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MOMENTUM RESOLUTION FOR MUONS USING THE FULL TRACKING SYSTEM OF SDC

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Momentum Resolution for Muons using the full tracking system of SDC†

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Abstract

The momentum resolution of the SDC detector for high momentum muons was estimated assuming that the full tracking system is available for track fitting. The result of G. Feldman shown at Snowmass '90 was independently reproduced, and a comparison was done between the Type-S and the Type-L magnet configurations presented in the EoI. The non-uniform field and the return field in the calorimeter area of the Type-S magnet do *not* significantly degrade the momentum resolution.

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1. Introduction

The design goal of the tracking system of the SDC detector has been set to be $\sigma_p/p = 0.2p_T(\text{TeV}/c)$ in the central rapidity region $|\eta| \lesssim 1.5$. This resolution is not satisfactory for detecting the $\mu^+\mu^-$ decay of a possible multi-TeV Z' production. G. Feldman showed [1] that the resolution can be much improved if the inner trackers and the outer muon trackers can simultaneously be used for track fitting. One of the figures that he showed is shown in Fig. 1. The resolution is improved from 20% (Inner only) to 12% (Full System) for p = 1 TeV/c. This argument is based on simple assumptions on the detector set-up, which will be shown in the next section, and a uniform solenoidal magnetic field is assumed there.

As is well known, the Type-S magnet generates an apparently non-uniform magnetic field because it is almost an air-core magnet. It has been shown by H. Iwasaki [2] and Y. Takaiwa [3] that this non-uniformity is not serious for the tracking in the inner trackers. However, when the full tracking system is used for track fitting as G. Feldman did, the non-uniformity may become serious because the non-zero return field in the calorimeter area gives an additional effect to the track trajectories, which tends to degrade the momentum resolution.

In this report we examine the effect of the non-uniform field of the Type-S magnet to the momentum resolution by using a simple estimation method. The method is explained in the next section and applied to the set-up assumed by G. Feldman in order to confirm his result. The Type-S and Type-L are compared in section 3, and the degradation due to the non-uniform field is evaluated.

2. Reproducing the Results of G. Feldman

2-1. Detector Set-up

The detector set-up that G. Feldman used is as follows:

Magnets:	Solenoid	2.0 T from r = 0 to 2.0 m
	Return yoke	-1.75 T from r = 3.5 to 4.1 m
	Toroid	1.8 T from r = 6.5 to 8.0 m
Scatteres:	Calorimeters and Return yoke	
		iron from r = 2.1 to 4.1 m
	Toroid	iron from r = 6.5 to 8.0 m

Inner Trackers:	8 layers from $r = 0.7$ to 1.8 m with $\sigma(r\phi) = 70$ μm
	vertex constraints $\sigma(r\phi) = 10$ μm, $\sigma(z) = 50$ μm
Outer Trackers:	4 layers at $r = 5.2$, 6.2, 8.2 and 10.2 m
	$\sigma = 250$ μm for two directions in each layer

The parameters of the inner trackers are chosen so as to give a momentum resolution of $\sigma_p/p = 0.2\, p_T$(TeV/c). We follow this set-up in this section.

2-2. Methodology

The χ^2 of a hypothetical function $f(x;\vec{a})$ against independent measurements y_i at the points x_i ($i = 1,2,\ldots,n$) is defined as

$$\chi^2 = \sum_{i=1}^{n} \frac{1}{\sigma_i^2}\left[f(x_i;\vec{a}) - y_i\right]^2, \tag{1}$$

where σ_i is the estimated measurement error, and $\vec{a}$ symbolically denotes a set of parameters a_μ ($\mu = 1,2,\ldots,m$) characterizing the function. When the parameter set $\vec{a}$ is unknown, their optimum values can be estimated from the measurements by searching the parameter set that minimizes the χ^2, and the errors of the optimized values of the parameters can be estimated from the second derivatives of the χ^2. Defining the matrix G as

$$G_{\mu\nu} = \frac{1}{2}\frac{\partial^2\chi^2}{\partial a_\mu \partial a_\nu}, \tag{2}$$

the error matrix $E_{\mu\nu}$ of the parameters $\vec{a}$ can be obtained by inverting the matrix G; *i.e.* $E = G^{-1}$. The diagonal component of E is identified as the square of the error of the corresponding parameter, $\sigma(a_\mu)^2 = E_{\mu\mu}$, and the error $\sigma(a_\mu)$ is usually a good approximation of the resolution of a_μ.

Further, by using expression (1), the second derivative in eq. (2) can be written as

$$\frac{1}{2}\frac{\partial^2\chi^2}{\partial a_\mu \partial a_\nu} \cong \sum_i \frac{1}{\sigma_i^2}\frac{\partial f(x_i;\vec{a})}{\partial a_\mu}\frac{\partial f(x_i;\vec{a})}{\partial a_\nu} \tag{3}$$

if $\partial^2 f/\partial a_\mu \partial a_\nu \cong 0$. This is a good approximation unless the function is quite exotic, and the equality is exact when the function is linear. Therefore, if the derivatives $\partial f/\partial a_\mu$ are given at each measured points, the matrix G can be calculated by eq. (3). Though this is a good method for some simple cases, the calculation may become very complicated when we have to treat a large number of parameters or a complicated set-up.

The most easy-going and flexible way to calculate a derivative in computer programs is the finite-difference method; taking a finite difference Δa, a derivative is approximated as $df(a)/da = [\, f(a+\Delta a) - f(a)\,]/\Delta a$. The matrix G is evaluated with this technique in the subroutine HESSE of the widely-used optimization program MINUIT [4]. We borrow the algorithm of HESSE in the following estimations.

Here it should be noted that the right side of eq. (3) does not contain y_i. This means that we can take any values of y_i in the calculation of eq. (2). In the following calculation we take y_i to be ideal values, *i.e.* $y_i = f(x_i;\vec{a}_0)$, where $\vec{a}_0$ denotes the input values of the parameters. This choice makes the minimization of χ^2 trivial and the calculation of the derivatives very easy.

2-3. Multiple Scattering

The spatial and angular displacements due to the multiple Coulomb scattering of muons after passing through a material with the thickness L and the radiation length L_R can be described as [5]

$$\begin{aligned} \Delta y &= z_1 L\theta_0/\sqrt{12} + z_2 L\theta_0/2 \\ \Delta\theta &= z_2\theta\,, \end{aligned} \tag{4}$$

respectively, where

$$\theta_0 = \frac{14.1 MeV/c}{p\beta}\sqrt{\frac{L}{L_R}}\left[1 + \frac{1}{9}\log_{10}\frac{L}{L_R}\right].$$

The z_1 and z_2 are independent Gaussian random variables with mean zero and variance one. We treat these two variables as the track parameters that have unit measurement errors, and add the term

$$\chi^2_{MS} = \sum z_i^2 \tag{5}$$

to the χ^2 of the measurements, eq. (1).

The above approximation may differ from actual values by about a few 10% for very thick materials like the calorimeters in our set-up. However it does not matter in our present rough estimations.

2-4. Other Details of the Method

We chose the following five variables as the track parameters:

$1/p$, d_0, ϕ_0, z_0 and α_0,

where p is the track momentum. The other parameters are defined at the closest approaching point of the track to the beam line: the distance and the

direction in x-y projection, and the z coordinate and the z-r direction ($dz/dr = \cot\theta$), respectively. In a uniform solenoidal magnetic field along z-axis, the trajectory of a high momentum particle can be approximated as

$$\begin{aligned} \delta &= d_0 + \phi_0 r + \frac{aB}{2p_T} r^2 \\ z &= z_0 + \alpha_0 r \end{aligned} \qquad (6)$$

where δ is the deviation in x-y projection from the straight line drawn approximately along the track direction at the production point, and $a = 0.3$ (GeV/c)/(T·m). The trajectory in the iron toroid is also approximated with similar functions but the deflection is along the z-axis.

In addition to the above five parameters, we treat the parameters z_1 and z_2 of the multiple scattering as track parameters. Since we have to consider two directions for each material and we have two thick materials, the number of parameters becomes $5 + (2\times2\times2) = 13$ in total.

The parameters p and α_0 are given as inputs in our calculation. At the first step, the ideal hit points are left at all measured points according to eq. (6) and other functions for free spaces and for the iron toroid, where we set $d_0 = \phi_0 = z_0 = z_1 = 0$. Then the 13×13 matrix G given by eq. (2) is calculated by means of the finite difference method. The matrix G is inverted by using the utilities in the FACOM's scientific subroutine library SSL II, to obtain the error matrix E. And then we take $\sigma(1/p)/(1/p)$ to be the estimated momentum resolution σ_p/p.

2-5. Results

The resultant momentum resolution is shown in Fig. 2 as a function of the transverse momentum p_T. We obtain the resolution of ~12% for $p_T = 1$ TeV/c and ~30% for 5 TeV/c. These are in good agreement with the results shown by G. Feldman (Fig. 1). The resolutions when the inner and the outer trackers are separately used are also shown for references.

The dotted line in the figure shows the resolution when the multiple scattering is ignored. We can see a reasonable behavior of the *full* resolution; it is almost identical to the *inner only* resolution up to a few 100 GeV/c and gradually approaches to the *full* (*no MS*) resolution as p_T increases.

3. Effect of the Non-uniform Field of Type-S Magnet

3-1. Method of the Estimation

The magnetic field of the Type-S magnet is illustrated in Fig. 3, which was calculated by S. Terada according to the geometry shown in the EoI. A non-uniformity of the field is apparent. For such non-uniform magnetic field, the approximated trajectory of high momentum particles, eq. (6), is modified to be

$$\delta = d_0 + \phi_0 r + \frac{a}{p_T} I_B(r), \tag{7}$$

where

$$I_B(r) = \int_0^r dr' \int_0^{r'} [B_z(r'', z) - B_r(r'', z)\cot\theta] dr'' \tag{8}$$

and $z = r''\cot\theta$ in the integral. The z-r relation is unchanged in the first order because of the absence of B_ϕ. The third term of eq. (7) strongly relating to the momentum resolution; the larger it is, the better the momentum resolution is. Therefore a smaller B_z and a non-zero B_r in the forward region of the Type-S magnet tend to degrade the momentum resolution. And the existence of a return field in the calorimeter area may also result in a worse resolution when the inner and outer trackers are simultaneously used for track fitting. Such effects of the non-uniform magnetic field of the Type-S magnet is examined in the following by using the method described in the previous section. The results are evaluated by comparing with those for the Type-L magnet in the EoI.

The integral $I_B(r)$ is numerically calculated using the subroutine SDFMAP coded by Y. Takaiwa, which returns the field vector at a given spatial point according to a field map. The positions of the materials and the outer trackers are modified according to the geometries illustrated in the EoI. However the set-up of the inner trackers is not changed and the area for calorimeters and the return yoke is assumed to be filled with iron, for simplicity.

The estimation of the momentum resolution is separately done for two rapidity regions $|\eta| < 1.0$ and $1.4 < |\eta| < 1.6$ in order to avoid complications due to the existence of many detector edges. Thanks to the flexibility of the estimation method that we are using here, the calculation program for the central region can be applied to the forward region, $1.4 < |\eta| < 1.6$, without fundamental modifications. The outer trackers are

considered to provide the information on x and y coordinates in the forward region, instead of $r\phi$ and z in the central region. The magnetic field in the iron toroid is assumed to be uniform even in the forward region. And all the layers of the inner trackers are assumed to extend to $|z| = 4.5$ m; namely all tracks to be examined penetrate all the layers.

3-2. Results

The estimated momentum resolutions for $p_T = 1$ and 5 TeV/c muons are shown in Fig. 4 as a function of rapidity. We can see that the degradation due to the non-uniform magnetic field of the Type-S magnet is very small and practically negligible in the central region; the resolution of Type-S is only ~10% worse than Type-L even for $p_T = 5$ TeV/c at $|\eta| = 1.0$. In the forward region the degradation is not very small: ~20% for 1 TeV/c and ~60% for 5 TeV/c at $|\eta| = 1.5$. However $p_T = 5$ TeV/c corresponds to the absolute momentum of ~10 TeV/c at $|\eta| \sim 1.5$. Such enormously large momenta need not be considered even at SSC. Therefore, in practice, we can conclude that the degradation must be at most ~30% even in the forward region $|\eta| \sim 1.5$.

4. Summary

The results on the momentum resolution for high momentum muons shown by G. Feldman at Snowmass '90 were independently reproduced. The resolution σ_p/p is ~12% for $p_T = 1$ TeV/c and ~30% for 5 TeV/c if the full tracking system of the SDC detector can be used for track fitting and the spatial resolution of 250 μm can be achieved in the outer trackers.

We examined the possible degradation of the resolution due to the non-uniform magnetic field of the Type-S magnet configuration comparing with the Type-L configuration. In consequence the degradation is not significant: practically negligible ($\lesssim 10\%$) in the central rapidity region $|\eta| < 1.0$, and at most ~30% even at $|\eta| \sim 1.5$.

References

[1] G. Feldman, talk at Snowmass '90

[2] H. Iwasaki, talk at Snowmass '90

[3] Y. Takaiwa, talk at Snowmass '90

[4] F. James and M. Roos, Comput. Phys. Commun. **10** (1975) 343

[5] See p.65 of: Review of Particle Properties, Phys. Lett. **204B** (1988) 1

Figure Captions

Fig. 1. One of the figures that G. Feldman showed at Snowmass '90.

Fig. 2. Our estimations of the momentum resolution for high momentum muons. The detector set-up is the same that G. Feldman used to derive the results shown in Fig. 1 (see the text).

Fig. 3. The magnetic field of the Type-S magnet configuration calculated by S. Terada.

Fig. 4. The results of our estimation. The momentum resolution is shown for $p_T = 1$ and 5 TeV/c, and Type-S and Type-L, as a function of rapidity.

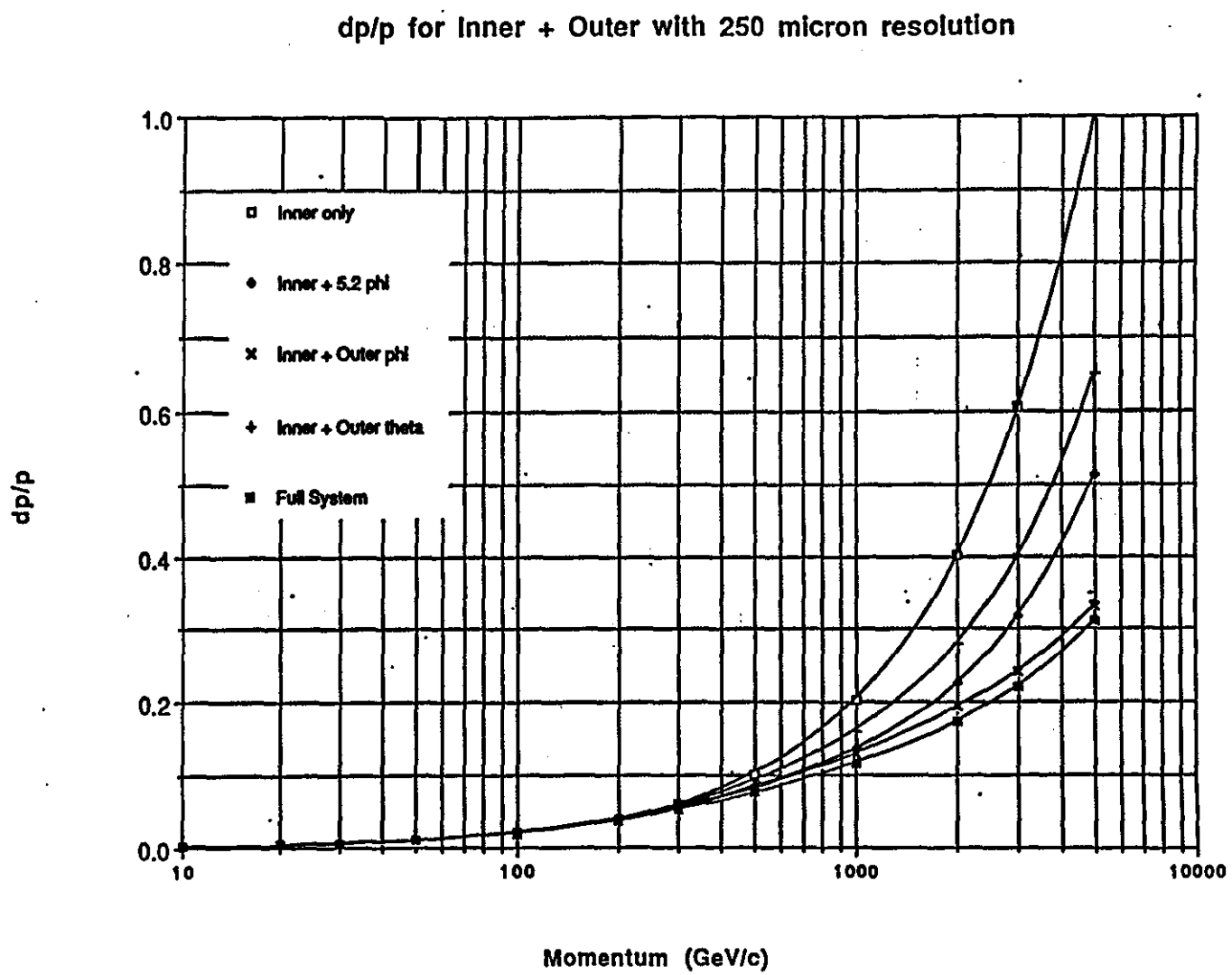
dp/p for Inner + Outer with 250 micron resolution
Inner only
Inner + 5.2 phi
Inner + Outer phi
Inner + Outer theta
Full System
dp/p
1.0
0.8
0.6
0.4
0.2
0.0
10
100
1000
10000
Momentum (GeV/c)

Fig. 1

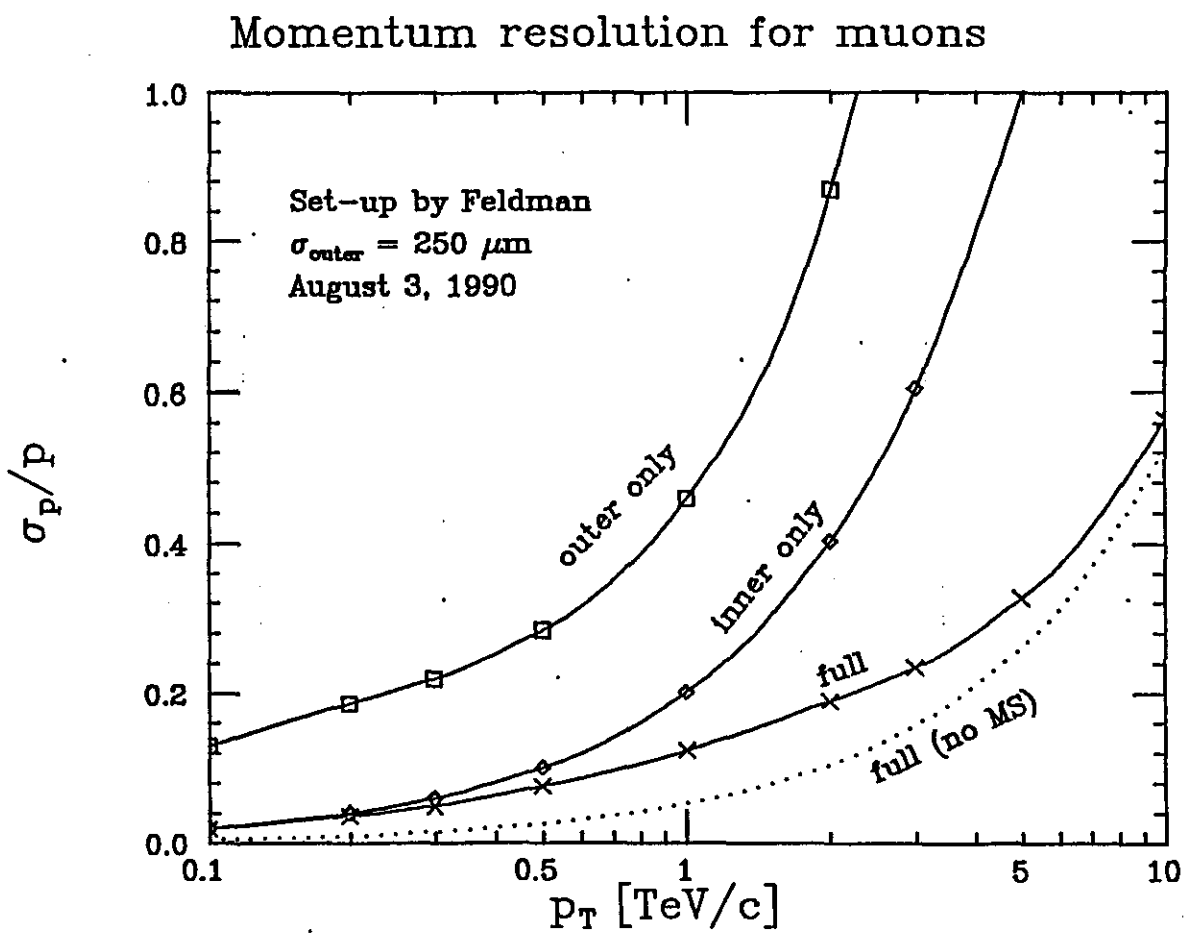
Momentum resolution for muons
Set–up by Feldman
σ_{outer} = 250 μm
August 3, 1990
outer only
inner only
full
full (no MS)
σ_p/p
p_T [TeV/c]
1.0
0.8
0.6
0.4
0.2
0.0
0.1
0.5
1
5
10

Fig. 2

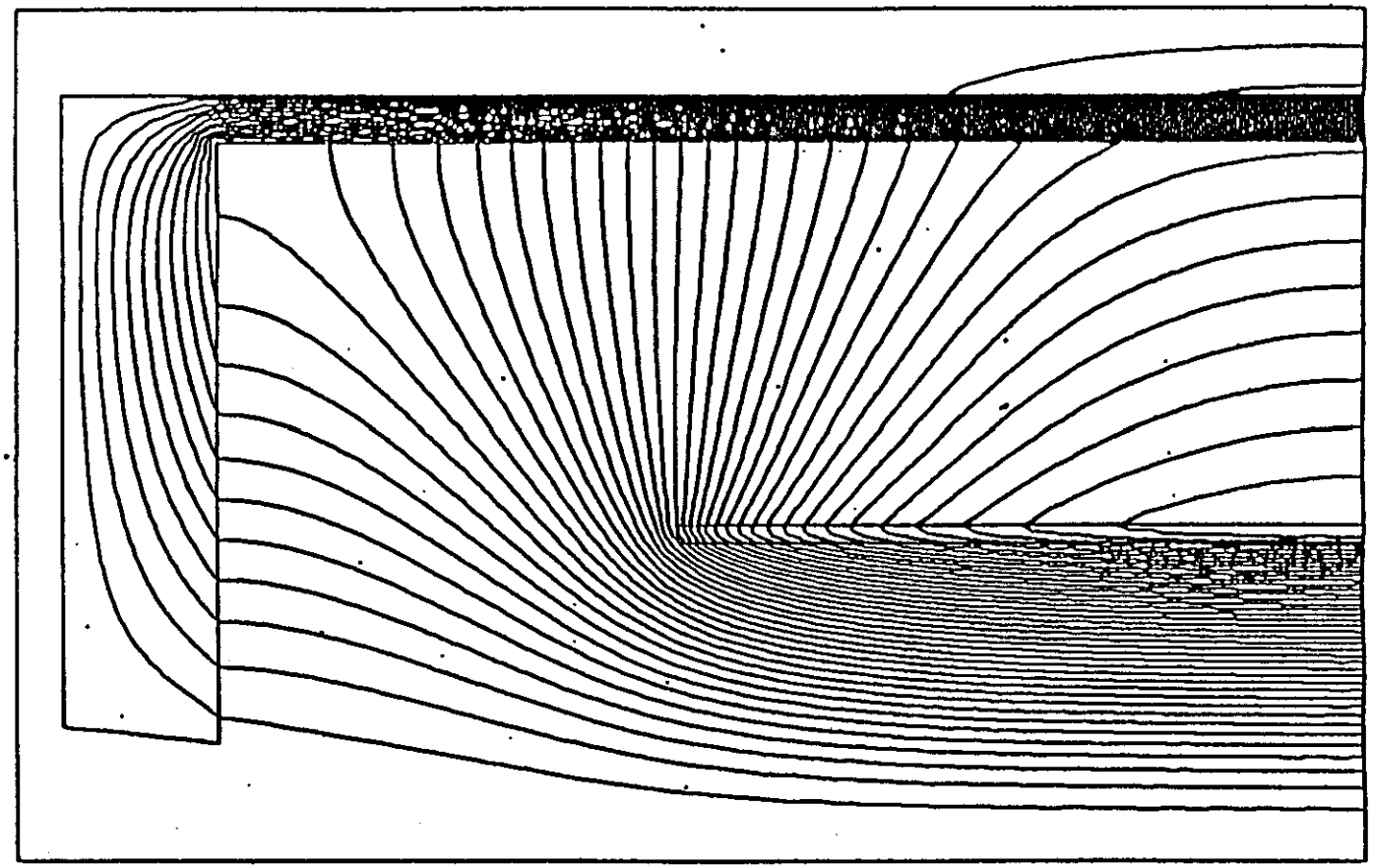

Fig. 3

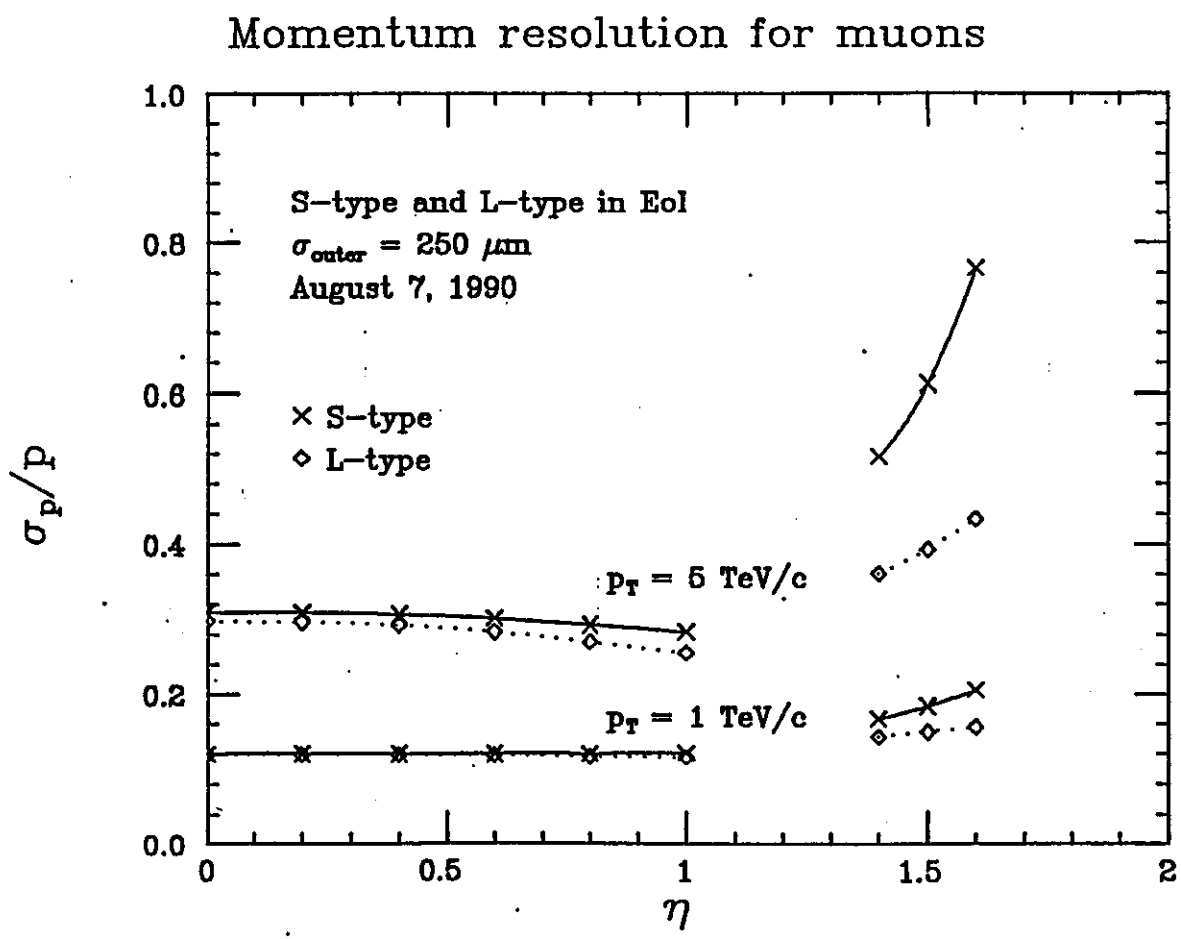
Momentum resolution for muons
S–type and L–type in EoI
σ_{outer} = 250 μm
August 7, 1990
× S–type
◇ L–type
p_T = 5 TeV/c
p_T = 1 TeV/c
σ_p/p
η
1.0
0.8
0.6
0.4
0.2
0.0
0
0.5
1
1.5
2

Fig. 4