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ABSTRACT

The response of a large, homogenous calorimeter to an incident hadron with energy E, relative to that for an incident electron or photon with the same energy, is adequately described by $R_h/R_e = [1-(1-\epsilon_h/\epsilon_e)(E/E_0)^{m-1}]$, where $m \approx 0.86$, $E_0 \approx 1$ GeV, and ϵ_e/ϵ_h is essentially the "intrinsic e/h" introduced by Wigmans. In a jet with energy E_J the averaged charged particle distribution is described by a fragmentation function $D(z)$, where (for particle energy $E \approx p_{\|}$) $E = zE_J$. The calorimeter response to a jet may be found by integrating the single-hadron response over this distribution:

$$R_J/\epsilon_e = E_J\left[1-(1-\epsilon_h/\epsilon_e)(E_J/E_0)^{m-1}\int_0^1 z^m D(z)dz\right]$$

The response to a jet is the same as that to a single hadron only if the integral is unity. For a low-multiplicity jet (such as the boosted jet from a B or W decay) the quantity is less than unity, and the calorimeter appears to be more compensating for a jet than it does for a single hadron with the same energy. For a high-multiplicity jet (*e.g.* from primary fragmentation) the opposite is true. The integral is greater than 0.9 and less than 1.2 for realistic jets, and produces errors of at most a few percent in jet energy. These are probably small compared with those of the low-z cutoff imposed by a possible solenoidal magnetic field and dead material in front of the calorimeter.

1. Introduction

Not all jets are created equal. Calorimeter response, measured in a test beam with incident hadrons, can be thought of as the response to the "jet" produced in the first collision of the hadron with a heavy nucleus in the calorimeter—or in a convertor out front, as is often done in test beam simulations of jet response. However, the incident jet might consist of just a few energetic particles, for example from the decay of a high-energy W or Z. At the other extreme, a high-energy jet from the primary collision at the SSC contains a large number of very soft particles. If the calorimeter is noncompensating, there is no guarantee that the response to these different kinds of jets with same energy will be the same.

We report here a study of the effect of differences in jet composition, as given by a range of fragmentation functions, on the response of an ideal calorimeter. By "ideal" we mean a calorimeter is large, in the sense that leakage effects can be ignored, and "homogenous," in the sense that the structure is the same throughout—if there is a plate structure, it is the same front-to-back.

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2. Response to a single incident hadron

In an earlier analysis[1], we pointed out that the level of low-energy hadronic activity in a cascade grows as a power law in the energy. A growth as E^m is observed in a variety of Monte Carlo studies, and is supported by a heuristic argument. The exponent is in the range $m \approx 0.85$–0.87, and may vary somewhat with energy. The less-than-linear rise is the result of the increasing fraction of the energy going into the electromagnetic channel through π^0 production. We may then write the response of a calorimeter to an incident hadron with energy E as

$$R_h = \epsilon_e E\left[1-(1-\epsilon_h/\epsilon_e)(E/E_0)^{m-1}\right] , \quad (1)$$

where

$R_h =$ Response of the calorimeter to an incident hadron, in coulombs or volts or whatever units are convenient;

$\epsilon_e =$ Efficiency with which electromagnetic energy deposition is converted to output signal;

$\epsilon_h =$ Efficiency with which energy deposition by low-energy hadrons (including ionization, energy lost in spallation and nuclear excitation, recoils from neutron scattering, nonlinear response, gate length, *etc.*) is converted to output signal. (The ratio

ϵ_e/ϵ_h is very close to Wigmans"s "intrinsic e/h");

$E_0 =$ Scale energy; very close to 1 GeV. In practice, only the product $(1-\epsilon_h/\epsilon_e)E_0^{1-m}$ can be measured.

In our idealized calorimeter, it may be assumed that the response to an electron or photon is linear in the energy:

$$R_e = \epsilon_e E \tag{2}$$

It then follows that the ratio R_e/R_h is Wigmans' "e/π," and that the "linearity" R_h/E is (within a constant) simply the reciprocal of "e/π."

Over the energy range usually explored in test beam runs, Eq. (1) describes a function virtually indistinguishable from the logarithmic form sometimes used. However, it preserves the all-important asymptotic behavior of R_h: "e/π" is asymptotic unity for any large homogenous calorimeter, and the response to hadrons is asymptotically proportional to the energy.

3. Response to an ensemble of particles

A jet with energy E_J consists of photons from π^0 decay and "stable" hadrons with different energies. (For our present purposes we ignore energy which might be carried by leptons.) We need only to sum the responses of all of these particles to obtain the response to a jet. Let R_{ej} be the response to the jth π^0 in the jet (with energy $E_{\pi^0 j}$) and R_{hk} the response to the kth stable hadron (with energy E_{hk}). Then the response to the jet is given by

$$R_J = \sum_{j=1}^{N_{\pi^0}} R_{ej} + \sum_{k=1}^{N_{\rm had}} R_{hk} \ . \tag{3}$$

Using Eqs. (1) and (2) to evaluate R_{hk} and R_{ej}, this reduces to

$$R_J = \epsilon_e E_J \Big[1-(1-\epsilon_h/\epsilon_e)(E_J/E_0)^{m-1} \times \sum_{k=1}^{N_{\rm ch}} (E_{hk}/E_J)^m\Big] \ . \tag{4}$$

If $m = 0$, the summation is equal to the stable hadron multiplicity If $m = 1$ it is equal to the fraction of the jet energy carried by stable hadrons, which on the average is somewhat greater than 0.6 and less than 2/3. If the summation is unity for $m \approx 0.86$, then the calorimeter has the same response for the jet as for an incident hadron. Imposition of this requirement for the "jet" from the first hadronic collision in the calorimeter results in recovery of the relationship between m, the mean multiplicity, and the average π^0 energy fraction in a single collision (F_{π^0}) which was given in Ref. 1.

4. Response to a jet described by a fragmentation function

It is convenient to replace the above summation by an integral over the fragmentation function describing the jet. It is usual to describe the charged particle number distribution in a jet by the fragmentation function $D(z) = dN_{\rm ch}/dz$, where z is the particle momentum component parallel to the jet axis, scaled by the total momentum of the jet. For the present analysis we treat z as the fractional energy, *i.e.* $E_{hk}/E_J \approx z$. The fractional *energy* distribution is given by $zD(z)$, and $D(z)$ has the properties

$$\int_0^1 D(z)dz = N_{\rm ch} \text{ and } \int_0^1 z\,D(z)dz = F_{\rm ch} \ . \tag{5}$$

Since $D(z)$ diverges as $z \to 0$ (at least if the π^0 mass is neglected), the first integral is interpreted as the multiplicity down to an experimental cutoff in z. The integral of $z^m\,D(z)$ is not divergent for $m > 0$.

Several problems arise in our use of the charged-hadron $D(z)$ as obtained from either experiment or simulation. In the first place, a calorimeter measures the energy of all hadrons and their decay products, with π^0's producing a different response than other hadrons. A significant fraction of the energy is carried by K_L^0's, neutrons, *etc.*, which are excluded in experiments, because the analysis uses only central-tracker information. In Monte Carlo simulations, the decay products of neutral hadrons such as K_S^0 and Λ's are excluded from the charged fraction unless special care is taken. This situation is illustrated in Fig. 1, where we show the jet momentum fraction for "charged" hadrons and for hadrons other than π^0's in one of the ISAJET[4] jet samples used in this study. Moreover, very low-energy particles are not observed an an experiment and may be poorly modeled in

simulation programs. These carry a significant fraction of the energy and contribute most of the multiplicity.

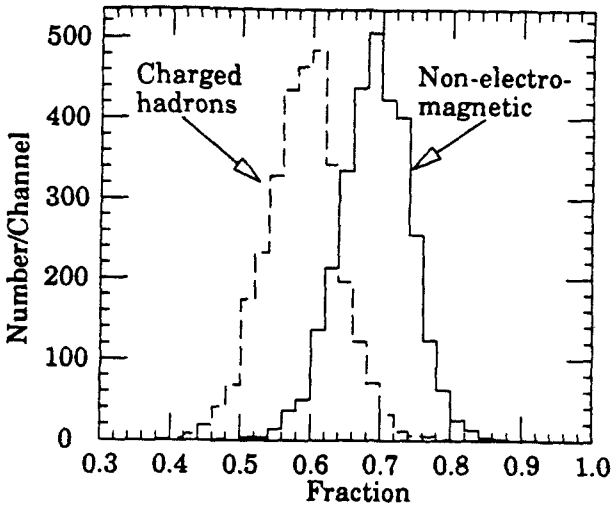

FIG. 1. Distribution of the momentum fraction in ISAJET TWOJET events at $\sqrt{s} = 40$ TeV, for 1000 GeV $< M_{JJ} <$ 2000 GeV, for (a) charged hadrons (excluding decay products of Λ, K^0, *etc.*) and (b) all hadrons excluding π^0's.

When the argument leading to Eq. (4) is repeated using the continuous function, one obtains

$$R_J = \epsilon_e E_J \Big[1 - (1 - \epsilon_h/\epsilon_e)(E_J/E_0)^{m-1} \times \int_0^1 z^m D(z) dz \Big] , \tag{6}$$

where $D(z)$ is for all non-electromagnetic hadronic particles, and includes (if necessary) an appropriate extrapolation to $z = 0$.

5. Examples

It remains to evaluate the integral in Eq. (6) for representative cases. We choose the following:

1. Jets from Z decay, as measured by the DELPHI collaboration at LEP[2]. The published fragmentation function is for the entire event, so we have normalized the function downward by a factor of two to describe the individual jets. Data were read from their Fig. 3(b) and extrapolated to $z = 0$.
2. CDF charged fragmentation function at $\sqrt{s} = 1800$ GeV[3]. $z dN_{\rm ch}/dz$ was extrapolated to $z = 0$ to force $\langle F_{\rm ch} \rangle = 0.65$, their reported value. (Since some of the energy is carried by neutrals, this value is probably too high for consistency with isospin conservation.)

Two samples of TWOJET ISAJET events at $\sqrt{s} = 40$ TeV. In both cases, all hadrons other than π^0's are used:

3. 3226 events with p_t (hard scatter) > 40 GeV/c, and 100 GeV $< M_{JJ} <$ 200 GeV. The mean jet momentum is 73 GeV/c, and the mean non-π^0 hadronic multiplicity is 26.
4. 3042 events with p_t (hard scatter) > 400 GeV/c, and 1000 GeV $< M_{JJ} <$ 2000 GeV. The mean jet momentum is 677 GeV/c, and the mean non-π^0 hadronic multiplicity is 70. The z distribution for these events is shown in Fig. 2.

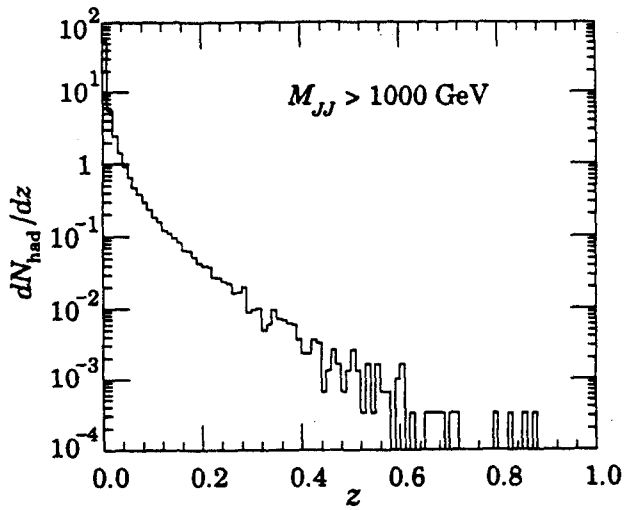

FIG. 2. Distribution in z for ISAJET TWOJET events at $\sqrt{s} = 40$ TeV, for 1000 GeV $< M_{JJ} <$ 2000 GeV.

The results are summarized in Table 1. Since one might naïvely expect $F_{\rm had} = 1 - F_{\pi^0} = 0.67$ from isospin considerations, some of the bias from the selection of charged hadrons can be removed by normalizing to this value (entries in parentheses). As can be seen, the integral is slightly under unity in the case of the similar low-energy LEP and Tevatron functions, but can be expected to reach values as high as 1.15 or so for the highest p_t jets at the SSC. Given the uncertainties in the calculation, we conclude that the correction factor for jets at the SSC will be bracketed by the values 0.90 and 1.20, and will

depend upon the kind of jet.

The correction factors given in Table 1 vary by about 0.03 with a change $\Delta m = 0.01$, introducing an additional uncertainty of perhaps 5%.

It is also interesting to look at the *distribution* of corrections for a given jet energy. This is shown for the $M_{JJ} \approx 100$ GeV ISAJET events in Fig. 3. For a noncompensating calorimeter this distribution causes a small resolution degradation, but this is nearly the same as the contribution of fluctuations in π^0 production in a single hadron's first collision in the calorimeter.

Table 1

Integrals over representative fragmentation functions. Numbers in parentheses are calculated for the non-electromagnetic energy fraction forced to be 0.67.

Source	Process	$\int_0^1 D(z)dz$	$\int_0^1 z^{0.86}D(z)dz$	$\int_0^1 zD(z)dz$
DELPHI	$Z \to jet\ jet$	11.0(12.1)	0.84(0.92)	0.61(0.67)
CDF	$\sqrt{s} = 1.8$ TeV	17.8*(19.9)	0.94(0.97)	0.65(0.67)
ISAJET	40 GeV, $\langle p_J \rangle = 73$ GeV/c	26.2(25.2)	1.04(1.00)	0.69(0.67)
ISAJET	40 GeV, $\langle p_J \rangle = 677$ GeV/c	69.8(64.7)	1.15(1.06)	0.72(0.67)

*The extrapolated low-momentum part of the function contributes 10 to this total.

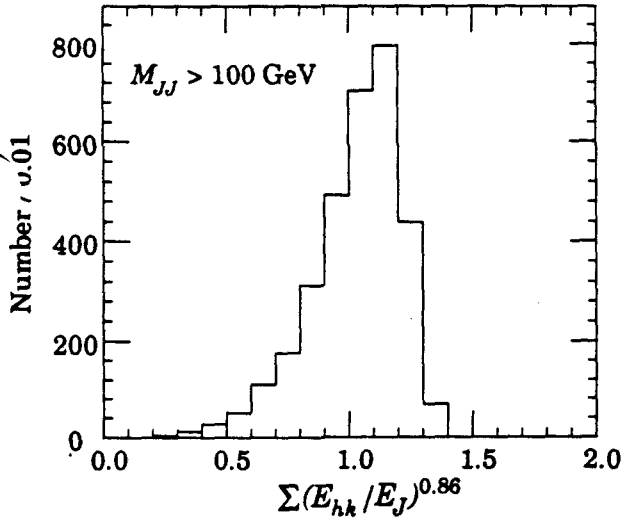

FIG. 3. Distribution of $\sum(E_{hk}/E_J)^{0.86}$ for a sample of ISAJET TWOJET events at $\sqrt{s} =$ 40 TeV with 100 GeV $< M_{JJ} <$ 200 GeV.

6. Effect of low-z cutoff

It can be seen from Fig. 2 that most of the particles in a typical jet have $z < 0.01$. The z dependence of the various integrals of interest in this study is shown in Fig. 4 for our 1000 GeV $< M_{JJ} <$ 2000 GeV ISAJET sample. It can be seen that 80% of the particles have $0.0 < z < 0.01$, and that these carry 17% of the energy. The other fragmentation functions we have considered have similar behavior.

We note the following:

a) While the curves for $a = 0.86$ and $a = 1.00$ change rapidly with the cutoff in z, their *ratio* does not. This means that when the integral for $a = 0.86$ is normalized to a given hadronic energy fraction, as is done for the entries in parentheses in Table 1, the compensation correction factor obtained in this way is not terribly sensitive to the cutoff.

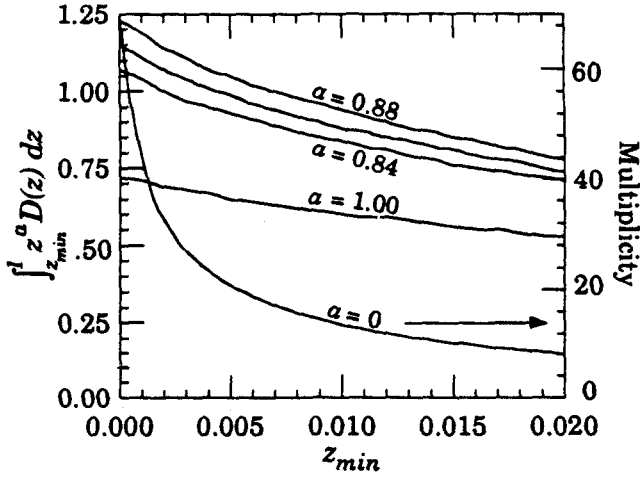

FIG. 4. Behavior of $\int_{z_{min}}^1 z^a D(z)dz$ for the 1000 GeV $< M_{JJ} <$ 2000 GeV ISAJET sample for small z, for $a = 0$ (multiplicity; axis on right), $a = 1$ (hadronic energy fraction), $a = 0.84$, $a = 0.86$ (unlabeled), and $a = 0.88$.

b) A substantial fraction of the energy is carried by very soft particles, and these preferentially deposit their energy in any dead material in front of the calorimeter.

c) For a 100 GeV jet, at least 3% of the energy is below 600 MeV in the ISAJET simulation; a particle with this energy cannot cross the proposed SDC detector tracking volume. Somewhat more energetic particles reach the calorimeter, but are not within the $\Delta\eta\Delta\phi$ region defining the jet.

7. Implications for calibration and compensation

The only reasonable way to calibrate a calorimeter is in terms of equivalent electron response, so for purposes of this section we take the first factor on the right of Eq. (1) (ϵ_e) as unity, and measure the response in GeV. For example, if $\epsilon_e/\epsilon_h = 1.30$ and $E_0 \approx 1$ GeV, then a 1000 GeV pion produces a response of 912.3 GeV (see Table 2). Conversely, if a 912.3 GeV signal is observed for an incident hadron, Eq. (1) can easily be inverted to find $E_h = 1000$ GeV. For an incident jet Eq. (6) must be used instead. The integral appears to be greater than 0.9 and less than 1.2 for realistic jets. The corresponding energies determined for jets in our $\epsilon_e/\epsilon_h = 1.30$ calorimeter are shown in Table 2. For example, if the integral in Eq. 6 is 0.90 and if the energy scale has been calibrated in a hadron test beam, then a 200 GeV jet will appear to have 202.4 GeV. The percentage errors shown in the table for the two limiting cases are plotted in Fig. 5, along with similar results for a calorimeter with $\epsilon_e/\epsilon_h = 1.15$. (A lead–liquid argon calorimeter with fast readout might have $\epsilon_e/\epsilon_h = 1.30$, and 1.15 might be obtained for a poorly-designed metal-scintillator calorimeter.) The error is 4% in the worst case at low energies, and decreases with energy and with the degree of compensation.

Another way to look at the situation is that the correction factor changes the effective degree of compensation for the calorimeter. This dependence is shown in Fig. 6.

Table 2

Energies found for jets with correction factors 0.90 and 1.20 on the basis of single-hadron calibration R_h in a calorimeter with $\epsilon_e/e_h = 1.30$. Energies and responses are in GeV.

E	R_h	$E_{0.90}$	Error	$E_{1.20}$	Error
10.0	8.3	10.2	1.95%	9.6	−3.91%
20.0	17.0	20.3	1.74%	19.3	−3.49%
50.0	43.3	50.8	1.51%	48.5	−3.02%
100.0	87.9	101.4	1.35%	97.3	−2.70%
200.0	178.0	202.4	1.21%	195.1	−2.43%
500.0	451.7	505.3	1.05%	489.5	−2.11%
1000.0	912.3	1009.5	0.95%	981.0	−1.90%
2000.0	1840.8	2017.1	0.85%	1965.8	−1.71%

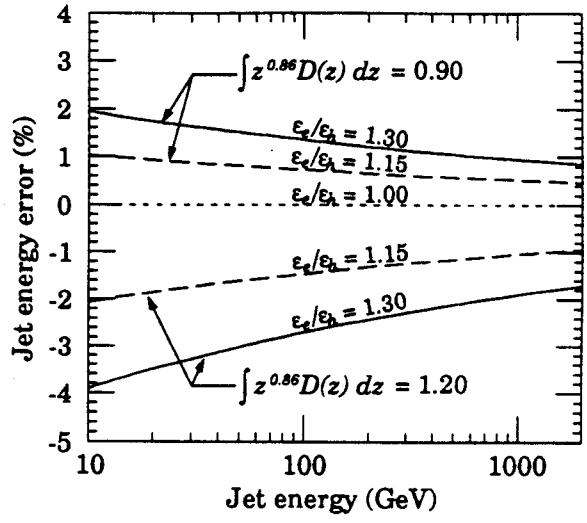

FIG. 5. Energy determination error as a function of jet energy for representative values of ϵ_e/ϵ_h, for the two extreme case of the correction factor: 0.90 (top curves, for hard, low-multiplicity jets) and 1.20 (bottom curves, for soft, high-multiplicity jets).

8. Conclusions

Jet composition variations cause small errors in energy determination in an ideal but noncompensated calorimeter calibrated with test-beam hadrons. These errors appear to be small, even for $\epsilon_e/\epsilon_h = 1.30$, in comparison with errors associated with the solenoidal field and dead matter in front of the calorimeter. These are related to the fact that much of a jet's energy is carried by very low-energy particles. Further studies of these effects are planned, as are studies of the effects of different compensation in electromagnetic and hadronic compartments.

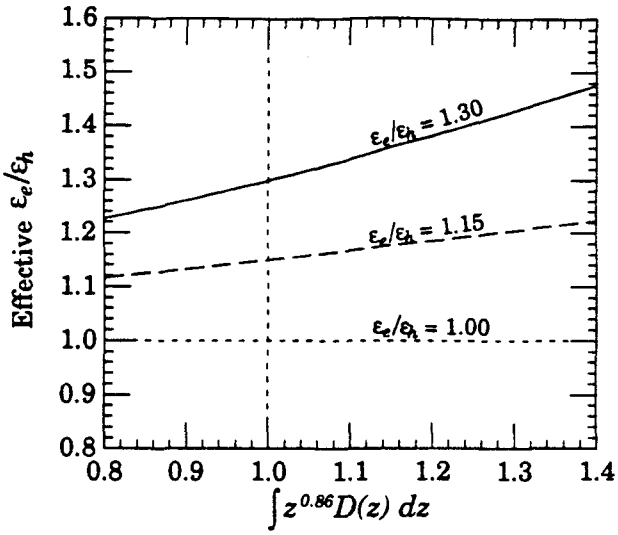

FIG. 6. Effective ϵ_e/ϵ_h for representative values of ϵ_e/ϵ_h as a function of the correction factor $\int z^{0.86} D(z) dz$ (see Eq. 6.)

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