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FOUR-COMPONENT APPROXIMATION TO CALORIMETER RESOLUTION

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FOUR-COMPONENT APPROXIMATION TO CALORIMETER RESOLUTION*

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ABSTRACT

A hadron with energy E cascades in a uniform calorimeter. With some probability, energy E_e enters the electromagnetic sector, mostly through π^0 production. It produces a visible signal E_e^{vis} via a second frequency function. To the approximation that escaped energy (carried by albedo particles, neutrinos, and muons) can be neglected, the remaining energy $E_h = E - E_e$ enters the hadronic sector. A third frequency function, with its variance dominated by binding energy loss fluctuations, describes the probability of obtaining a hadronic contribution E_h^{vis} to the visible energy deposition for fixed E_e. Finally a readout transducer such as photomultiplier samples the visible energy to produce an output signal, or observed energy, and in the process introduces further fluctuations. In this report, we describe the combined frequency function for all four processes. It turns out that they are uncorrelated, so that variances add in quadrature. As expected, the variance contribution due to the initial π^0 production is proportional to the degree of noncompensation in the Gaussian approximation. However, (a) there is a slow ($\ln E$) decrease in the "constant term," and (b) the familiar $1/\sqrt{E}$ for the sampling term is not quite correct.

1. Introduction

This study was motivated by ongoing discussions with several people. One discussion has to do with the energy scaling of the fractional resolution of a hadronic calorimeter, σ/E. It is usual to write the contribution of sampling fluctuations as to $1/\sqrt{E}$, and the contribution of other effects, notably noncompensation, as having little or no energy dependence. The combined fractional standard deviation behaves as $1/\sqrt{E}$ at low energies and is asymptotically constant. In general, one should be able to write the variance as a general quadratic form

$$\left(\frac{\sigma}{E}\right)^2 = \frac{C_1^2}{E} + \frac{2\rho C_1 C_2}{\sqrt{E}} + C_2^2 \quad , \qquad (1)$$

where C_1 is due to sampling and C_2 is the result of noncompensation, cracks, channel-to-channel gain variations, nonuniformities, and the like; in the present discussion only noncompensation contributions are considered. The correlation is ρ, and $0 \le \rho \le 1$. If the processes leading to C_1 and C_2 are completely uncorrelated, $\rho = 0$ and the contributions to σ/E add in quadrature, while if they are totally correlated, $\rho = 1$ and $\sigma/E = C_1/\sqrt{E} + C_2$. The conclusion of this report is that $\rho = 0$, so that the two terms add in quadrature. This follows from the mathematical structure of the problem as formulated here; I have been unable to find any mechanism which would result in correlation between the sampling and noncompensation contributions.

The consequences are far from academic, since results for C_1 and C_2 are usually quoted without respect to how the fits were made. As an example, two fits to some preliminary data[1] are shown in Fig. 1. The "constant term" is more than 60% higher when the fit is made assuming linear addition. (The dotted curve will be discussed later.)

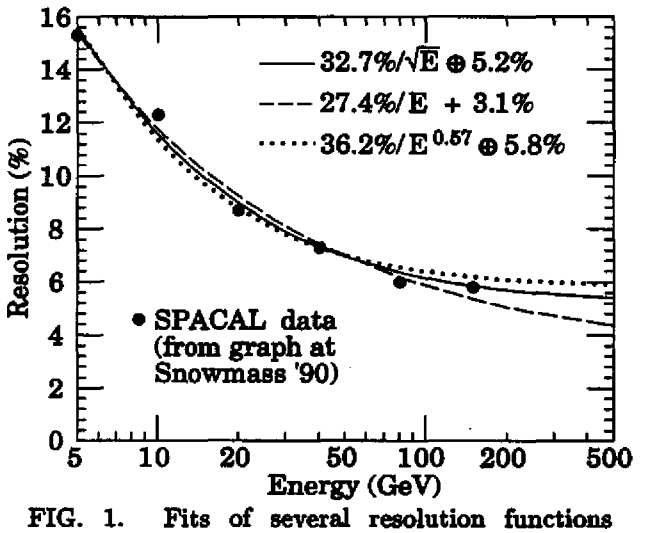

FIG. 1. Fits of several resolution functions to highly unofficial SPACAL data, showing the sensitivity of the "constant term" to the assumptions.

A second motivation has to do with a challenge from Tut Alsmiller. In previous papers I have considered the behavior of the means of various distributions, arguing that the mean response of a simple calorimeter is described by only three quanti-

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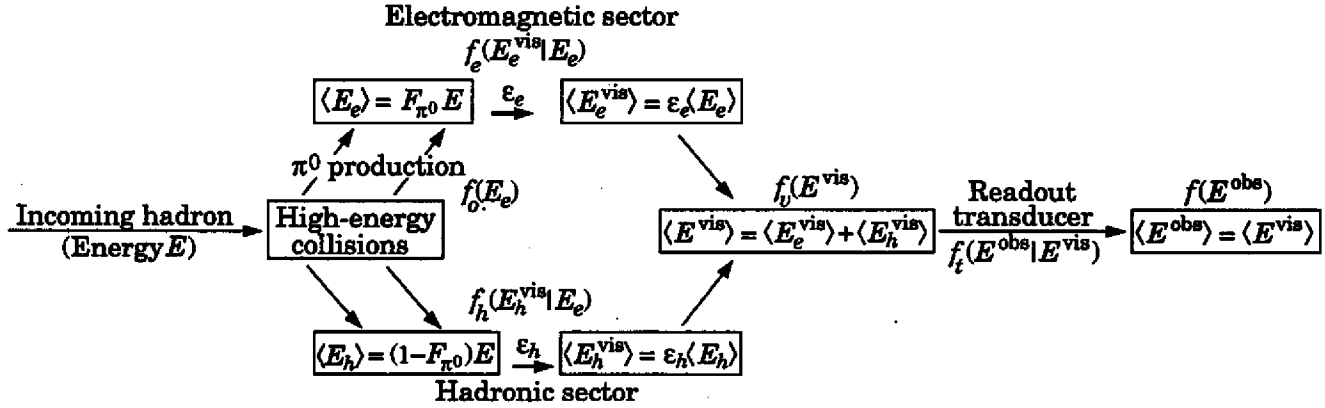

FIG. 2. Logic flow in a hadron-induced cascade in a calorimeter. Transfer of energy to the electromagnetic sector occurs in many high-energy collisions. ϵ_h (ϵ_e) is the efficiency with which energy in the hadronic (electromagnetic) sector is converted to visible energy. Included are all effects of saturation, gate width, *etc.* The visible energy is converted to the energy (signal) finally observed by a photomultiplier or other readout transducer, which introduces further fluctuations.

ties, and that only one of these has energy dependence. The picture should also yield a description of resolution, and Tut asked if I could provide it. This paper is the result, except that I now challenge him to find one crucial distribution function, the hadronic response function called $f_h(E_h^{vis}|E_e)$ in the following discussion. This quantity is subject to large fluctuations, evidently because of a small number of nuclear spallation events in which perhaps 30% of the hadronic energy is for all practical purposes lost. These fluctuations are the reason hadronic resolution is always much worse than electromagnetic resolution. A careful study of this distribution function would be very valuable. Here we make only a Gaussian approximation, relying as usual on the central limit theorem for salvation.

The function describing the distribution of the electromagnetic fraction in a cascade is explored in more detail than before, albeit only in the context of FLUKA simulations. A parallel HETC study of this distribution would be very valuable.

As in other papers, I restrict the discussion to the response of a uniform calorimeter to an incident high-energy hadron with energy E. By "uniform" I mean that the structure of the calorimeter (usually a sandwich structure) is the same from front to back, and that the dimensions are sufficient to contain the cascade. In addition, plate thickness is small compared with the interaction length of a ~1 MeV neutron (~10 cm), but not necessarily small compared with a radiation length.

2. Formulation of the problem

The logic flow in a hadronic cascade is shown in Fig. 2. It is simplified in two respects: escaped energy (carried by albedo, neutrinos, and muons) is neglected, and complicated processes are lumped together. For example, in the low-energy hadronic sector, the effects of nuclear binding losses, saturation, and gate width are combined in relating the hadronic energy E_h to the visible hadronic energy E_h^{vis}.

Energy is transferred to the electromagnetic sector (the EGS world) in high-energy collisions, mostly through π^0 production. In a given cascade the energy transfer is E_e, but the mean value is $F_{\pi^0}E$.* With some efficiency (ϵ_e, on the average) this energy appears as a visible signal E_e^{vis}. Similarly, $E_h = E - E_e$ enters the hadronic sector, and E_h^{vis} contributes to the visible signal. E_h contains all of the hadronic energy below the π^0 threshold as well as ionization loss by above-threshold hadrons. It also contains the energy of photons from nuclear processes, since this energy scales with the incident energy in the same way as other hadronic contributions.

The four distributions under consideration are as follows:

1. The distribution of E_e, given by the frequency function $f_0(E_e)$. (It is convenient to leave the incident energy E implicit.)

* We reserve f_{π^0} for the energy transfer to the electromagnetic sector in a single collision.

2. The distribution of visible electromagnetic energy is given by the conditional frequency function $f_e(E_e^{\rm vis}|E_e)$.

3. Similarly, the distribution of visible hadronic energy is given by $f_h(E_h^{\rm vis}|E_e)$. Since $E_h = E - E_e$, it is sufficient to express the condition as on E_e.

The distribution of the total visible energy, $f_v(E^{\rm vis})$, can be calculated from f_0 and the two conditional distributions. Only the total visible energy can be observed:

$$E^{\rm vis} = E_e^{\rm vis} + E_h^{\rm vis} \tag{2}$$

The problem is to find the distribution $f_v(E^{\rm vis})$ for an incident hadron with energy E. We first combine f_e and f_h, subject to the constraint provided by Eq. (2):

$$f^{\rm vis}(E^{\rm vis}|E_e) = \int f_e(E_e^{\rm vis}|E_e) \times f_h(E^{\rm vis} - E_e^{\rm vis}|E_e) dE_e^{\rm vis} \tag{3}$$

This distribution is then summed over intermediate values of E_e:

$$f_v(E^{\rm vis}) = \int f_0(E_e) f^{\rm vis}(E^{\rm vis}|E_e) dE_e \quad . \tag{4}$$

4. Finally, the distribution of observed signal, or energy, is obtained by folding this with whatever photoelectron statistics or electronic noise distribution characterizes the readout transducer, as described by the conditional distribution $f_t(E^{\rm obs}|E^{\rm vis})$. The separation of $f_v(E^{\rm vis})$ and the final distribution $f(E^{\rm obs})$ is in many ways artificial, but it serves a useful purpose: The distribution of $E^{\rm vis}$ is in some sense irreducible, except for the choice of sampling fraction, it is beyond the control of the experimenter. Given a configuration, it can in principle be calculated. Its variance thus defines the best that a given calorimeter can do. A proper designed readout should have a readout which introduces a variance which is small, but not too much smaller, than the variance of $E^{\rm vis}$.

This final step does not change the mean of the distribution (by definition!), but the variance adds to the variance of $f_v(E^{\rm vis})$, and the skewness is slightly modified. Accordingly, most of the conclusions and analysis is based on $f_v(E^{\rm vis})$, and this distribution is (appropriately) included as an afterthought.

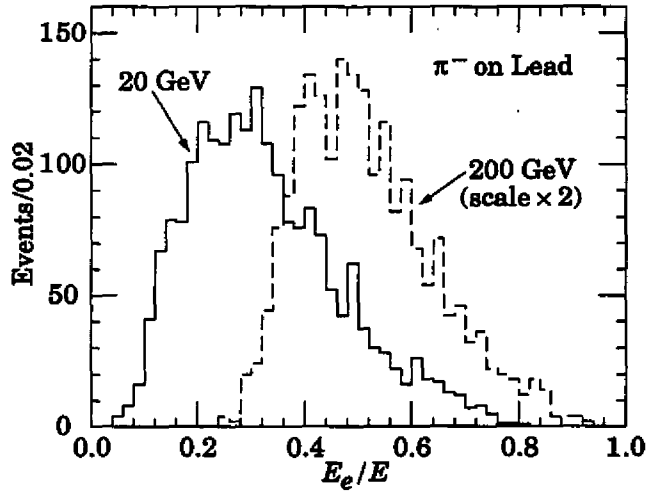

FIG. 3. Distributions of E_e for 20 GeV and 200 GeV negative pions incident on lead, as simulated with FLUKA90.

3. The distribution $f_0(E_e)$

The function $f_0(E_e)$ has been generated with the aid of the high-energy hadronic cascade simulation program FLUKA90[2]. Typical distributions, at 20 GeV and 200 GeV, are shown in Fig. 3. As expected, the mean moves to the right with increasing energy, as more of the hadronic energy is transferred to the electromagnetic sector as the number of hadronic generations increases. The width scaled by the incident energy is almost independent of energy, and the distribution has positive skewness.

The methodology and geometry used in simulating these events is described elsewhere[3]. The first π^- interaction was permitted only after the first 1.7 λ_I had been traversed, so that there was essentially no albedo loss. In accord with the observations made in SDC-91-00002[4], simulations were limited to the high-energy sector, where π^0 production and consequently all of the energy transfer to the electromagnetic sector occurs. The time required to produce the results shown in Figs. 3 and 4 was exceedingly modest; less than an hour was required for the 500 events simulated at 1 TeV, and the CPU time was proportionally less at lower energies.

The energy dependences of the mean and standard deviation are shown in Fig. 4, where we show the mean divided by incident energy (F_{π^0}) and the square root of the variance divided by

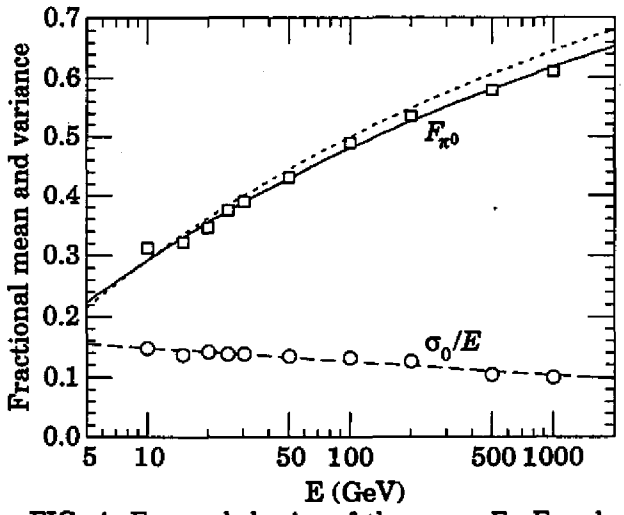

FIG. 4. Energy behavior of the mean $F_{\pi^0}E$ and width σ_0, scaled by the energy E, as simulated using FLUKA90 for negative pions incident on a lead "calorimeter." The function $[1-(E/E_0)^{m-1}]$ is shown for the comparison with F_{π^0}, for $m = 0.866$ and $E_0 = 0.764$ GeV (solid curve, least-squares fit to data above 10 GeV) and for the nominal values $m = 0.85$ and $E_0 = 1$ GeV (dotted curve). The dashed line through the σ_0/E points is the least-squares fit $0.171 - 0.010 \ln E$.

energy ($\sigma_{nc} = \sigma_0/E$). It is expected from previous work[5] that the mean hadronic energy ($(1-F_{\pi^0})E$) can be described by a constant times E^m, where $m \approx 0.85$. Accordingly, the curves drawn through the means are of the form $[1-(E/E_0)^{1-m}]$. The solid curve is a least-squares fit (excluding the 10 GeV point) for which $m = 0.866$ and $E_0 = 0.764$ GeV. Nominal values from the Tuscaloosa paper are used for the dotted curve.

The width σ_0/E is remarkably constant, as was first pointed out by Wigmans[6]. (This is the famous "constant term" in the resolution, or at least that part intrinsic to cascade production.) Dan Green has observed that most of the variance is simply the result of fluctuations in the number of π^0's produced in the first collision, and he has been able to make a rough estimate of the width. Since the multiplicity increases logarithmically with energy, we should expect σ_0/E to decrease slightly as the energy increases. The dashed line is a least-squares fit, $0.171-0.010 \ln E$. This function has the value 0.14 at 20 GeV and 0.12 at 200 GeV.

The dimensionless "coefficient of skewness" is defined as the third moment about the mean divided by the variance to the 3/2 power (σ^3). Within the Monte Carlo statistics, this quantity is independent of energy and has a value near 0.6.

The mean and width have been studied previously[3, 5], but in a rather second-hand way. The present method is more sensitive since the distributions of E_e are studied directly. These results are the best to date for examining the veracity of the power-law hadronic yield and for observing the slope of the "constant term."

4. Characteristic functions

Further analysis is considerably simplified by using a mathematical development not (yet) given in the *Review of Particle Properties*. The characteristic function $\phi(u)$ associated with the frequency function $f(x)$ is essentially its Fourier transform, or the expectation value of $\exp(iux)$:

$$\phi(u) = E(e^{iux}) = \int e^{iux} f(x)dx \qquad (5)$$

It has many useful properties, several of which we exploit here.

If $f_1(x)$ and $f_2(y)$ have characteristic functions $\phi_1(u)$ and $\phi_2(u)$, then the characteristic function of the weighted sum $ax+by$ is $\phi_1(au)\phi_2(bu)$. (This is just the statement from usual Fourier theory that the Fourier transform of the convolution of two functions is the product of the two Fourier transforms.)

Let the (partial) characteristic function corresponding to the conditional distribution $f_2(x|z)$ be $\phi_2(u|z)$, and the probability distribution of z be $f_1(z)$. The characteristic function after integration over the intermediate value is given by

$$\phi(u) = \int f_1(z)\phi_2(u|z)dz \quad . \qquad (6)$$

Suppose we can write ϕ_2 in the form

$$\phi_2(u|z) = A(u)e^{ig(u)z} \quad . \qquad (7)$$

Then

$$\phi(u) = A(u)\phi_1(g(u)) \quad . \qquad (8)$$

Finally, we note from Eq. (5) that the nth moment α_n of the distribution $f(x)$ is given by

$$\left.\frac{d^n\phi}{du^n}\right|_{u=0} = i^n \int x^n f(x)dx = i^n \alpha_n \quad . \qquad (9)$$

As a result, it is often easy to calculate all the

moments of a distribution defined by $\phi(u)$ even when the inversion is not carried out.

Several other definitions are useful. The nth moment about the mean is given by

$$\mu_n = \boldsymbol{E}[(x-\alpha_1)] \; . \tag{10}$$

The semi-invariants κ_n are defined by

$$\phi(u) = \exp\left(\sum_1^\infty \frac{\kappa_n}{n!}(iu)^n\right) \quad . \tag{11}$$

The α_n's, μ_n's, and κ_n's are related algebraically, and the first few are familiar:

$$\begin{aligned} \kappa_1 &= \alpha_1 \text{ (the mean)} \\ \kappa_2 &= \mu_2 \; (= \sigma^2) \\ &= \alpha_2 - \alpha_1^2 \\ \kappa_3 &= \mu_3 \\ &= \alpha_3 - 3\alpha_1\alpha_2 + 2\alpha_1^2 \end{aligned} \tag{12}$$

Any odd moment about the mean is a measure of skewness; the simplest of these is the coefficient of skewness $\gamma_1 = \mu_3/\mu_2^{3/2}$.

The characteristic function of a Gaussian frequency function with mean m and variance σ^2 is

$$\phi(u) = e^{imu - \frac{1}{2}\sigma^2 u^2} \quad , \tag{13}$$

so the Gaussian is that unique distribution for which all semi-invariants beyond the second vanish. In this study I also use the simplest skewed distribution,

$$\phi(u) = e^{imu - \frac{1}{2}\sigma^2 u^2 - \frac{i}{3!}\mu_3 u^3} \; . \tag{14}$$

5. Results

Let $\phi^{\text{vis}}(u|E_e)$ be the (partial) characteristic function corresponding to the conditional visible energy distribution $f^{\text{vis}}(E^{\text{vis}}|E_e)$ introduced in Eq. (2). Then

$$\phi^{\text{vis}}(u|E_e) = \phi_e(u|E_e)\phi_h(u|E_e) \quad , \tag{15}$$

and

$$\phi_v(u) = \int f_0(E_e)\phi^{\text{vis}}(u|E_e)dE_e \quad . \tag{16}$$

The electromagnetic response is expected to be fairly Gaussian. The hadronic response is less well known, but because the variance is dominated by a small number of collisions with large nuclear binding energy losses, it is likely to deviate substantially from a Gaussian distribution. However, the distributions are not yet available, so I make a Gaussian approximation here also.

For an ensemble of events with electromagnetic energy E_e, we would not expect the variance of the electromagnetic signal to be constant, but instead to be proportional to E_e. The variance of the the hadronic fraction should be proportional to $(E - E_e)$. Accordingly, we consider the characteristic functions

$$\begin{aligned} \phi_e(u|E_e) &= e^{iu\epsilon_e E_e - \frac{1}{2}\sigma_{e0}^2 \epsilon_e E_e u^2} \\ \phi_h(u|E_e) &= e^{iu\epsilon_h(E-E_e) - \frac{1}{2}\sigma_{h0}^2 \epsilon_h (E-E_e) u^2} \quad , \end{aligned} \tag{17}$$

so that

$$\phi^{\text{vis}}(u|E_e) = e^{iu\epsilon_h E - \frac{1}{2}\sigma_{h0}^2 \epsilon_h E u^2} \times e^{iE_e(u\Delta\epsilon - \frac{i}{2}u^2\Delta\sigma^2)} \; , \tag{18}$$

where $\Delta\epsilon \equiv \epsilon_e - \epsilon_h$, and $\Delta\sigma^2 \equiv \sigma_{h0}^2\epsilon_h - \sigma_{e0}^2\epsilon_e$. $\Delta\sigma^2$ is usually positive. Noting that Eq. (18) has the same form as Eq. (7), we see that

$$\phi_v(u) = e^{iu\epsilon_h E - \frac{1}{2}\sigma_{h0}^2 \epsilon_h E u^2} \phi_0(u\Delta\epsilon - \tfrac{i}{2}u^2\Delta\sigma^2) \; , \tag{19}$$

where ϕ_0 is the characteristic function corresponding to f_0.

The distribution of E_e was discussed in Section 3. Based on that analysis (and with reference to Eq. (14)), I approximate it as

$$\phi_0(u) \approx e^{iF_{\pi^0}Eu - \frac{1}{2}\sigma_0^2 u^2 - \frac{i}{3!}\mu_{30}u^3} \; , \tag{20}$$

where F_{π^0} and σ_0 are given in section three, and $\mu_{30} \approx 0.6\sigma_0^3$. Then Eq. (19) becomes

$$\begin{aligned} \phi_v(u) = \exp\Big(& iu\epsilon_e E\{\epsilon_h/\epsilon_e + F_{\pi^0}\Delta\epsilon/\epsilon_e\} \\ & - \tfrac{1}{2}u^2\{\epsilon_e E[(1-F_{\pi^0})(\sigma_{h0}^2\epsilon_h/\epsilon_e - \sigma_{e0}^2) + \sigma_{e0}^2] \\ & \quad + (1-\epsilon_h/\epsilon_e)^2\sigma_0^2\epsilon_e^2\} \\ & - \tfrac{i}{3!}u^3\{\mu_{30}\Delta\epsilon^3 - 3\sigma_0^2\Delta\sigma^2\Delta\epsilon\} + \ldots\Big) \quad . \end{aligned} \tag{21}$$

(Since only the first three moments of f_0 were evaluated, we drop terms of order u^4 and higher.)

Two housekeeping chores are necessary before interpreting these results: In the first place, it was pointed out in Section 3 that σ_0 is proportional to energy, except for a slow logarithmic decrease: $\sigma_0 \approx \sigma_{nc}E$. We therefore make this substitution. Secondly, with the notation used above the mean visible energy for an incident electron (or for a hadron if $\epsilon_e = \epsilon_h$) is $\epsilon_e E$. In a real situation, one calibrates the device so that the measured mean

is E. Accordingly, we replace $\epsilon_e E$ by E.

With these substitutions and the aid of Eqs. (12) and (14), the mean, variance, and skewness of the distribution given by Eq. (21) can be written down.

After some rearrangement, the mean can be written as:

$$\langle E^{\rm vis} \rangle = E[1 - (1 - F_{\pi^0})(1 - \epsilon_h/\epsilon_e)] \quad . \qquad (22)$$

This is a convenient form, since the second term, the product of the hadronic fraction and the degree of noncompensation, expresses the energy-dependent correction for noncompensation. The quantity in square brackets is the "linearity curve" (the ratio of response to energy) for hadronic response, and, alternatively, the reciprocal of the usual "e/π" ratio.

Similarly, the variance is the coefficient of $\frac{1}{2}u^2$ in Eq. (21), or the quantity in curly brackets. With the substitutions given above, the fractional variance becomes

$$\left(\frac{\sigma}{E}\right)^2 = \frac{(1 - F_{\pi^0})(\sigma_{h0}^2 \epsilon_h/\epsilon_e - \sigma_{e0}^2) + \sigma_{e0}^2}{E} + (1 - \epsilon_h/\epsilon_e)^2 \sigma_{nc}^2 \quad . \qquad (23)$$

This is *almost* of the form given by Eq. (1), with two important differences:

1. $\rho = 0$. There is no cross term; there is no correlation.

2. If F_{π^0} were not a function of energy, the numerator of the first term would be just the square of the usual coefficient of $1/\sqrt{E}$. However, F_{π^0} *is* energy dependent. This means that the familiar $1/\sqrt{E}$ dependence never did exist!

To understand this statement, it is important to remember the relative size of the different variances. For purposes of evaluating SDC detector requirements, the design goal assumed in the LoI was $\sigma_{had}/\sigma_{EM} = 0.7/0.25 = 2.8$. We identify this ratio with $(\sigma_{h0}\sqrt{\epsilon_h/\epsilon_e})/\sigma_{e0}$. The hadronic variance (scaled by ϵ_h/ϵ_e) is thus about 8 times larger than the electromagnetic variance, and to a good approximation σ_{e0}^2 can be dropped. Eq. (23) then becomes

$$\begin{aligned}\left(\frac{\sigma}{E}\right)^2 &\approx \frac{(1 - F_{\pi^0})\sigma_{h0}^2 \epsilon_h/\epsilon_e}{E} + (1 - \epsilon_h/\epsilon_e)^2 \sigma_{nc}^2 \\ &\approx \frac{(E/E_0)^{m-1}\sigma_{h0}^2 \epsilon_h/\epsilon_e}{E} + (1 - \epsilon_h/\epsilon_e)^2 \sigma_{nc}^2 \\ &\approx \left(\frac{\sigma'}{E^{0.57}}\right)^2 + (1 - \epsilon_h/\epsilon_e)^2 \sigma_{nc}^2 \quad .\end{aligned} \qquad (24)$$

A fit to the final form is shown by the dotted curve in Fig. 1. It is doubtful if this small deviation from $1/\sqrt{E}$ behavior would have been noticed.

The physical reason for deviation from $1/\sqrt{E}$ behavior is simple. As the energy increases, more and more of the cascade moves from the hadronic to the electromagnetic sector. The resolution therefore improves for this reason alone, and so improves more rapidly with increasing energy than would be expected just from sampling statistics.

Finally, we note that the "constant term" is explicitly proportional to the degree of noncompensation. We have previously concluded that the contribution (in quadrature) to the fractional resolution σ/E was $(1 - \epsilon_h/\epsilon_e) \times 14\%$. In this analysis σ_{nc} decreases from 14% at 20 GeV to 12% at 200 GeV.

6. Inclusion of transducer contributions

For illustrative purposes we take the distribution $f_t(E^{\rm obs}|E^{\rm vis})$ as Gaussian also, with mean $E^{\rm vis}$ (by definition) and variance $\sigma_t^2 E^{\rm vis}$ Following the same arguments as above, the characteristic function of the final distribution, $\phi(u)$, is obtained by evaluating $\phi_v(u)$ as given by Eq. (21) at $u + \frac{i}{2}\sigma_t^2 u^2$:

$$\phi(u) = \phi_v(u + \tfrac{i}{2}\sigma_t^2 u^2) \qquad (25)$$

The mean remains the same (as planned) and the variance increases by $\sigma_t^2 \langle E^{\rm vis} \rangle$. The third moment about the mean is increased by $3\sigma_t^2\sigma^2$, where σ^2 is the variance of f_v, as given by Eq. (23). The change in skewness comes about because the variance is not constant, but is proportional to the input signal; the same "feedup" of moments has occurred in earlier parts of this discussion.

The inclusion of f_t is mathematically hollow, but it is of importance for designers: The variance of f_v can be obtained by careful simulation. The only part within the control of the experimenter is the sampling fraction, since this variance is obviously smaller if the sampling fraction is large.

The burden on the experimenter is to make σ_t^2 smaller than σ^2 (for example, by efficient light collection and good photocathode sensitivity, in the case of photomultiplier readout), but there is no point in making σ_t^2 very *much* smaller than σ^2.

7. Summary and discussion

It follows from the mathematical structure of the problem as presented here that the sampling and noncompensation contributions to the resolution are not correlated; nor have we been able to identify any mechanism which would provide correlation. In general, correlation occurs only in the case of multivariate distributions (for example, the correlation between energy deposited in the first interaction length and the energy deposited elsewhere). As I understand a calorimeter in which a single number is measured, there are a series of cascaded and convoluted distributions but never a coupling among them.

On the other hand, it has been found that

1. The sampling term decreases a little faster than $1/\sqrt{E}$, and
2. The noncompensation contribution decreases slowly with increasing energy.

It will be of great interest to understand the distribution $f_h(E_h^{\rm vis}|E_e)$, as this will permit a reasonably complete quantitative solution to the problem.

6. Appendix: Summary of variable definitions

$f_0(E_e)$: Distribution of electromagnetic energy for incident energy E (implicit in all distributions)

$f_e(E_e^{\rm vis}|E_e)$: Distribution of visible EM energy if E_e is in EM sector

$f_h(E_h^{\rm vis}|E_e)$: Distribution of visible hadronic energy if $E - E_e$ is in hadronic sector

$f^{\rm vis}(E^{\rm vis}|E_e)$: Distribution of total visible energy if E_e is in EM sector

ϵ_e: Average efficiency with which EM energy is converted to visible EM energy; $\approx F_{\pi^0}/\langle E_e^{\rm vis}\rangle$

ϵ_h: Average efficiency with which hadronic energy is converted to visible hadronic energy; $\approx (1 - F_{\pi^0})/\langle E_h^{\rm vis}\rangle$

$f^{\rm vis}(E^{\rm vis})$: Distribution of total visible energy

$f_t(E^{\rm obs}|E^{\rm vis})$: Distribution of observed response after readout transducer, given $E^{\rm vis}$ visible energy (defined to have same mean)

$\phi_0(u)$: Characteristic function corresponding to $f_0(E_e)$. In all cases, subscripts and conditions label the ϕ corresponding to each f.

F_{π^0}: Fraction of energy entering the EM sector (mean value of $f_0(E_e)$ divided by E)

σ_0^2: Variance of $f_0(E_e)$ divided by E^2

σ_{nc}: σ_0/E (the noncompensation contribution to the "constant term)

α_n: nth moment of a distribution

μ_n: nth moment about the mean

σ_0^2: Variance of $f_0(E_e)$ divided by E^2

μ_{30}: Third moment of f_0 about its mean

σ_{e0}^2: Variance of $f_e(E_e^{\rm vis}|E_e)$ is taken to be $\sigma_{e0}^2\epsilon_e E_e^{\rm vis}$, or, after final scale changes, as $\sigma_{e0}^2 E_e^{\rm vis}$

σ_{h0}^2: Variance of $f_h(E_h^{\rm vis}|E_e)$ is taken to be $\sigma_{h0}^2\epsilon_h E_h^{\rm vis}$, or, after final scale changes, as $\sigma_{h0}^2 E_e^{\rm vis}\epsilon_e/\epsilon_h$

$\Delta\epsilon$: $\epsilon_e - \epsilon_h$

$\Delta\sigma^2$: $\sigma_{h0}^2\epsilon_h - \sigma_{e0}^2\epsilon_e$

References

1. These data were obtained from a graph of preliminary SPACAL data, obtained from R. de Salvo at Snowmass '88. As such, they are not results made available by the collaboration, and should be considered as illustrative only.
2. A. Ferrari, A. King, and G. R. Stevenson, oral tradition (1991).
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