SDC-91-145

SDC
SOLENOIDAL DETECTOR NOTES

ANGLE DEPENDENCE OF FORWARD MUON MOMENTUM RESOLUTION

September 12, 1990

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This memo presents arguments which justify a parameterization of the momentum resolution as a function of the pseudo-rapidity η for the forward and muon system using either an air core or iron core toroid as either a stand alone system or for use in conjunction with forward CTD tracking. We also present a parameterization for the momentum resolution of the iron barrel toroid used as a stand alone system. Several of these arguments have emerged from discussions with Gary Feldman.

1. General Considerations

The basic parameterization we use is of the form:

$$\frac{\sigma(P_\perp)}{P_\perp} = \sqrt{\mathcal{A}^2 + (\mathcal{B}\ P_\perp)^2} \tag{1}$$

where $\mathcal{A}$ and $\mathcal{B}$ depend on the muon polar angle ,θ.

$$\theta = 2\ tan^{-1}\ (\ e^{-\eta}\ )$$

The $\mathcal{B}$ term, which dominates at high momentum, represents the effects of the finite position resolution capability of the detector; while the $\mathcal{A}$ term which dominates at low momentum is due to the effects of multiple coulomb scattering (MCS).

The situations for the barrel and forward toroid are highly analogous and are illustrated below.

The muon is incident to the toroidal slab at an angle α. The muon enters the toroid and travels with a momentum always normal to the toroidal magnetic field. The muon bends through an angle β due to the Lorentz force while simultaneously scattering through a (projected) MCS scattering angle γ. Both β and γ are angles with respect to the entering direction of the muon. For a given bending (or scattering) thin slab of thickness Δ, the bending and RMS scattering angles are given by:

$$\beta = \frac{.3\ B\ (\Delta/cos\ \alpha)}{P} \quad , \quad \sigma_\gamma = \frac{.014}{P}\sqrt{\frac{(\ \Delta/cos\ \alpha)}{x_o}} \tag{2}$$

The $cos\ \alpha$ dependences come from the fact that the muon travels a distance of ($\Delta/cos\ \alpha$) through the magnetic field or scattering material of the slab.

The momentum is determined using coordinate information from planes parallel to the slab for both the cases of forward and barrel (stand alone) systems. These detectors measure the momentum by essentially measuring the difference between incident and final muon slopes (δs) or the difference between the *tangents* of the exiting and entering muon angle with respect to the toroid normal. In the absence of MCS effects, the measured slope difference can be related to the total momentum P via:

$$\delta s = tan(\alpha + \beta) - tan(\alpha) \approx \frac{\beta}{cos^2\ \alpha} \approx \frac{.3\ B\ \Delta}{P\ cos^3\ \alpha} \tag{3}$$

We note two of the powers of $cos\ \alpha$ in Eqn. (3) come from the derivative of $tan\ \alpha$ (or $sec^2\ \alpha$) and the remaining power of $cos\ \alpha$ comes from the expanded path length through the magnetic field for a muon of non-normal incidence.

In light of the inverse relationship between the slope difference (δs) and P we have:

$$\frac{\sigma(P_\perp)}{P_\perp} = \frac{\sigma(P)}{P} = \frac{\sigma_s}{\delta s} = \frac{P\ cos^3\ \alpha}{.3\ B\ \Delta}\ \sigma_s \tag{4}$$

Fluctuations in δs come both from MCS (σ_{mcs}) and from finite chamber granu-

larity (σ_*):

$$\sigma_s = \sqrt{\sigma_{mcs}^2 + \sigma_*^2} \tag{5}$$

We assume a constant value of σ_* . The α dependence of σ_{mcs} is more complicated:

$$\delta s - \delta s_{no\ mcs} = tan(\alpha + \beta + \gamma) - tan(\alpha + \beta) \approx \frac{\gamma}{cos^2\ \alpha}$$

$$\text{Hence}\ \ \sigma_{mcs} = \frac{\sigma_\gamma}{cos^2\ \alpha}$$

Inserting Eqn.(3) for a single scattering slab we get:

$$\sigma_{mcs} = \frac{1}{cos^2\ \alpha}\ \frac{.014}{P}\ \sqrt{\frac{(\ \Delta/cos\ \alpha)}{x_o}} \propto \frac{\sqrt{\Delta(\theta)}\ \ cos^{-5/2}\ \alpha}{P} \tag{6}$$

where $\Delta(\theta)$ allows for the possibility that the slab thickness might be position dependent (such as likely to be the case for the air core toroid).

By comparing the momentum dependences of Eqn. (4) , Eqn. (6), to Eqn.(1), one finds that σ_{mcs} contributes to the $\mathcal{A}$ term and σ_* contributes to the $\mathcal{B}$ term. We begin by accumulating the α dependences of the $\mathcal{A}$ term:

$$\mathcal{A} \propto \frac{P\ cos^3\ \alpha}{B(\theta)}\ \sigma_{mcs} \propto \frac{\sqrt{\Delta(\theta)\ \ cos\ \alpha}}{B(\alpha)} \tag{7}$$

where $B(\alpha)$ gives any angular dependence in the magnetic field (eg in the air core forward toroid). Now we turn to the $\mathcal{B}$ term.

$$\mathcal{B}P_\perp \propto \frac{P\ cos^3\ \alpha}{B(\theta)}\ \ ,\ \ P_\perp = P\ sin\ \theta$$

$$\text{Hence}\ \ \mathcal{B} \propto \frac{cos^3\ \alpha}{B(\theta)\ sin\ \theta} \tag{8}$$

2. Parameterizations for Stand-Alone Toroids

2a. Forward Iron Core Toroid

The forward iron toroid and measuring system lies in Z planes. This means a muon with polar angle θ makes an incident angle of $\alpha = \theta$ with respect to the toroid normal. We assume a uniform thickness, uniform field toroid so that $\Delta(\alpha)$ and $B(\theta)$ are constants. Making these replacements in Eqns. (7) and (8) we have:

$$\mathcal{A} \propto \sqrt{cos\ \theta}\ \ ,\ \ \mathcal{B} \propto \frac{cos^2\ \theta}{tan\ \theta}$$

The coefficients of these terms for the EoI muon system are given in a summary at the end of this memo. These coefficients are computed using my complete matrix technique computer program.

2b. Forward Air Core Toroid

The forward air core toroid and measuring system lies in Z planes so again $\alpha = \theta$. The magnetic field of the air core toroid has a polar angle dependence of:

$$B(\theta) \propto \frac{1}{tan\ \theta}$$

We **assume** that the thickness of the air core toroid support structure is **proportional to B** as well.

$$\Delta(\theta) \propto \frac{1}{tan\ \theta}$$

Making these replacements into Eqns. (7) and (8) we have:

$$\mathcal{A} \propto \sqrt{sin\ \theta}\ \ ,\ \ \mathcal{B} \propto cos^2\ \theta$$

The $\mathcal{B}$ term for the air core toroid which I calculate is the same as that first obtained by Tom Fields.

2c. Barrel Iron Toroid

The barrel toroid and measuring system lies parallel to the Z axis. We thus have $\alpha = \pi/2 - \theta$. We further assume that both the magnetic field, and scattering planes are independent of θ. Thus:

$$\mathcal{A} \propto \sqrt{sin\ \theta}\ \ ,\ \ \mathcal{B} \propto sin^2\ \theta$$

3. Forward Toroids with Inner tracking help

The inner tracking help which I've considered is provided by forward microstrip and possible radial chambers of either straws or scintillating fiber detectors. We assume that these detectors measure the x and y coordinates as the track intercepts the Z plane of the detector. Momentum analysis is provided by the solenoidal field which bends track trajectories in the ϕ direction. Under these assumptions, the momentum measurement is essentially the measurement of the track sagita as all x and y coordinates determined from individual Z detector planes are are projected on a common Z plane. The momentum error of such a circle fit scales as inverse square of the cord length over which the circle is measured. The cord length for the forward measuring system. is proportional to $tan\ \theta$. This means that the momentum error measured by these Z planes scales as:

$$\frac{\sigma(P_\perp)}{P_\perp} \propto \frac{P_\perp}{tan^2\ \theta}$$

We then assume the inner tracker and toroid information can be combined as independent measurements:

$$\left(\frac{\sigma(P_\perp)}{P_\perp}\right)^{-2} = \left(\frac{\sigma(P_\perp)}{P_\perp}\right)^{-2}_{\text{toroid}} + \left(\frac{\sigma(P_\perp)}{P_\perp}\right)^{-2}_{\text{inner}}$$

In fact, my complete computer calculations differ slightly from the parameterization presumably due to correlations introduced by ϕ measurements made in the forward toroids.

4. Summary

$$\frac{\sigma(P_\perp)}{P_\perp} = \sqrt{\mathcal{A}^2 + (\mathcal{B}\ P_\perp)^2}$$

System	$\mathcal{A}$	$\mathcal{B}$
Forward Iron Toroid	$0.118\ \sqrt{cos\ \theta}$	$9.42 \times 10^{-5}\ cos^2\ \theta / tan\ \theta$
Air core Toroid	$0.0335\ \sqrt{sin\ \theta}$	$2.07 \times 10^{-4}\ cos^2\ \theta$
CTD forward tracking		$5.2 \times 10^{-5}\ tan^{-2}\ \theta$
Iron barrel toroid	$\propto \sqrt{sin\ \theta}$	$\propto sin^2\ \theta$