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S. Margulies

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# THE SHAPE OF THE VLPC PHOTOELECTRON SPECTRUM FROM A SCINTILLATING FIBER OF CIRCULAR CROSS-SECTION

S. Margulies*

Department of Physics
University of Illinois at Chicago

ABSTRACT

This note investigates the shape of the VLPC photoelectron spectrum expected from a scintillating fiber of circular cross-section. Two types of phenomena contribute: 1) a continuous distribution due to particles traversing different thicknesses in the circular fiber, and 2) fluctuations due to various physical processes. The two types of phenomena are combined in a simple illustrative calculation. The resulting photoelectron spectrum does not have a Poisson shape, being considerably wider relative to its mean.

## 1. Introduction

Understanding the shape of the photoelectron spectrum produced by a visible-light photon counter (VLPC) is an important aspect of developing a tracker using scintillating-fiber technology. With this in mind, a recent note[1] investigated the effect of the cross-sectional shape of a scintillator on the resulting pulse-height distribution. The fact that the fibers under consideration are round was expected to have an effect on the VLPC photoelectron spectrum.

Using simple assumptions -- in particular, that the scintillation light is directly proportional to the thickness traversed by a minimum-ionizing particle,

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and neglecting fluctuations -- the pulse-height spectrum from a scintillator of circular cross-section was shown to exhibit a long tail at low values and to rise to a peak at the maximum pulse amplitude.[1] This result is applicable directly only to situations where relatively large amounts of light reach the photodetector. In fiber/VLPC situations, where just a few photons reach the photodetector, fluctuations become very important. Nevertheless, the fact that scintillation light is generated non-uniformly in a circular fiber should result in a non-Poisson VLPC photoelectron distribution.

The present note continues the investigation by introducing fluctuations in a simple way. A numerical calculation of the corresponding VLPC photoelectron spectrum is then made using parameters consistent with recent measurements.[2] The result, while not exact, serves to illustrate the non-Poisson nature of the photoelectron spectrum.

## 2. Pulse-height spectrum from a round scintillator

Reference 1 considered a uniform, parallel flux of minimum-ionizing particles incident on a scintillator of circular cross-section. Assuming that the scintillation light produced is directly proportional to the thickness traversed, that the efficiency of light collection and transmission to the photodetector is independent of production point, that fluctuations are negligible, and that each particle is detected, the pulse-height spectrum was shown to be

$$f(V) = \frac{1}{N_{total}} \frac{dN}{dV} = \frac{1}{V_o\sqrt{(V_o/V)^2 - 1}} \quad . \tag{1}$$

Here, $V_o$ is the maximum pulse-height corresponding to a particle crossing along a diameter. The dependence of f(V) on V is shown in Fig. 1.

The approximations used in obtaining Eq. (1) influence the result in significantly different ways. While the geometrical simplifications affect the spectrum, they do so in a smooth way; neglecting fluctuations omits essential stochastic features. Among the fluctuations that influence the VLPC photoelectron distribution are 1) Landau fluctuations in the energy loss, which affects the number of excitations produced by an ionizing particle traversing a given thickness of scintillator, 2) fluctuations in photoemission following excitation, 3) fluctuations in light transport due to imperfections in the fiber core and the core-cladding interface, 4) fluctuations in photoelectron production in a detector with quantum efficiency less than unity, and 5) fluctuations in the charge amplification process in the VLPC.

## 3. Fluctuations

All types of fluctuations need not be included to demonstrate that the VLPC photoelectron spectrum deviates from the Poisson shape. For simplicity, fluctuations of type 1), 3), and 5) are neglected in the calculation presented in the next section. Of the remaining fluctuations, first consider light from a source of constant intensity such that, on average, $\bar{n}$ photons reach the VLPC during a detection time T. Let fluctuation type 2), manifested as a variation in the incident photon flux, be described by $p_{\bar{n}}(m)$, the probability that m photons arrive during T. If the VLPC has quantum efficiency $\eta \leq 1$, then detection, which involves fluctuation type 4), is a binary process described by the binomial distribution. Hence, the probability of detecting n photoelectrons is

$$p_{det}(n) = \sum_{m=n}^{\infty} \frac{m!}{n!(m-n)!} \eta^n (1-\eta)^{m-n} p_{\bar{n}}(m) \quad . \tag{2}$$

There is a lack of unanimity regarding $p_{\bar{n}}(m)$. Conventional wisdom--supported by some authors[3] -- states that the incoming photons obey Poisson statistics, so that

$$p_{\bar{n}}(m) = \frac{\bar{n}^m}{m!} e^{-\bar{n}} \quad . \tag{3}$$

Others[4,5,6] state that only perfectly coherent light has this form, and light from incoherent sources -- certainly the case for scintillation -- obeys Bose-Einstein statistics, so

$$p_{\bar{n}}(m) = \frac{1}{1+\bar{n}} \left(\frac{\bar{n}}{1+\bar{n}}\right)^m \quad . \tag{4}$$

One author[7] takes a middle position, writing that Eq. (4) is valid for long-wavelength light, but that Eq. (3) is a good approximation for visible light.

The two distributions are quite different (except for $\bar{n} << 1$). The Poisson distribution peaks near $m = \bar{n}$ and has a variance $\sigma^2 = \bar{n}$; the Bose-Einstein distribution is maximum at $m = 0$ and its variance is $\sigma^2 = \bar{n} + \bar{n}^2$. Nevertheless, each has the interesting property that, when convoluted with a binomial distribution as in Eq. (2), the <u>form</u> of the distribution is preserved but the mean changes to $\bar{n}' = \eta\bar{n}$.[7,8,9] Since the recent results from fiber/VLPC measurements at UCLA[10] appear Poisson-like, Eq. (3) is used in the illustrative calculation which follows. Thus, using the above property, the result of Eq. (2) is taken to be

$$p_{det}(n) = \frac{\bar{n}'^n}{n!} e^{-\bar{n}'} \quad . \tag{5}$$

Equation (5) is valid for a source of constant intensity. Suppose, however, that the VLPC is illuminated by light from independent sources, the $i^{\underline{th}}$ source

producing, on average, $\bar{n}_i$ photons, and contributing with relative weight $w_i$. Then, Eq. (5) may be generalized to

$$P_{det}(n) = \sum_i w_i \frac{\bar{n}_i'^{\,n}}{n!} e^{-\bar{n}_i'} , \tag{6}$$

where $\bar{n}_i' = \eta\bar{n}_i$.

## 4. VLPC photoelectron spectrum

To obtain the expected VLPC photodetection spectrum, fluctuations must be added to the smooth distribution calculated for a scintillator of circular cross-section. This can be accomplished by a "quantization" procedure. That is, the pulse-height distribution of Eq. (1), shown in Fig. 1, is a smooth function of the continuous variable V. However, the pulse-height is directly proportional to the number of photons reaching the photodetector, and this is a relatively small integer value for a fiber/VLPC system.

Since pulse-height is equivalent to photon number, let the interval $0 \leq V \leq V_o$ be divided into equal segments. The $i^{\underline{th}}$ segment can then be thought of as originating from a source that produces, on average, $\bar{n}_i'$ photoelectrons with a relative weight $w_i$. Use of ten equal divisions produces values consistent with recent results. That is, let the circular-scintillator spectrum (see Fig. 1) be "quantized" so that the first segment, $0 \leq V \leq 0.1\ V_o$, corresponds to $\bar{n}_1' = 1$, the second segment, $0.1\ V_o \leq V \leq 0.2\ V_o$, corresponds to $\bar{n}_2' = 2$, and so on, up to the tenth segment, $0.9\ V_o \leq V \leq V_o$, which corresponds to $\bar{n}_{10}' = 10$. The weights for these intervals are found from the areas

$$w_i = \int_{0.1(i-1)V_o}^{0.1iV_o} f(V)dV \quad , \quad i = 1, 2, \ldots, 10,$$

where f(V) is given in Eq. (1). A graph of $w_i$ as a function of $\bar{n}_i'$ -- the "quantized" circular-fiber distribution -- is shown in Fig. 2. This distribution has a mean value

$$\bar{n}' = \sum_{i=1}^{10} w_i \bar{n}_i' = 8.3 \quad ,$$

consistent with recent measurements[2] made using a 4-m-long clear fiber to transport photons from a 1-mm-dia. scintillating fiber, excited by electrons from $^{207}$Bi, to a VLPC.

The VLPC photoelectron spectrum may now be calculated by using the weights from the "quantized" distribution in Eq. (6). The result is shown in Fig. 3a. This spectrum has a mean of 8.2, an rms value of 3.6, and a full width at half-maximum of 8.6. For comparison, a Poisson distribution whose mean is $\bar{n}' = 8.3$ -- with rms value of $\sqrt{8.3} = 2.9$ and full width at half-maximum of 6.9 -- is shown in Fig. 3b. As expected, the two spectra have almost the same mean values but have rather different shapes. The distribution obtained from a fiber of circular cross-section has relatively long tails and is significantly wider than the corresponding Poisson spectrum.

## 5. Summary

The shape of the VLPC photoelectron spectrum expected from a scintillating fiber of circular cross-section, a question first raised in Ref. 1, has been in-

vestigated. Two types of phenomena contribute to the spectrum. The first is a continuous distribution due to the circular cross-section. The second consists of fluctuations due to various physical processes. For the purpose of a simple illustrative example, only two sources of fluctuation were included: the fluctuation in the number of photons produced by the scintillation, taken to have a Poisson form, and the fluctuation in the binary detection process, which has a binomial form.

These two types of phenomena were combined by first "quantizing" the pulse-height spectrum in a way consistent with recent measurements, and then combining the result with the fluctuations. As expected, the calculated VLPC photoelectron spectrum does not exhibit a Poisson shape. Instead, because of the different amounts of light produced by particles traversing different thicknesses in the circular fiber, the distribution is considerably wider relative to its mean. Including Landau fluctuations would probably widen the spectrum even more.

Although the calculation is rather simple, neglecting various processes and approximating others for ease of calculation, it serves to illustrate that the VLPC photoelectron spectrum from a round fiber should not be expected to have a Poisson form.

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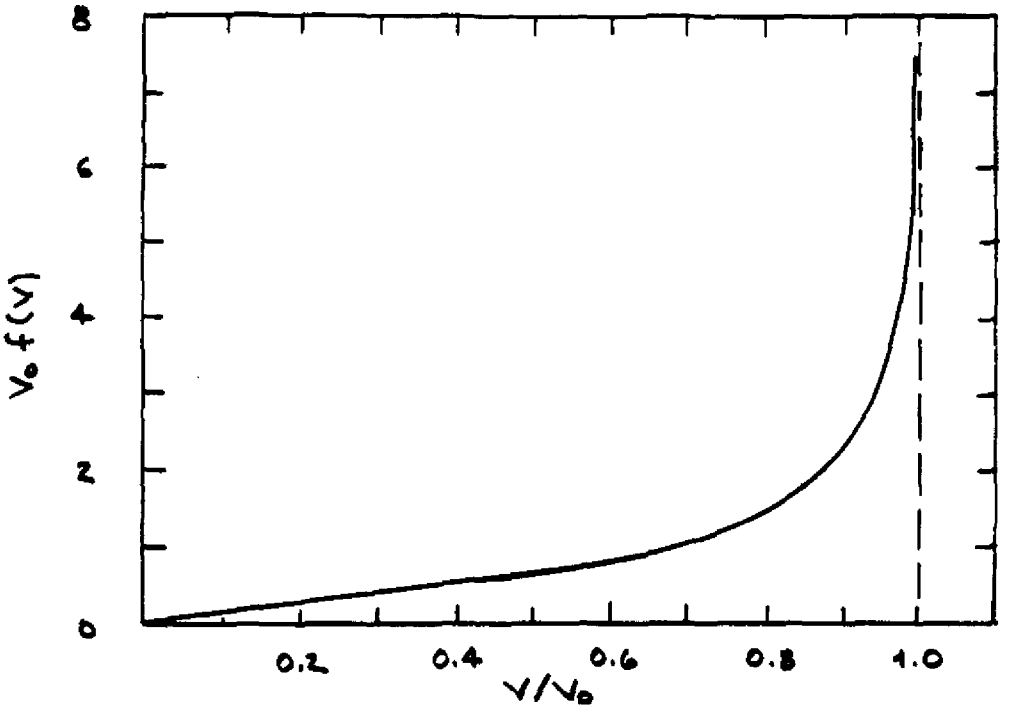


Figure 1. Pulse height distribution from a scintillator of circular cross-section, as calculated in Ref. 1.

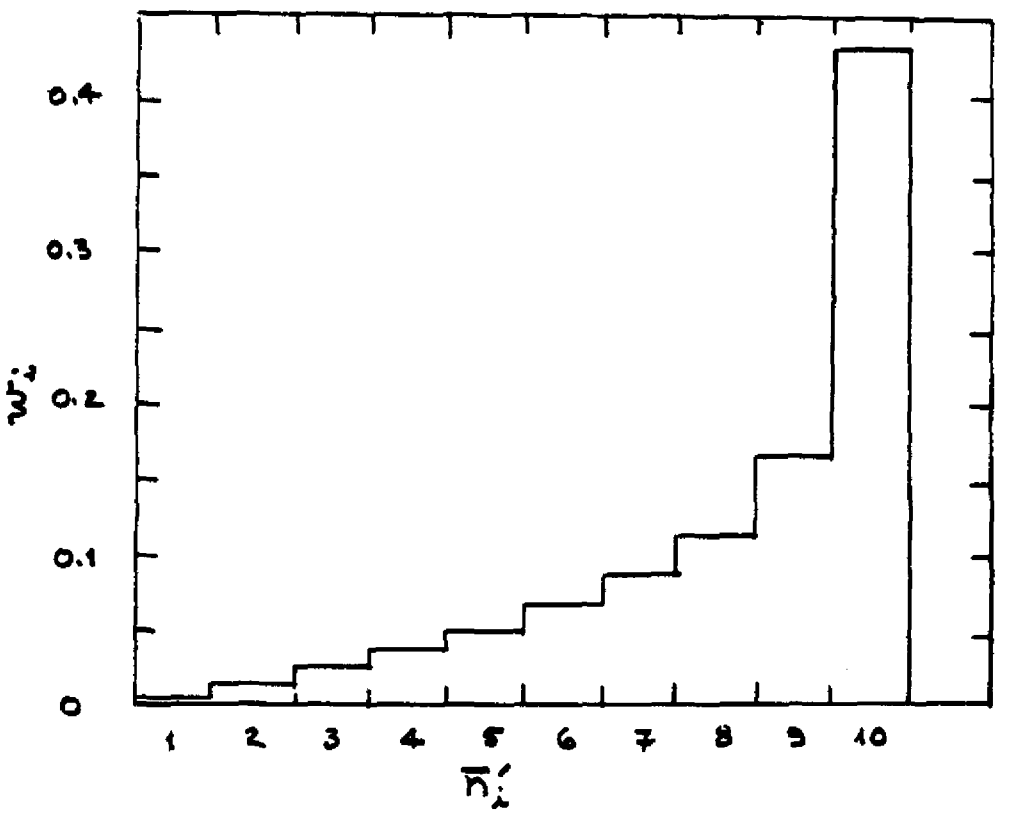


Figure 2. Relative weights as a function of average photoelectron number obtained by "quantizing" the circular-fiber distribution.

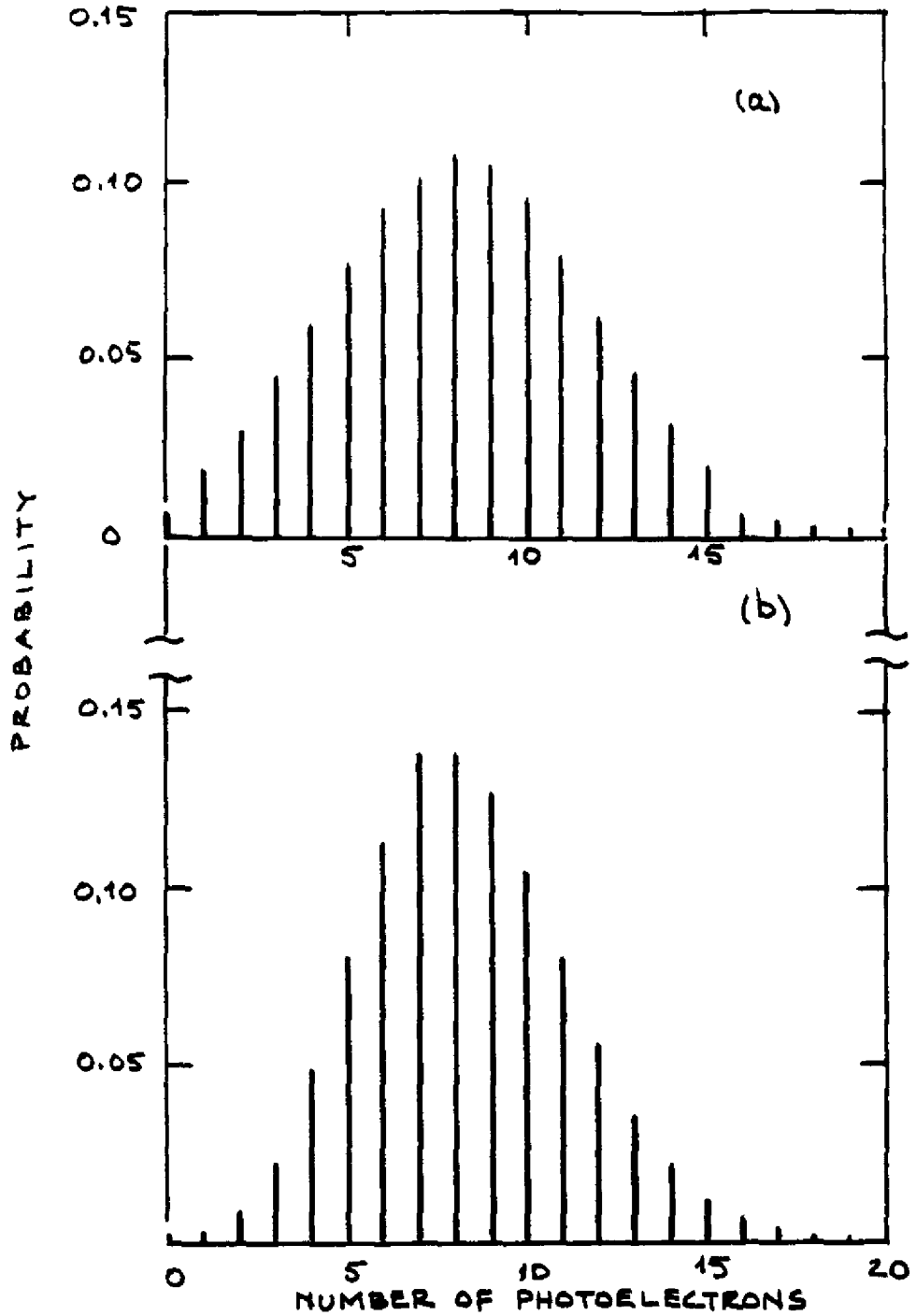


Figure 3(a) Calculated VLPC photoelectron spectrum.
(b) Poisson distribution with a mean of 8.3.