1}\ \left(T_{j\beta} t_\beta^{(p)} - T_{j\beta} t_\beta^{(\mu)} - \delta w_j\right)$$

Taking the expectation value and autocorrelating we get:

$$< \chi^2 >= \Delta t^t\ H^{(p)}\ \Delta t + Tr\ C^{(\mu)}\ C^{(p)\ -1} \tag{1}$$

where:

1. Δt^t is the transpose of the vector describing the difference between the muon and putative track track parameters.
2. $H^{(p)}$ is the fit matrix for the putative track. The inverse of the fit matrix is the putative track parameter error matrix. The components are $H_{\alpha\beta}^{(p)} = C_{ij}^{(p)\ -1}\ T_{i\alpha} T_{j\beta}$
3. $C^{(\mu)}$ and $C^{(p)}$ are the coordinate covariance matrices for the muon and putative tracks including measurement and multiple scattering effects.

The second term of Eqn.(1) has a straightforward physical interpretation. If the covariance matrix of the putative track is the same as that of the muon which caused the hits, the second term will be the trace of the unit matrix which is just the the number of muon system coordinates or the number of degrees of freedom in the χ^2 matching test. If, in addition, $\Delta t = 0$, one would obtain the familiar result: $< \chi^2 >= N_{\rm dof}$. If, on the other hand, the putative track is much better measured than the muon (*eg* resolution is mcs dominated and the putative track has a much higher momentum than the muon), the second term will insure

$< \chi^2 > \gg N_{\rm dof}$ even if $\Delta t = 0$. In this hypothetical case, even though putative track extrapolates the same as the muon, the spread in the hits around the track is much larger than expected given the putative track momentum and thus the match will typically be unacceptable.

2. "Analytic" Results

In this section we compare the matching confusion characteristics of various possible muon chamber layouts which differ in the number and placement of ϕ layers in BW1 $\rightarrow$ BW3. Each layout places two (projective) θ layers[3] in each of the stations BW1 $\rightarrow$ BW3. Each layer is assumed to offer a spacial resolution of 250 μm. The positions and thicknesses of the BW1 $\rightarrow$ BW3 chamber stations are those suggested by the integration group[4], **except** we use a 60 cm thick BW1 rather than the 35 cm allowed in integration group layout. The below tables summarize the assumed geometry and various layouts used in these studies.

Table 1 : Radial Geometry

	R_{in}	R_{out}
Calorimeter[5]	2.1 m	4.32 m
BW1	5.67	6.27
Toroid	6.45	7.95
BW2	8.18	8.53
BW3	9.78	10.66

Table 2 : Layout Description[1,3]

Layout	BW1	BW2	BW3
1	2ϕ , 2θ	2θ	2θ
2	2θ	2ϕ , 2θ	2θ
3	2θ	2θ	2ϕ , 2θ
4	2ϕ , 2θ	2ϕ , 2θ	2ϕ , 2θ

We next compare the level of matching confusion for the 4 possible layouts. A convenient comparison measure of matching confusion is the volume of the space of parameter differences which gives a χ^2 change of 1 unit. By Equation 1, the $\chi^2 = 1$ boundary will form an ellipsoid in $\Delta 1/P$, $\Delta\phi$, and $\Delta\theta$. The volume of this ellipsoid will be proportional to the reciprocal square root of the product of the eigenvalues of the $H^{(p)}$ matrix[6]. In principle one can match a putative track to muon hits using just the ϕ layers ($\Delta\phi$), or just θ layers ($\Delta 1/P\ \Delta\theta$), or all layers ($\Delta 1/P\ \Delta\phi\ \Delta\theta$).

Figure 1a compares the $\Delta\phi$ confusion volume for putative tracks at $\eta = 0$ as a function of $P_\perp$ for Layout 1 (ϕ layers in BW1: solid curve) , Layout 2 (ϕ layers in BW2 : dash) , Layout 3 (ϕ layers in BW3 : dot), and Layout 4 (ϕ layers in all BW : dash dot). We actually plot $P_\perp \times \Delta\phi$ since $\Delta\phi \propto 1/P_\perp$ at low $P_\perp$ where mcs effects dominate measurement error. The η dependence of the $P_\perp \times \Delta\phi$ is proportional to $1/\sqrt{sin\ \theta}$ at low $P_\perp$. A useful summary of Figure 1a and the foregoing is that the effective ϕ resolution in the low $P_\perp < 20\ GeV$ limit is:

$$\sigma_\phi = \begin{pmatrix} \frac{31\ mrad-GeV}{P_\perp\ \sqrt{sin\ \theta}} & (2\phi \text{ in BW1 }) \\ \frac{80\ mrad-GeV}{P_\perp\ \sqrt{sin\ \theta}} & (2\phi \text{ in BW2/3}) \end{pmatrix}$$

At low $P_\perp$, cases where ϕ layers are placed in BW1 (Layouts 1 and 4) have about 2.6 times less confusion than cases where BW1 contains no ϕ layers (Layouts 2 and 3). Our Monte Carlo studies of $B\overline{B}$ production (discussed in Section 3) show that low $P_\perp$ is the most relevant limit. We note that the confusion volume for Layout 2 is exactly the same as the volume for Layout 3. If one must put the ϕ layers outside the toroid, the radius choice for the outer ϕ layers makes no difference at low $P_\perp$.

We note the double plateau structure of the volume versus $P_\perp$ curve (Figure 1a) which is most pronounced for Layout 1 (solid curve). The double plateau structure is crucial to the case for placing ϕ layers in BW1. Figure 1b hints at the explanation for this effect by showing the effects of removing one of the two

ϕ layers in Layout 1 (lower curves) or in Layout 3 (upper curves). The solid curves have two ϕ layers; while the dashed curves have only one. Apparently the lower $P_\perp$ plateau uses information from both ϕ layers, while the higher $P_\perp$ plateau effectively uses information from just one layer. A ϕ match can be made using both the ϕ intercept (which can be measured with just one layer) and the ϕ slope (which requires two ϕ layers). Figure 1a,b suggests that the slope information provided by two ϕ layers in BW1 is vastly superior in discriminating among matches at low $P_\perp$ but owing to the short lever arm $\approx$ 30 cm between the two planes, the slope information rapidly deteriorates with increasing $P_\perp$ as measurement information begins to dominate over mcs. Reduction in the ϕ lever arm below 30 cm creates an even more rapid deterioration in the slope information. For example if the radial separation between the two ϕ layers is reduced to 10 cm, a 52% degradation in ϕ matching resolution will occur relative to the 30 cm case even at $P_\perp$ as low as 25 GeV.

Figure 1c shows the matching confusion volume $P_\perp \times (\Delta P_\perp\ \Delta\phi\ \Delta\theta)$ as a function of $P_\perp$ of the putative track at $\eta = 0$ for the 4 layouts. The same solid, dash, dot, dash dot code is used in Figure 1c as 1a. Again in the low $P_\perp$ limit, layouts 1 and 4 which feature two ϕ layers in BW1, have about a factor of 2.6 times less confusion than layers 2 and 3 which have no ϕ layers in BW1. A factor of $sin\ \theta\ P^2$ was included in the calculation of the $(\Delta P_\perp\ \Delta\phi\ \Delta\theta)$ volume to convert from $\Delta 1/P$ to $\Delta P_\perp$. To facilitate comparisons we include an additional factor of $P_\perp$ since again the volume $(\Delta P_\perp\ \Delta\phi\ \Delta\theta) \propto 1/P_\perp$ in the low $P_\perp$ limit where resolution is mcs dominated. The η dependence of the $P_\perp \times (\Delta P_\perp\ \Delta\phi\ \Delta\theta)$ is proportional to $\sqrt{sin\ \theta}$ at low $P_\perp$.

Finally we consider the effects of an station to station misalignment on our matching confusion studies. We assume that all the layers within a BW1 $\rightarrow$ BW3 station are well aligned with respect to each other, however each station is miss aligned with an rms error (in $R\ \phi$) of $\sigma = 1mm$ with respect to the CTD. Figure 1d is a plot of $P_\perp \times \Delta\phi$ versus $P_\perp$ at $\eta = 0$ for the 4 layouts (using the same solid , dashed , dot , dash dot code as in Figure 1a) under the misalignment assumption.

In the low $P_\perp$ region the misalignment has no effect. We believe this is because the matching information comes from the ϕ slope rather than the intercept and the slope measurement is unaffected by station to station misalignment.

3. Monte Carlo Studies

The latest version of ISAJET (V6.43) was used to generate $B\overline{B}$ pair TWO-JET events for $\sqrt{s} = 40$ TeV p-p collisions. The B-meson/baryon in each ISAJET event was FORCE1 semi-muonically decayed to allowed B-decay modes as specified in the latest DECAY.DAT table. ISAJET runs varying the B-quark Pt interval were made. Information for charged, stable particles with $P_\perp > 10.0$ GeV/c and $|\eta| < 3.0$ were written to a separate file for input to the confusion analysis program(s). Table 3 (below) summarizes the results of these MC runs. Figure 2 shows the $B\overline{B} \rightarrow 1\mu$ cross section $\times$ semi-muonic branching ratio versus $P_\perp(B)$.

Table 3- Isajet V6.43 SSC $B\overline{B} \rightarrow 1\ \mu$ Simulation Results

# Events	$P_\perp(B)$ Limits	$\sigma\ (mb)$	$\int \mathcal{L}\ dt\ \ (mb^{-1})$
10000	10- 50 GeV/c	0.6033E-02	0.1658E+07
10000	50-100 GeV/c	0.5710E-04	0.1751E+09
10000	100-200 GeV/c	0.5223E-05	0.1914E+10
10000	200-300 GeV/c	0.3309E-06	0.3022E+11
10000	300-400 GeV/c	0.5503E-07	0.1817E+12
5000	400-500 GeV/c	0.1412E-07	0.3539E+12

For all charged, stable particles generated/event, require:
a.) $P_\perp(trk) > 10.0$ GeV/c
b.) $|\eta(trk)| < 3.0$

Analysis of the 400< $P_\perp(B)$ <500 GeV/c data set is presented first, followed by results for the remaining $P_\perp(B)$ intervals. In Figure 3, distributions for the accepted event multiplicity, the muon $P_\perp$, and the muon η are shown which characterize the event sample. Only muons satisfying $|\eta| < 1.5$ were subjected to the matching analysis.

For computing expediency, tracks falling within ± 100 $m$rad (in both $\Delta\phi$ and $\Delta\theta$) of the muon candidate were considered putative tracks for the confusion test. **Computation of muon and putative track confusion is restricted to tracks within the fiducial cone.**[7] Typical values for the number of of tracks close enough to the muon to fall into the putative track category ranged between 0 and 15, as illustrated in Figure 3d. To gauge the level of confusion, putative track parameters were used to compute the expectation value of the matching χ^2 described above. If a putative track was found to have a confidence level greater than 2% then the associated muon was declared confused.

The results of the confusion analysis on the 400< $P_\perp(B)$ <500 GeV/c run confirm the analytic results regarding placement of ϕ layers. We consider first matching tests performed using information from the ϕ layers only. The four geometry layouts of Table 2 and an additional three layouts, which are identical to layouts 1-3 except that each ϕ chamber consists of only one left-right unambiguous layer, are used as test configurations. A histogram of the number of tracks which are confused is shown for layout 1 in Figure 4a. The points in Figure 4b give the average number of confused tracks corresponding to the each of the layout schemes. The case of a single ϕ layer placed in BW1 is layout 5, a single ϕ layer in BW2 is layout 6, and a single ϕ layer in BW3 is layout 7. The increase in confusion that results when the ϕ layers are moved from BW1 to BW2 is about a factor of 2.2, consistent with the analytic result given the average momentum of putative tracks is 25 GeV/c. Note also that placement of the ϕ layers in either BW2 or BW3 makes little difference in the average number of confused tracks for this $P_\perp$ region. When only one ϕ layer per BW station is used (the last three bins of Figure 4b) the confusion is 15% to 90% worse than the corresponding deployment with two ϕ layers.

Similarly, when matching is performed using information from all layers ($\Delta Q/P$ $\Delta\phi$ $\Delta\theta$) the average number of confused tracks is consistent with analytic calculations. Figures 4c,d give the number of confused putative tracks and the average number of confused putative tracks for each of the layouts. Confusion rates for layouts 2 and 3, which have no ϕ layers in BW1, are found to be a factor of 2.4 greater than those having ϕ layers in BW1.

The dependence of the confusion on the $P_\perp$ and η of the muon is shown in

Figures 5a,b for ($\Delta\phi$) matching using layout 2. (The results for layout 1 are similar but with reduced statistics). The η dependence reflects the reduced resolving power of ($\Delta\phi$)-only measurements for tracks traversing greater lengths of material. For ($\Delta Q/P$ $\Delta\phi$ $\Delta\theta$) matching the corresponding dependencies are shown in Figures 5c,d. Curvature matching improves with large $P_\perp$ and η resulting in a flat η dependence.

In Figure 6 the results of the confusion analysis of the remaining ISAJET Monte Carlo runs are shown. Each row in the figure corresponds to one of the data sets (the $10 < P_\perp(B) < 50$ GeV/c and $50 < P_\perp(B) < 100$ GeV/c samples have been combined) and gives the number of tracks falling within the fiducial cone as well as the average number of confused tracks using the two matching criteria. Note that the number of confused tracks inside the fiducial cone is a strong function of $P_\perp(B)$. For either full matching or ($\Delta\phi$) matching, the average confusion tends to increase with $P_\perp(B)$. Events in the highest $P_\perp(B)$ sample have confusion rates which are a factor of 4 to 7 times that of the lowest $P_\perp(B)$ sample when ($\Delta\phi$) matching is used. When full matching is employed the corresponding confusion growth is typically a factor of 3 for most geometry configurations. Figures 7,8 give the confusion dependence on $P_\perp(B)$ according to layout scheme employed for both matching criteria.

We repeated the analysis for a sample of decays $T\overline{T} \rightarrow \mu X$ generated with a top quark mass of 130 GeV. The average number of confused tracks in the fiducial cone were found to differ only by a few percent from the $B\overline{B}$ samples when full matching was used; however the number of tracks within the fiducial cone is much smaller than in the $B\overline{B}$ samples.

4. Summary

We have presented studies on the ability to match putative tracks from the CTD to hits in the muon system. We have concentrated on how the placement of ϕ layers in the three chamber stations affects the level of confusion. These results have potential applications to both the offline analyses such as B physics, inclusive muon studies, as well as in the 3rd level trigger. In the interests of expediency, we have simplified the problem by ignoring the effects of δ-rays, left/right ambiguity, muon showering etc. and hence our absolute estimates of

confusion are really lower bounds; we hope the comparisons of confusion between different layout scenarios are valid.

Our central conclusion is that the placement of two ϕ layers in BW1 significantly ($\times 2.6$) reduces matching confusion in the low $P_\perp$ limit compared to layouts without ϕ layers in BW1. At sufficiently high $P_\perp$ (for the putative track), the station choice and number of ϕ layers becomes irrelevant to the muon - CTD matching issue. Beyond $P_\perp = 10TeV$, ϕ layers in BW3 is actually preferable to ϕ layers in BW1. If two ϕ layers are placed in BW1, shortening the lever arm between the two ϕ layers will significantly reduce the range of $P_\perp$ with excellent ϕ matching resolution. These conclusions follow from resolution calculations, and are confirmed in high $P_\perp$ $B\overline{B}$ simulations.

In addition, analysis of the ISAJET simulations showed that reduction in confusion levels acheived by placement of ϕ layers in BW1 with adequate lever arm were significant over a large $P_\perp(B)$ momentum range. Typically the average confusion level for putative tracks is 0.1% to 0.5% when ϕ layers are placed in BW1 and full matching is employed. If only ϕ matching is used putative and muon tracks are confused 20% to 50% of the time on average. The confusion levels were also found to exhibit reasonable behavior as a function of $\eta(\mu)$; the confusion grows with $|\eta(\mu)|$ when only ϕ information was used while becoming essentially independent of $|\eta(\mu)|$ with full matching.

Footnotes

(1)We model this as two independent ϕ measurements separated by an $\approx 30cm$ lever arm. In reality four physical ϕ layers might be used because of drift time and ambiguity considerations.

(2)We hope that comparisons between one possible layout and another are still valid, however!

(3)One θ layer is placed close to the inside radius of each BW package and the other is placed on the outside radius. The ϕ layers are sandwiched in between the two θ layers with a maximum possible spacing between them.

(4)Dated 9/23/91 , distributed at the October ORNL SDC meeting

(5)We model the calorimeter (for scattering purposes) as a 2.22 meter thick slab of iron.

(6)Or the reciprocal square root of the $H^{(p)}$ determinant.

(7) The effect of the cone definition on putative track selection is $\approx 5\%$ underestimation of the absolute confusion level over a large $P_{\perp}(B)$ region.

FIG 1: CONFUSION VOLUME VERSUS Pt

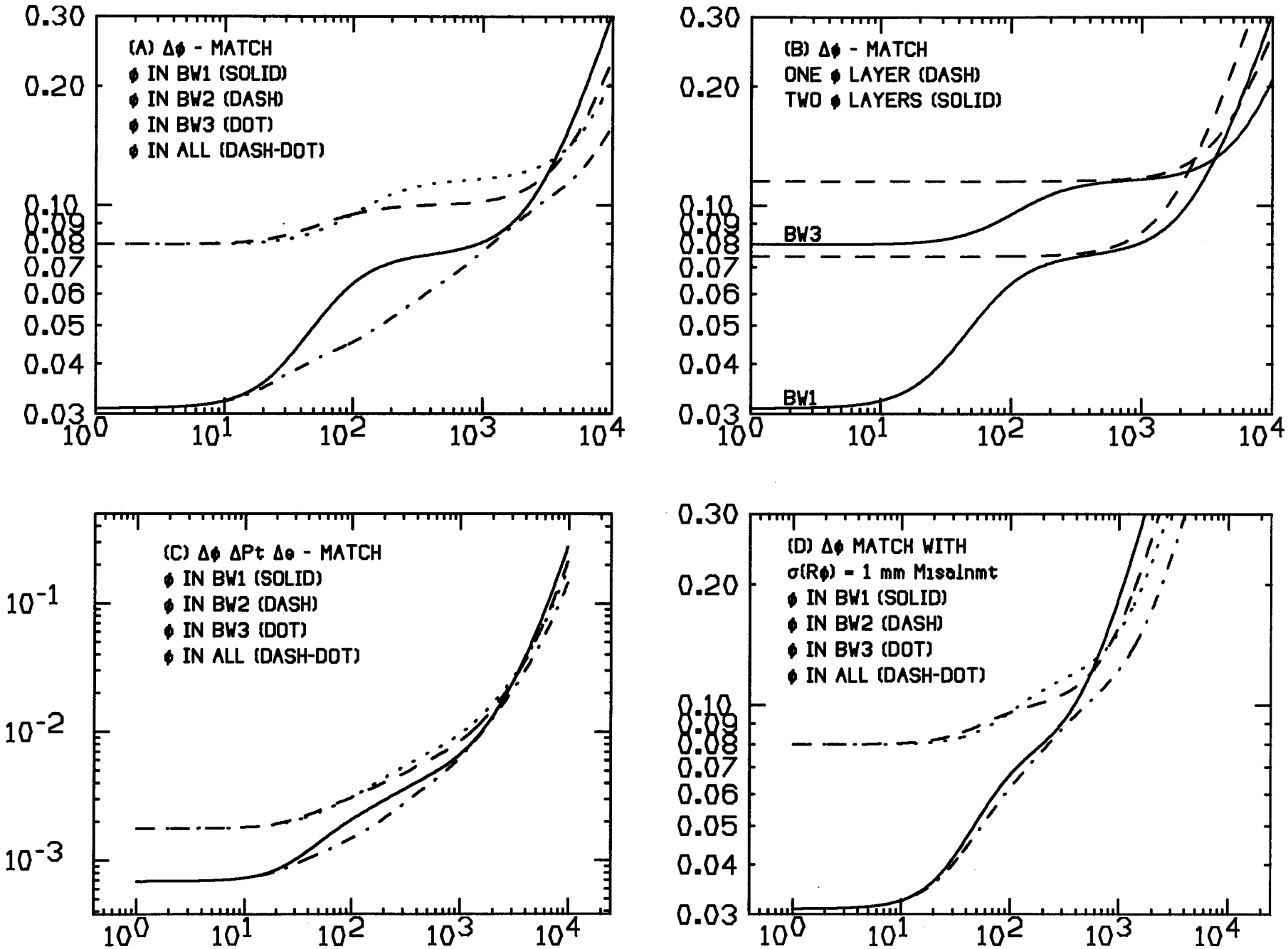
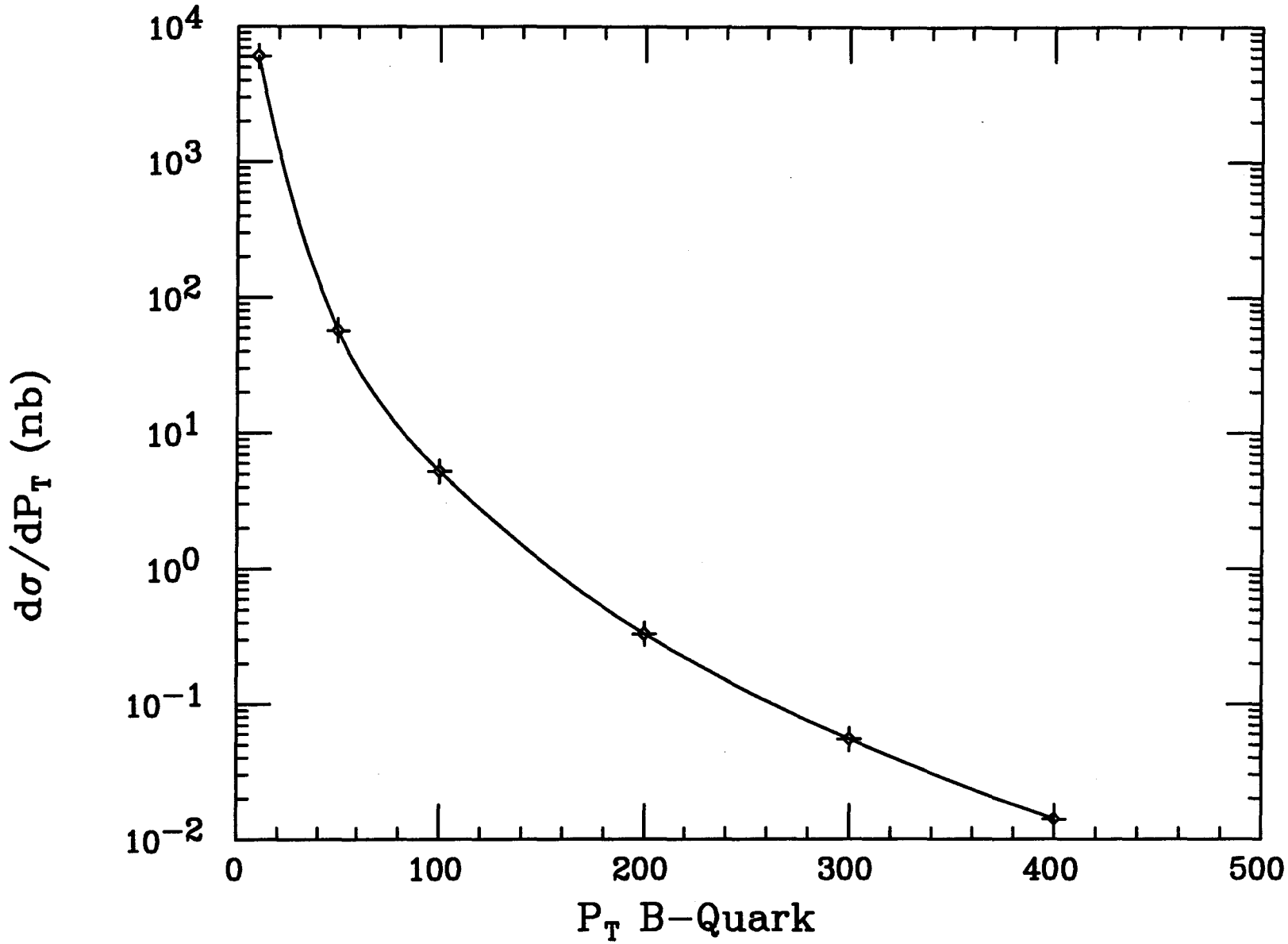


Figure 2:BBbar -> 1 μ Cross Section



ISAJET V6.43

Track P_T > 10.0 GeV/c, $|\eta|$ < 3.0

Fig 3a: NTRAK

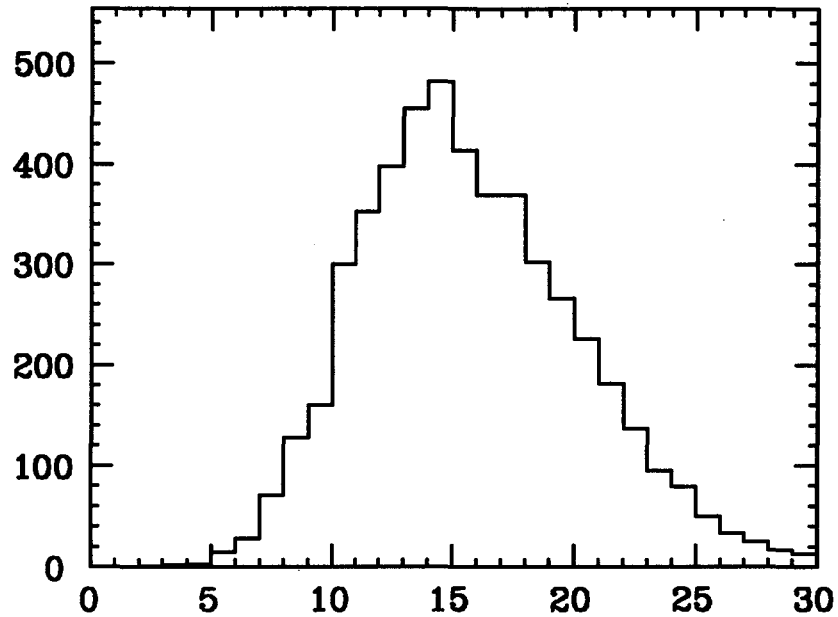


Fig 3b: $\eta(\mu)$

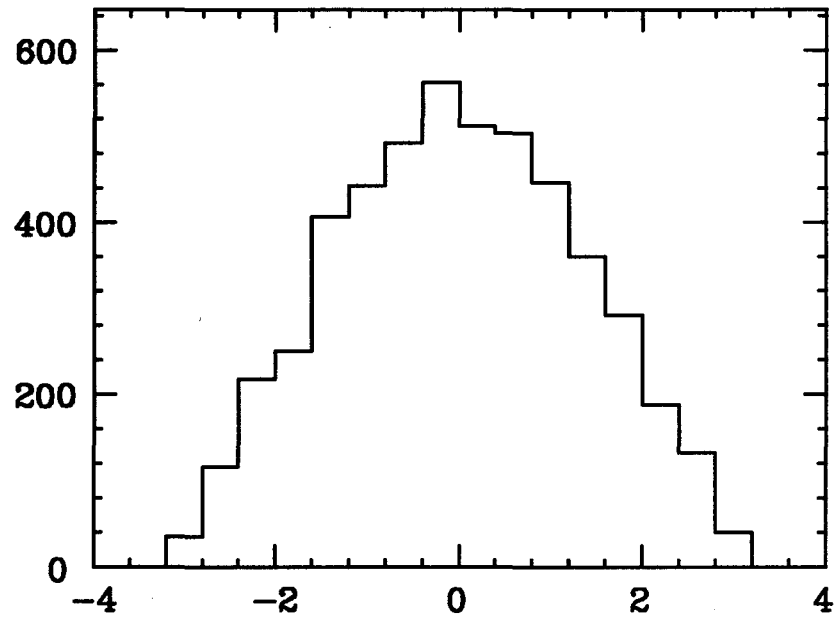


Fig 3c: Pt(μ)

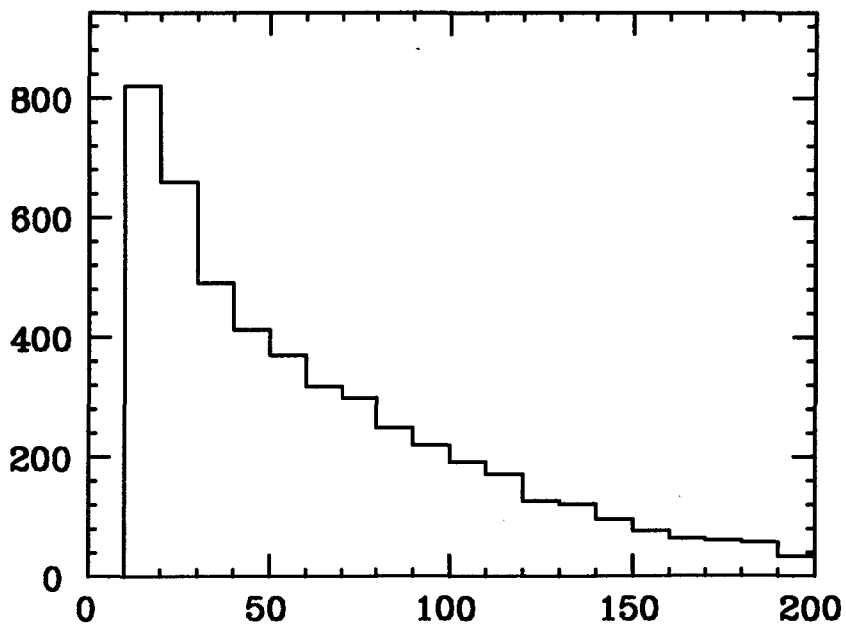


Fig 3d: NCONE ($-1.5<\eta(\mu)<1.5$)

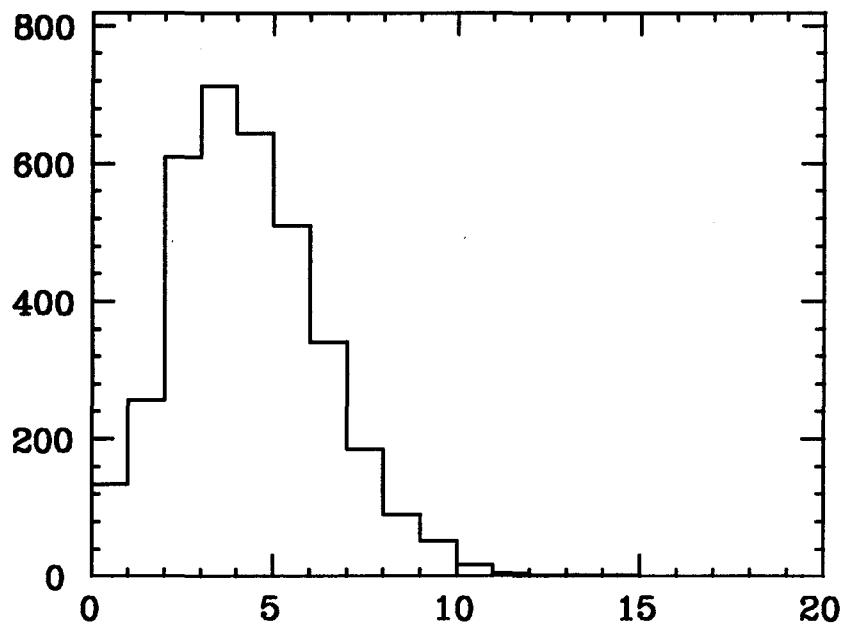


Fig 4a: # confused ($\Delta\phi$)−match

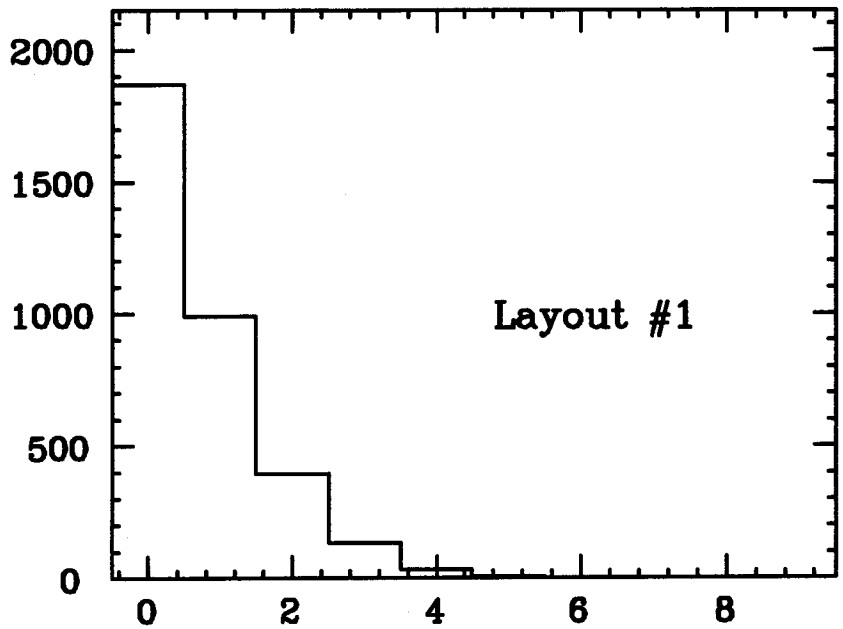


Fig 4b: ($\Delta\phi$)−match

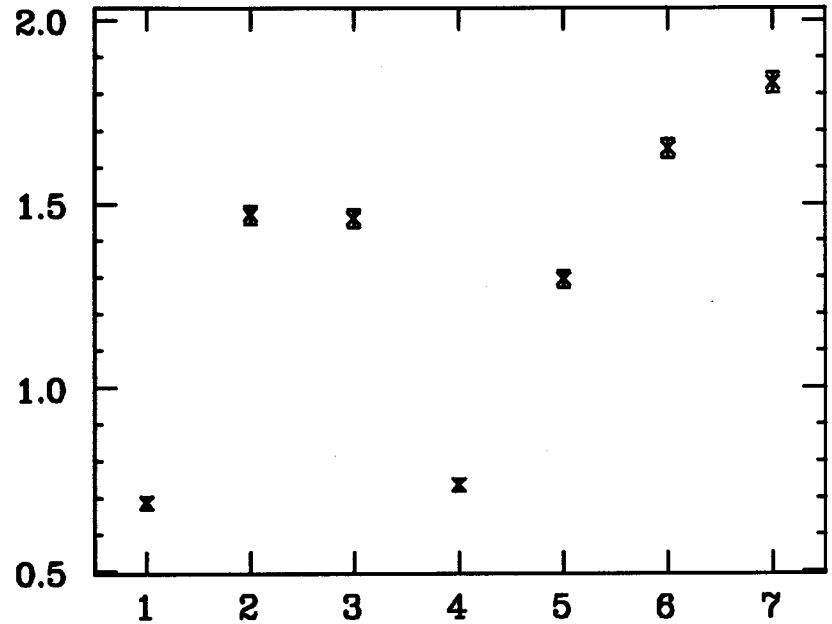


Fig 4c: # confused ($\Delta\phi$ $\Delta Q/P$ $\Delta\theta$)−match

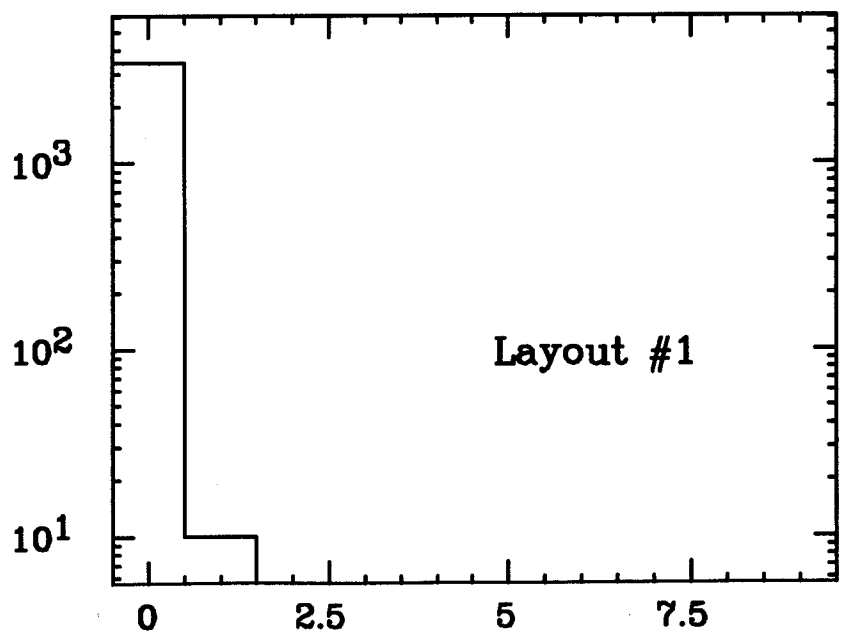
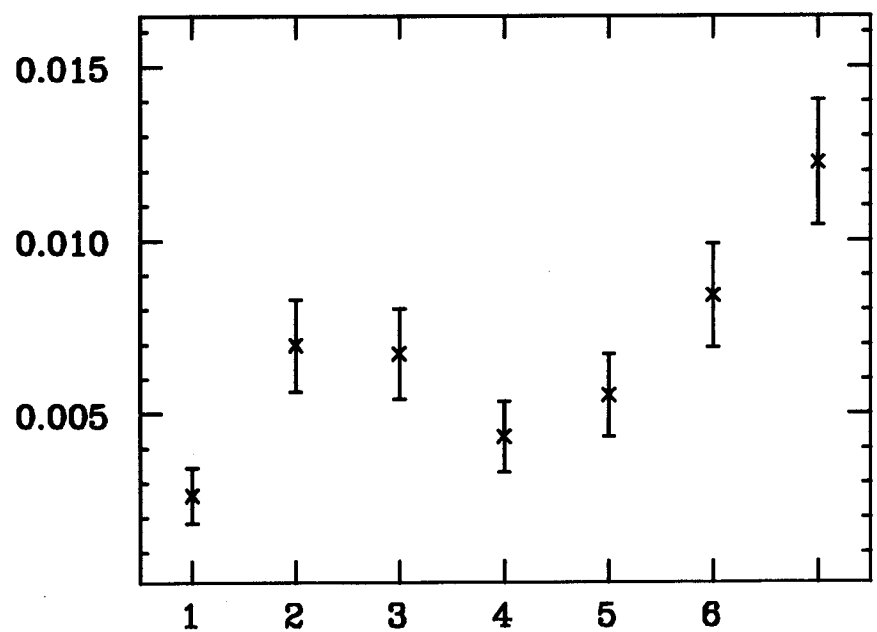


Fig 4d: ($\Delta\phi$ $\Delta Q/P$ $\Delta\theta$)−match



(Layer #2)

Fig 5a: conf vs Pt(μ) ($\Delta\phi$)−match

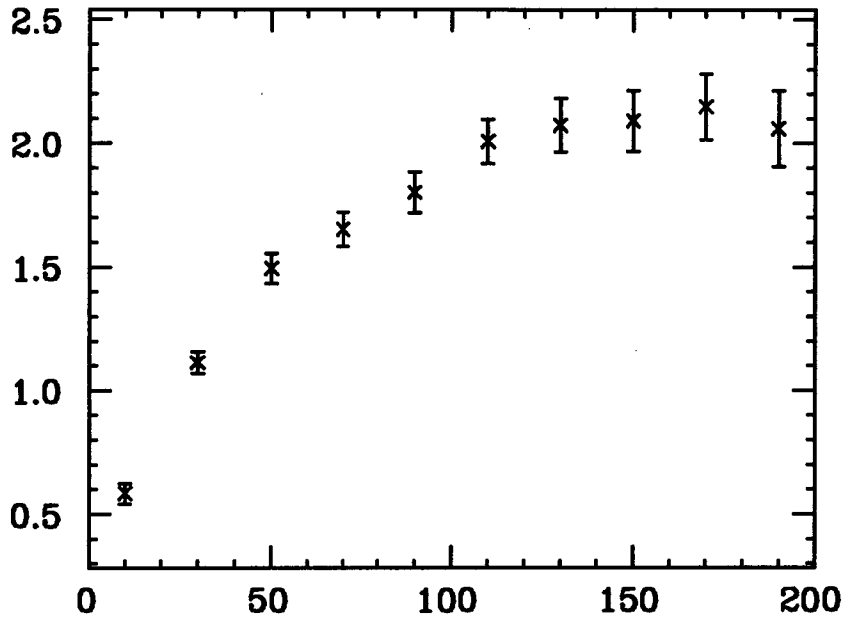


Fig 5b: conf vs η ($\Delta\phi$)−match

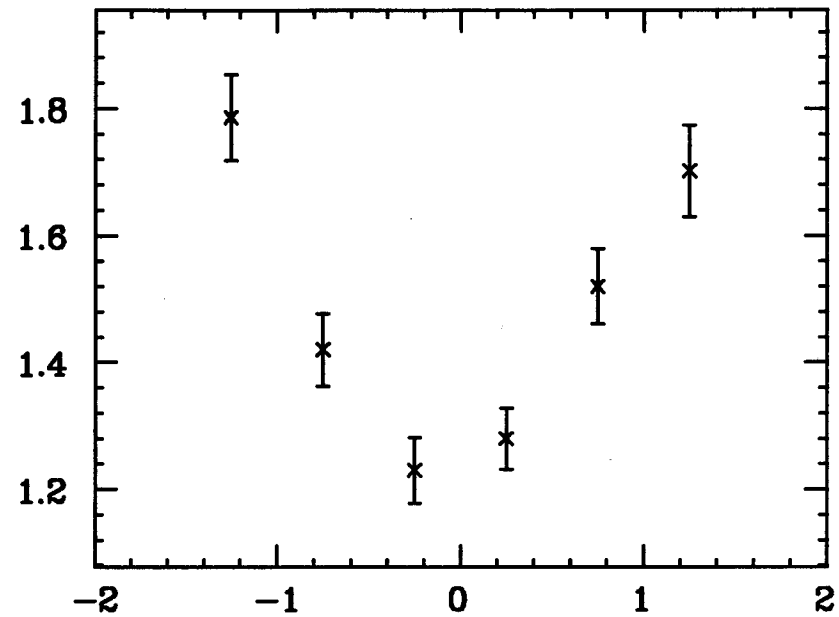


Fig 5c: conf vs Pt(μ) ($\Delta\phi$ $\Delta Q/P$ $\Delta\theta$)−match

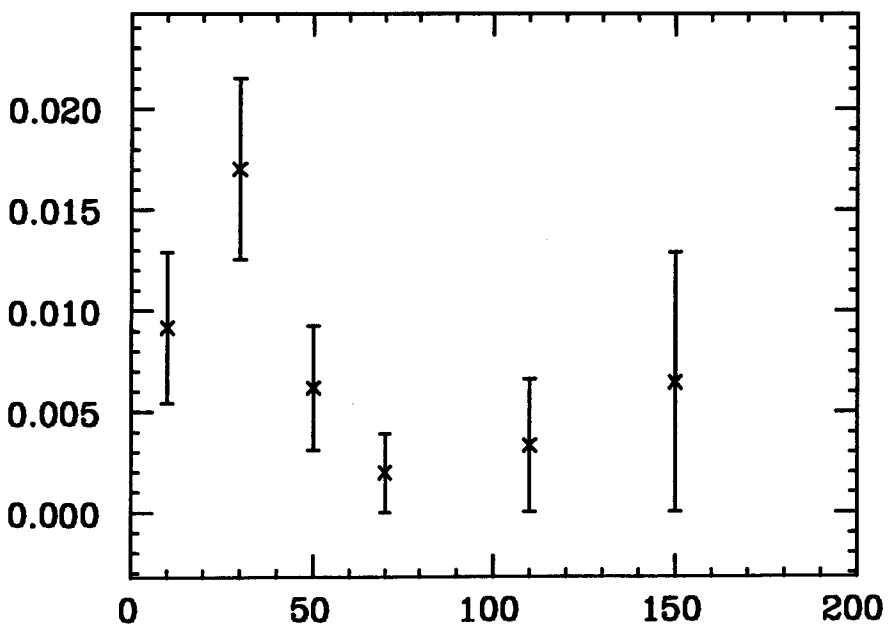


Fig 5d: conf vs η ($\Delta\phi$ $\Delta Q/P$ $\Delta\theta$)−match

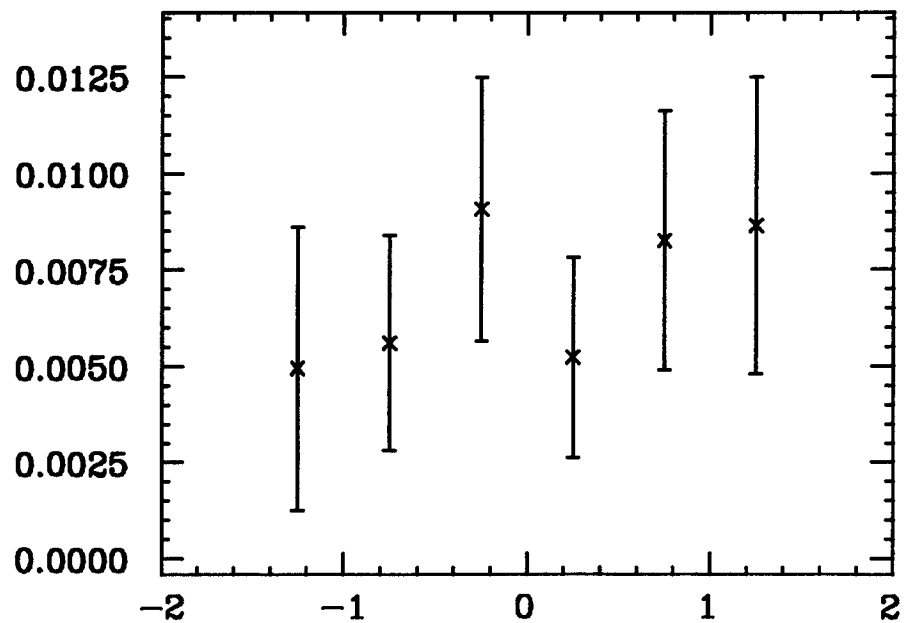
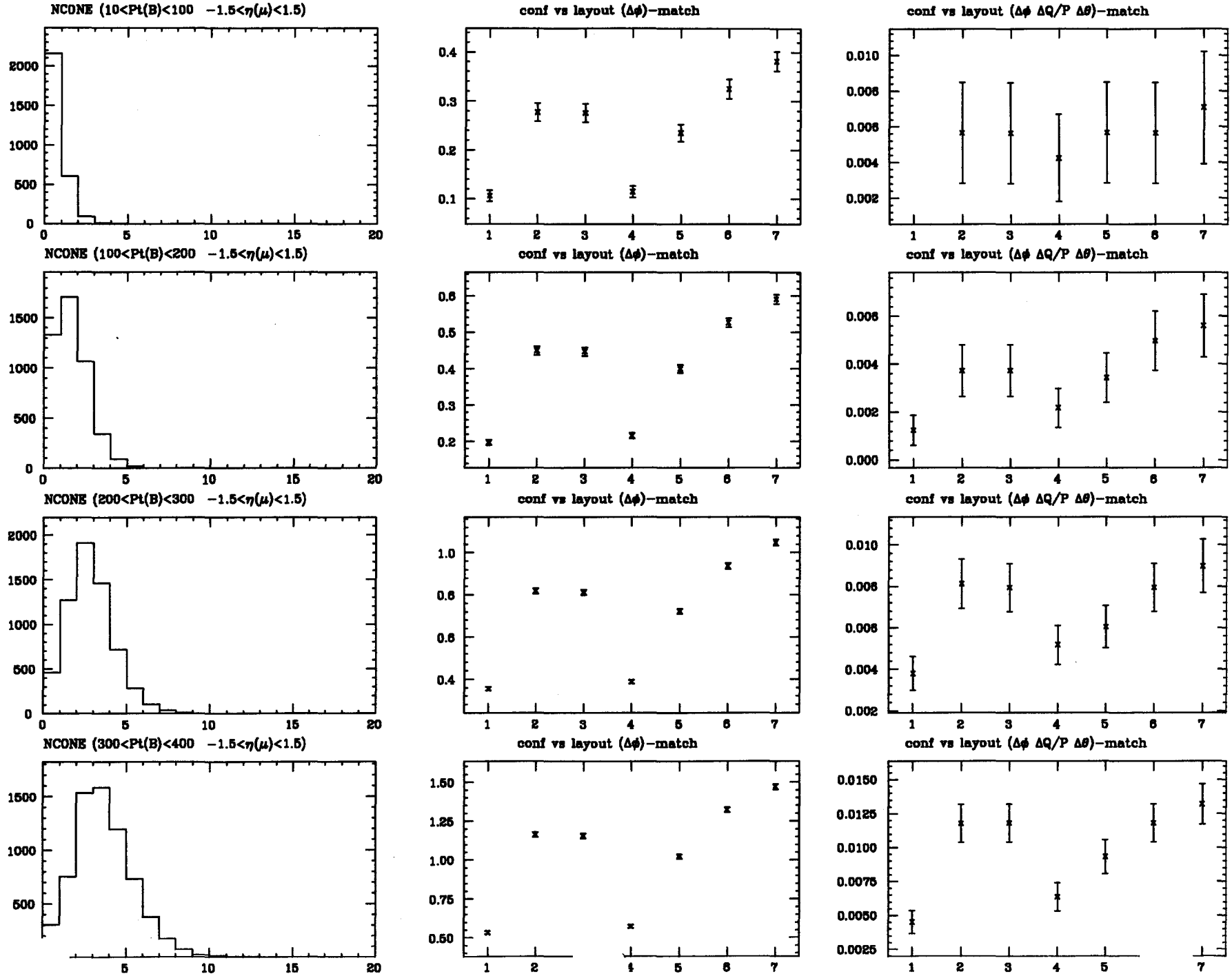


Fig 6: ISAJET BBbar -> 1 μ Events:10 < Pt(B) < 400 GeV/c



ig 7: Confusion vs B P_T

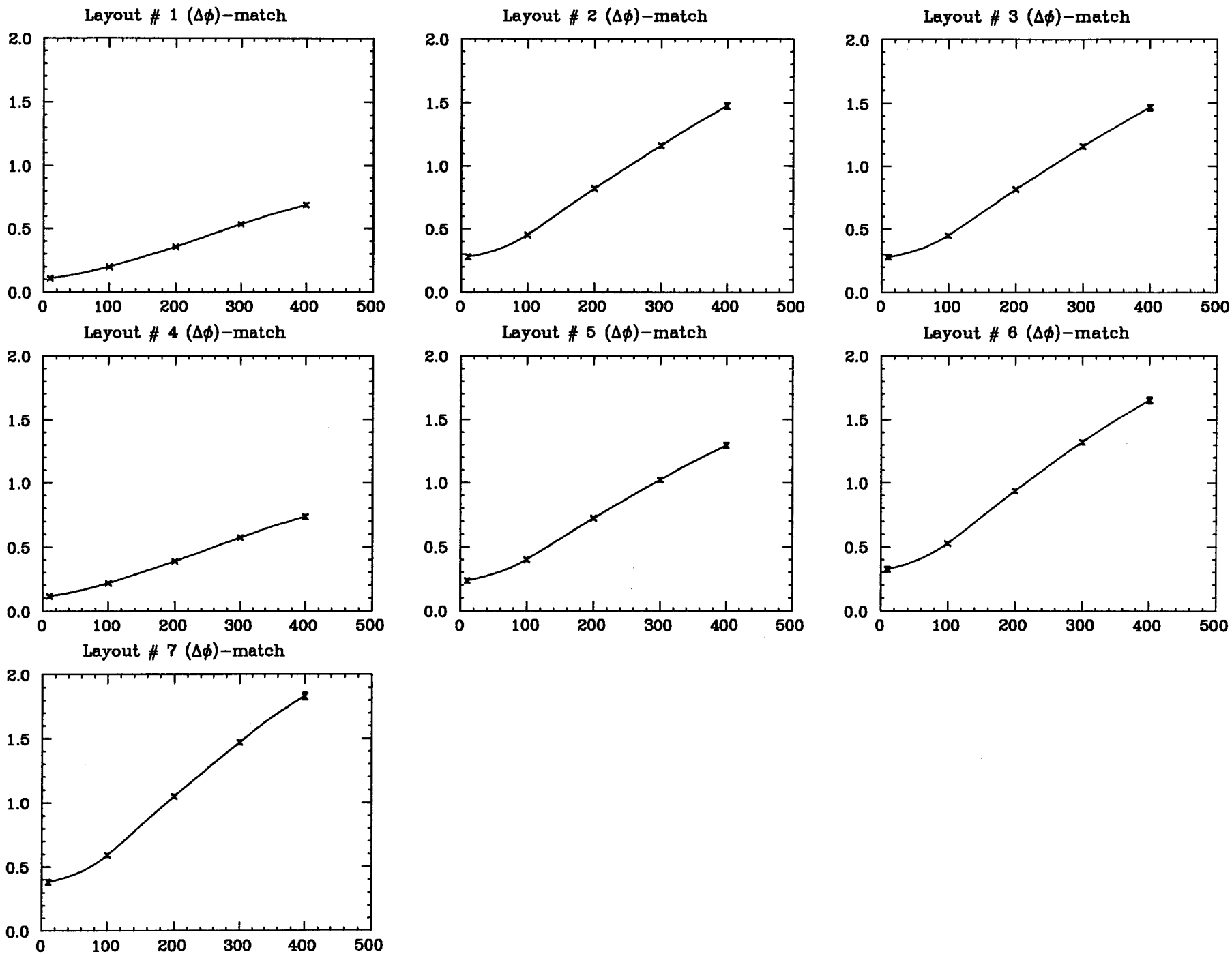
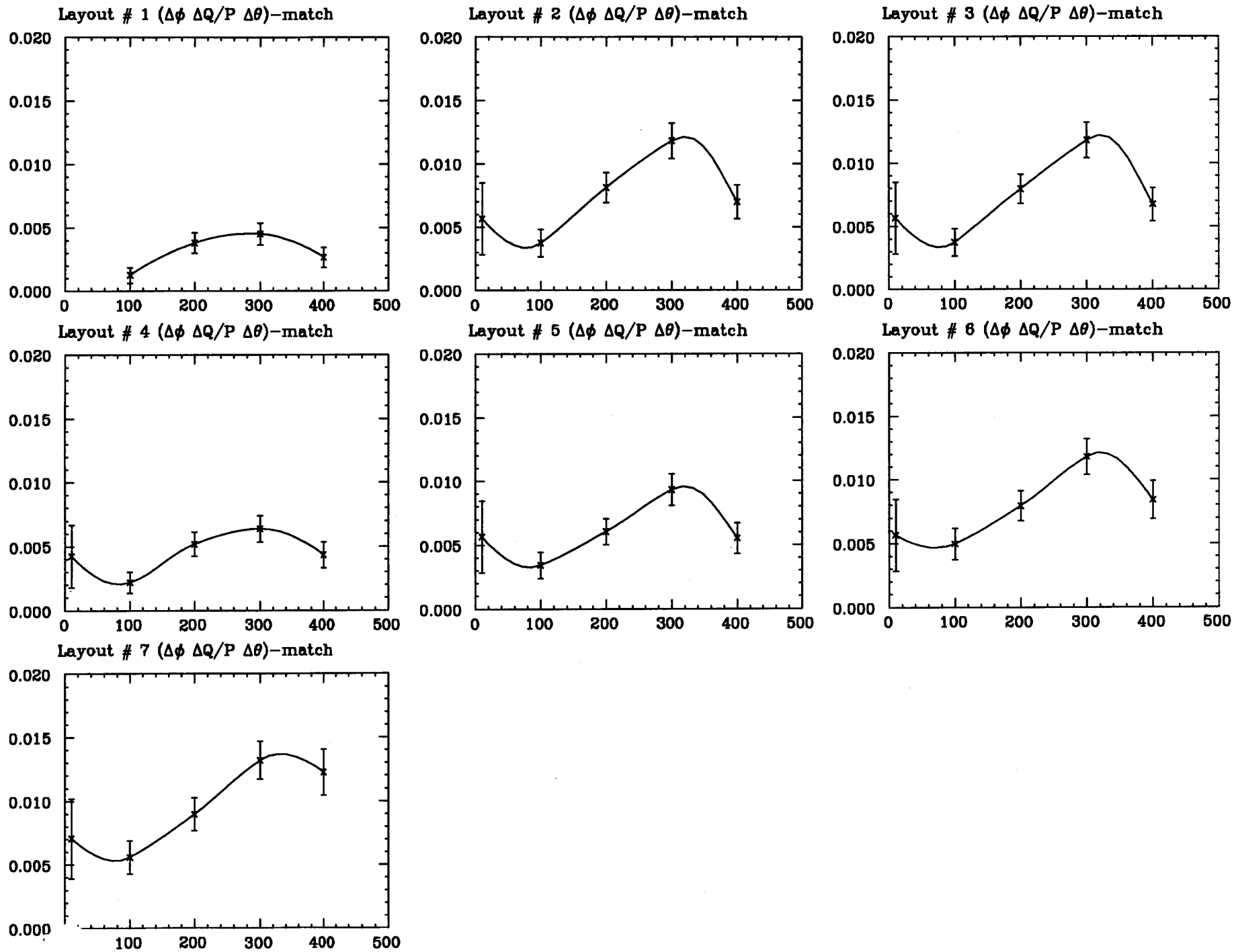



Fig 8: Confusion vs B P_T