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SOLENOIDAL DETECTOR NOTES

DIJET SPECTROSCOPY AT HIGH LUMINOSITY

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A study of the dijet mass resolution has been made appropriate to high luminosity operation. As a benchmark, the mass resolution of $W \to jj$ for a Higgs boson of 800 GeV has been optimized for no, eight, and sixteen overlapping minbias events. A factor of 2.5 degradation in M_{jj} width is seen.

Introduction

One question to address in considering high luminosity operation is whether or not jet spectroscopy is degraded in a major way. In order to study this question, the FNAL product called SSCSIM[1] was used to examine the mass resolution of dijets, dM_{jj}, for various clustering cone sizes, R, and cuts to the LEGO plot. The main question was, what limits are imposed on dijet spectroscopy by physics alone, independent of detector details? This being the case, all smearing of energies, transverse and longitudinal smearing and extent of showers, e/h effects, cracks, and clustering were turned off. Note that these effects do exist as options in SSCSIM, so that detector dependent effects may be studied in the future, if desired. The sole effect of experimental apparatus was to bin the LEGO plot in bins of size, $\delta y = \delta\phi = 0.05$. The claim that this binning has no effect was substantiated by running SSCSIM a few times with bins of size 0.025, and observing no change in the dijet mass distribution.[2]

Fragmentation Functions

In order to build up some feeling for the physics before jumping into the Monte Carlo world, one can begin by constructing a "hand calculation" of the possible effects of fragmentation on M_{jj}. One starts by looking at the mean multiplicity in e^+e^- collisions,[3] and observing that $\langle n \rangle \sim \ln(M)$, where M is the virtual photon mass. Assuming that $M \to q + \bar{q}$, then the parton momentum p is $\sim M/2$. Further assuming that the quark and antiquark fragment independently, then a 100 GeV mass M "decays" into 2 partons with $p = 50$

GeV in the rest frame, which each fragment into ~ 9 hadrons. These hadrons have a distribution of momenta k defined by the fragmentation function D(z). The kinematic limits are:

$$\begin{aligned} z_0 &< z < 1 \\ z_0 &\sim m_\pi/p \\ k &\equiv zp \quad . \end{aligned} \tag{1}$$

Given that <n> goes as ln(M), we need D(z) ~ 1/z. Since D(1) = 0, we assume a factor of $(1-z)^\alpha$ in D(z). The value for <n> is taken from e^+e^- collisions assuming that the parent parton momenta are as follows:

$$\begin{aligned} M &\to q + \bar{q} \\ p &\sim M/2 \sim \sqrt{s}/2 \quad . \end{aligned} \tag{2}$$

All that remains is to enforce a normalization. The function D(z) must satisfy two sum rules; the sum over all fragments is the mean multiplicity, and the sum over all daughter momenta is the parent momentum:

$$\begin{aligned} D(z) &= (\alpha + 1)\ (1 - z)^\alpha/z \\ \int zD(z)dz &= 1 \\ \int D(z)dz &= \langle n \rangle \\ &= (\alpha + 1)\ [\ln\ (1/z_0) - (1 + 1/2 + 1/3 + \ldots 1/\alpha)] \quad . \end{aligned} \tag{3}$$

The resultant function D(z) is shown in Fig. 1. Also shown are functions which are roughly equal to D(z) over some range of z. One finds that D(z) ~ 2/z works well at low z, while D(z) ~ 30 exp(- 5.5z) works well for 0.2 < z < 0.8.

One can then integrate D(z) in order to find at what z the first, second, etc. fragment occurs. This procedure wipes out the fluctuations, but is still useful in providing an average picture of the fragmentation process. The series in z_i is 0.617, 0.172, 0.088, 0.054, 0.033, 0.02, 0.012, ... This series sums to ~ 1.0, which confirms the momentum conservation sum rule in the case of discrete fragments. As regards fluctuations, one can assume that one fluctuates ± one unit in the fragmentation chain, i.e., i = i ± 1.

Consider now the decay of a W at rest into a u and d quark. These quarks will have ~ 50 GeV momentum. The quarks fragment into hadrons. Our previous work on D(z) allows us to state that each quark fragments into 9 hadrons on average. Assuming that the hadrons have a limited transverse momentum, ~ 0.7 GeV, with respect tBo the parton axis, then the vector momentum of the quark is roughly just the scaler sum of the longitudinal momenta of the hadrons directed along the direction of the parent quark.

$$M^2 = 2p_q p_{\bar{q}}(1 - \cos\theta_{q\bar{q}}) \qquad (4)$$

$$p \sim \sum_i k_i, \sum_i z_i = 1.$$

As seen in Fig. 2a, there is a distribution of momenta for the 9 fragments of the quark. Typically, only 5 of the hadrons have momenta larger than the background due to minbias, kt ~ 0.7 GeV. In Fig. 2b, is plotted the angle of the fragment with respect to the quark axis, assuming kt = 0.7 GeV and using $k_{||} \sim k = zp$. Clearly the fragments with higher fragment number have both large angles and low momenta. This implies that fragments with number greater than ~ 5 will be lost in the confusion of soft tracks from the underlying event. Note that this conclusion has nothing to do with detector specifics.

Using the results shown in Fig. 2, we can estimate the errors on M_{jj} caused by this confusion of soft tracks between fragments and underlying event tracks. Define a cone clustering size for the jets as follows:

$$R = \sqrt{\delta y^2 + \delta\phi^2} \quad . \qquad (5)$$

Using Fig. 2b, we can truncate the series shown in Eq. 4, and calculate the mass of the pair of resultant sums. The deviation of this truncated mass from the generated mass is an indication of the mass error caused by using a finite

It is also instructive to make hand estimates for W boosted by being decay products of heavy parents. For example, a 800 GeV Higgs boson yields Ws whose truncation error is equal to its natural width at a smaller cone radius of R ~ 0.3. One then expects boosted Ws to be more robust in the presence of backgrounds.

Obviously, with no background, one can just make an arbitrarily large cone radius. However, as R increases, the number of spuriously included tracks increases. Assuming a "plateau" height of 8 tracks per unit of rapidity, uniformly distributed in ϕ, then for W at rest, the momentum lost to truncation is equal to the momentum gained from spurious tracks when R ~ 0.3 if 20 minbias events (kt ~ $\langle kt \rangle$ = 0.7 GeV) are overlapped with the decaying W. Obviously, boosted W are less susceptable, as they are boosted out of the minbias zone of confusion for more of their fragments than W at rest. These simple estimates are useful in understanding overlapped events as we will see later.

Low Transverse Momentum W Dijet Mass Reconstruction

Fortified by some preliminary estimates of the magnitude of the effects, one can plunge into SSCSIM. The first sample looked at was W from decays of 150 GeV top quarks. First, one looks at the reconstructed mass, M_{jj}, using all hadronic fragments within a cone of radius R. In all of what follows, in order to remove the finite width of the W, one plots the ratio of the reconstructed to the generated mass:

$$\delta = Mr/Mg. \tag{6}$$

Since the resulting distributions are rather non-Gaussian, one adopts the convention of defining the standard deviation to be the FWHM/2.4. The resulting histogram of δ for R = 0.5 using only hadrons from W fragments is shown in Fig. 3. Note that δ is almost always < 1, since one can only lose hadrons in this case, and hence can only underestimate M_{jj}.

In order to make contact with the previous estimates, in Fig. 4a is shown the shift in the mean of M_{jj}, $\langle\delta\rangle$ - 1, as a function of R when only hadrons from the fragmentation of W are used. The Monte Carlo results are compared to the hand estimates. In both cases only losses can occur, so that $<\delta> - 1$ is always negative. The agreement is adequate. Assuming that the fluctuations in the losses are ~ the losses, one can plot in Fig. 4b the standard deviation of δ for both the hand estimate and the results of SSCSIM. Again the agreement is not bad. The Monte Carlo confirms that only for $R > 1.0$ will the error be less than the error due to the finite width of the W state.

The next step is to find if one can indeed use $R > 1.0$, given the confusion of soft fragments and soft tracks from the event itself and/or from events which overlap the temporal resolution of the detector. The idea is to play losses off against spurious gains so as to optimize the resolution. As we will see, this is a complex process; the optimal value of R and the best resolution depends on the boost of the jet, the number of overlapping events, and the cuts one imposes on entries in the LEGO plot.

For slow W, the shift in the mean of δ is shown in Fig. 5a. The three curves are for W fragments only, for all tracks in the event, and for all tracks with $kt > 1.0$ GeV. Clearly for only W fragments $<\delta> - 1$ is always negative. For all tracks, spurious gains are ~ losses for R ~ 0.45. By attempting to kill soft tracks (at the expense of truncating real fragments) with a kt cut,[4] one shifts the radius where losses ~ gains out to R ~ 0.6. The resulting error in δ is shown in Fig. 5b for the three conditions. Without the kt cut, there is a minimum (optimal) resolution of ~ 0.05 at R ~ 0.5. The kt cut allows us to push out to R ~ 0.7 with a minimal resolution of ~ 0.035. In either case, one is unable to approach the natural width scale (resolution ~ 0.01) which occurs at R ~ 1.0. As stated above, this conclusion has nothing to do with detector specifics; it is merely kinematics. As an example, the distribution of δ is given in Fig. 6 for $R = 0.5$ and with the kt cut imposed. Looking at Fig. 3, one can obviously see the degradation in shape which comes unavoidably when the tracks from the entire event are included.

High Transverse Momentum W Dijet Mass Reconstruction

In an attempt to avoid this increase in resolution, one can look at boosted Ws. In what follows, a sample of events from a Higgs boson of mass 800 GeV was used. One expects from elementary considerations, that problems arising from confusions with soft, unassociated tracks will be alleviated in this case with respect to problems with slow Ws.

By way of comparison, in Fig. 7a is plotted the error in δ as a function of R for the Ws from 150-GeV top and from 800-GeV Higgs. Clearly, as expected, the boosted Ws have smaller errors (at the same R) than the slow Ws. As seen in Fig. 7b, when all tracks in the Higgs event with kt > 1.0 GeV are included the error, at all R, is less than the error for slow Ws from 150 GeV top. This result is not surprising, since the boosted Ws have more fragments above the soft kt cut by virtue of their boost. Hence one expects reduced errors at low R (the jet size is smaller) and at large R (the kt scale for W fragments is higher). In this case the optimal resolution of ~ 0.035 occurs for R ~ 0.7. The distribution of δ for R = 0.5 and with the kt > 1.0 GeV cut imposed on all tracks is shown in Fig 8. A glance at Fig. 6 is sufficient to convince one that the resolution in δ is indeed improved for boosted W.

High Luminosity Overlaps For W From H(800)

Given the problems with just the event itself, what problems arise if there are multiple events overlapping within the time resolution of the detector in question? This question must be addressed for slow detectors and/or for detectors which seek to operate at luminosities beyond the design value. It is clear that a luminosity in excess of $10^{34}/(\mathrm{cm}^2\mathrm{sec})$ is feasible from an accelerator standpoint. It is equally clear that such an elevated luminosity is needed if one is to push 2 gauge boson amplitudes to the unitarity limit of mass scales of a few TeV.[5]

First consider the transverse energy density appropriate to a minbias event (or ISAJET 2 jet event with pt ≥ 3 GeV). The "plateau" height for the sum of charged and neutral particles is ~ 8.

$$1/\sigma(d\sigma/dy) \sim 8. \tag{7}$$

This means there are 48 tracks in a detector spanning ± 3 units of rapidity, y. Taking cell sizes in rapidity and azimuthal angle;

$$\delta y \ \delta\phi = (0.05)^2, \tag{8}$$

one finds 0.003 tracks/cell. Assuming kt ~ <kt> ~ 0.7 GeV/track, then one has 30 GeV/event. However this is spread over the ~ 15,000 cells, so that one has only 0.002 GeV/cell. If 20 minbias events are overlapped [as is the case, on average, for a luminosity of $10^{34}/(\mathrm{cm}^2\mathrm{sec})$], then one finds 960 tracks. However, this ~ 600 GeV of transverse momentum causes only 0.06 tracks/cell, or 0.04 GeV/cell. It is this sort of calculation which leads us to believe that the pileup is small; we can put a cell threshold of kt = 1.0 GeV per cell which will remove the majority of the minbias tracks, and retain all W fragments above that threshold. The real question is the fluctuations in the background. To address that question requires a Monte Carlo study.[6]

In order to study high luminosity dijet spectroscopy, the Higgs(800) data was selected as a signal sample. Background was chosen to be dijets with a 3 GeV threshold. These events were overlapped, cuts were applied, and the jet 4 vector within a cone size R was computed. Finally the mass of the dijet was computed, M_{jj}, and compared to the generated W mass. The results for a kt cut of 1.0 GeV are shown in Fig. 9a. The error as a function of R is shown for only W fragments used, for all tracks in the event, for 8 additional overlapped events, and for 16 additional events. Obviously there is no optimal R for the first case. For R > 1.0, however, the error is comparable to the error due to the natural width, so that larger R values are useless. As seen before, the total event has a minimum at R ~ 0.8, with an error ~ 3 times the error due to the natural width. With 8 overlapped events, the optimal R occurs at ~ 0.5, while for 16 overlaps the minimum occurs at R ~ 0.3. Clearly, the case with 8 overlapped events has a degraded error with respect to the single event. The minimum error is ~ 0.05, while that for 16 overlaps is ~ 0.1.

It may be that the kt threshold is not optimized. To explore this possibility, the cut was changed to kt > 2.0 GeV. The results analagous to those of Fig. 9a are shown in Fig. 9b. Obviously the <n> = 1 and <n> = 9 cases are not improved. However, one has some improvement in the <n> = 17 case; the minimum error location is pushed out to R ~ 0.5, and the minimum error is reduced to a value ~ 0.08. In fact, the present study indicates that the kt cuts which are indicated are close to the optimal cuts.

Roughly optimal mass distributions are shown in Fig. 10. All have R = 0.5. In Fig. 10a is shown the $\langle n \rangle = 1$ case, with a kt cut of 1.0 GeV. In Fig. 10b, is shown the $\langle n \rangle = 9$ case, also with kt > 1.0 GeV. Some minor worsening is observed. Finally, Fig. 10c shows the $\langle n \rangle = 17$ case with kt > 2.0 GeV. Clearly, the error is worsened, making dijet spectroscopy more difficult at elevated luminosities or with slow detectors.

Detector Effects

As stated above, it is expected that the dijet mass resolution is dominated by kinematics for any reasonable detector. A detailed study of such effects is beyond the scope of this note. However, some simple hand calculations make this assertion plausible. Referring to Eq. 4, one can differentiate with respect to k_i in order to look at the effects of calorimetric resolution. One finds that;

$$dM/M = dk_i/2p. \tag{9}$$

Thus the leading (i = 1) fragment should dominate the resolution. Since that fragment is fast, the "constant term" in the resolution should dominate, $dk_i/k_i \equiv a/\sqrt{k_i} + b$. If that is so, then,

$$dM/M = bz_1/2. \tag{10}$$

For example, if dM/M is no better than 3% (see Fig. 7) then ($z_1 = 0.62$) the energy resolution will contribute equally if the constant term is ~ 10%. Since most detectors aim for a much smaller resolution, energy smearing is not relevant here.

In a similiar vein, the cell granularity is also not expected to be a problem:

$$dM/M = d\theta_{q\bar{q}}/[\tan\ (\theta_{q\bar{q}}/2)]. \tag{11}$$

Given that the opening angle is large, and that $d\theta_{q\bar{q}} \sim dy = d\phi$ one expects little effect for cells sizes equal to or less than those considered in this note. These small cell sizes are needed to reduce pileup effects in any case.

Summary and Conclusions

A summary of the detector independent conclusions for dijet spectroscopy are given below.

a. Hand estimates give useful guidance and insight to the problem and confirm the Monte Carlo results.

b. Low transverse momentum Ws have errors comparable to those due to their natural width for R ~ 1.0 if only W fragments are used to compute M_{jj}.

c. When all the tracks in the event are used, the minimum error for low pt Ws occurs at R ~ 0.5 with a kt > 1.0 GeV cut. For W due to H(800) decays, the best error of 0.035 occurs at R = 0.7.

d. The optimum clustering radius R depends on the pt of the W and on the number of overlapping events. The cut to suppress minbias tracks also depends on the number of overlapping events. For the H(800) sample, for $<n>$ = 1, kt > 1.0 GeV, the optimum is R ~ 0.7, error ~ 0.035. For $<n>$ = 9 it is R ~ 0.5, error ~ 0.05. For the case of $<n>$ = 17, the cut must be raised to kt > 2.0 GeV, in order to achieve an optimum at R ~ 0.5, with error ~ 0.08.

e. Dijet spectroscopy is worsened by overlapping events. At a luminosity of $10^{34}/(cm^2 sec)$ for a detector that can resolve one SSC bunch (16 nsec) or for a detector which integrates over 16 crossings (256 nsec) at design luminosity, one loses in resolution by a factor of 0.08/0.035 ~ 2.5. This loss is with respect to operation at design luminosity in the former case, and with respect to optimal operation in the latter case.

References

[1]SSCSIM was largely written by A. Beretvas and A. Para. Documentation is available through them.

[2]W. Wu has studied t mass resolution, and A. Para has studied W mass resolution. The results will appear as an SDC/FNAL Note.

[3]Review of Particle Properties, Particle Data Group.

[4]A. Bay et al., proceedings of the Summer Study on High Energy Physics in the 1990s, ed. S. Jensen, World Scientific, 882 (1988).

[5]M. Chanowitz, "Electroweak Symmetry Breaking: Unitarity, Dynamics, and Experimental Prospects," Ann Rev Nucl Part Sci, 38, 323 (1988).

[6]J.H. Mulvey, CERN 88-02 (1988).

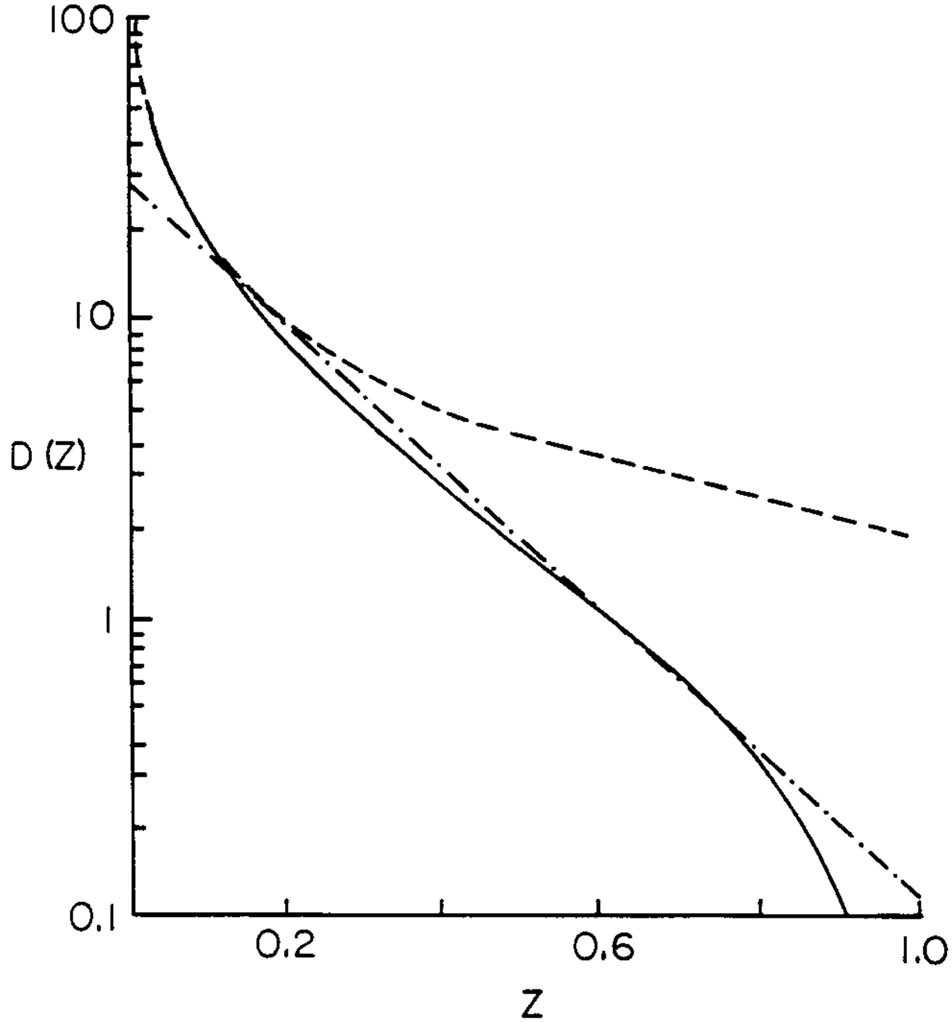

Fig. 1. Simplified model for D(z) as a function of z(solid). Approximate functions 2/z (dashed) and 30 exp(- 5.5z) (dashed-dotted) are also plotted.

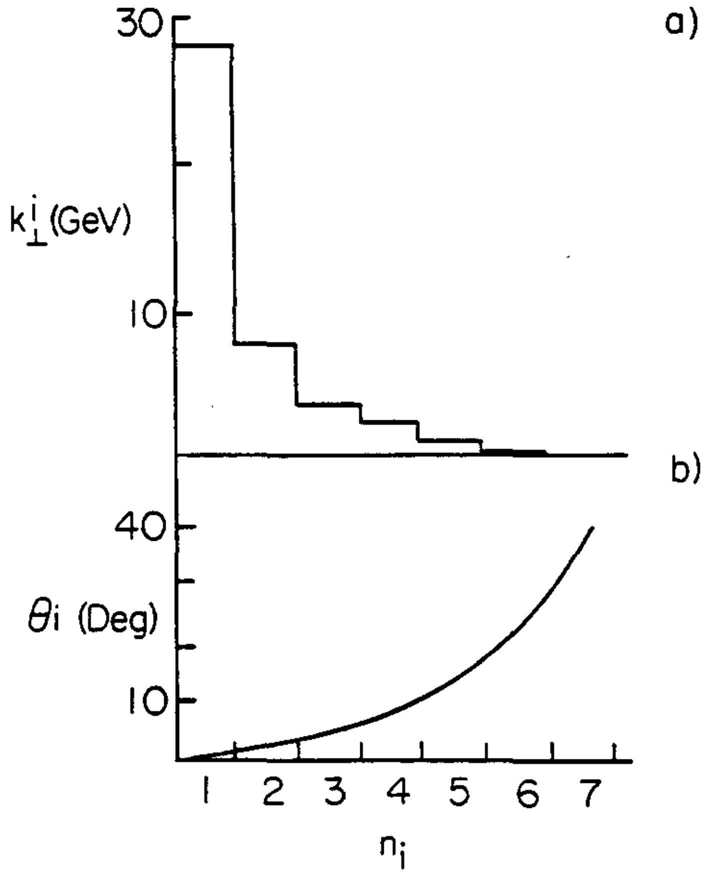

Fig. 2a. Histogram of fragment momentum as a function of fragment number.

Fig. 2b. Plot of fragment angle with respect to the parton direction as a function of fragment number.

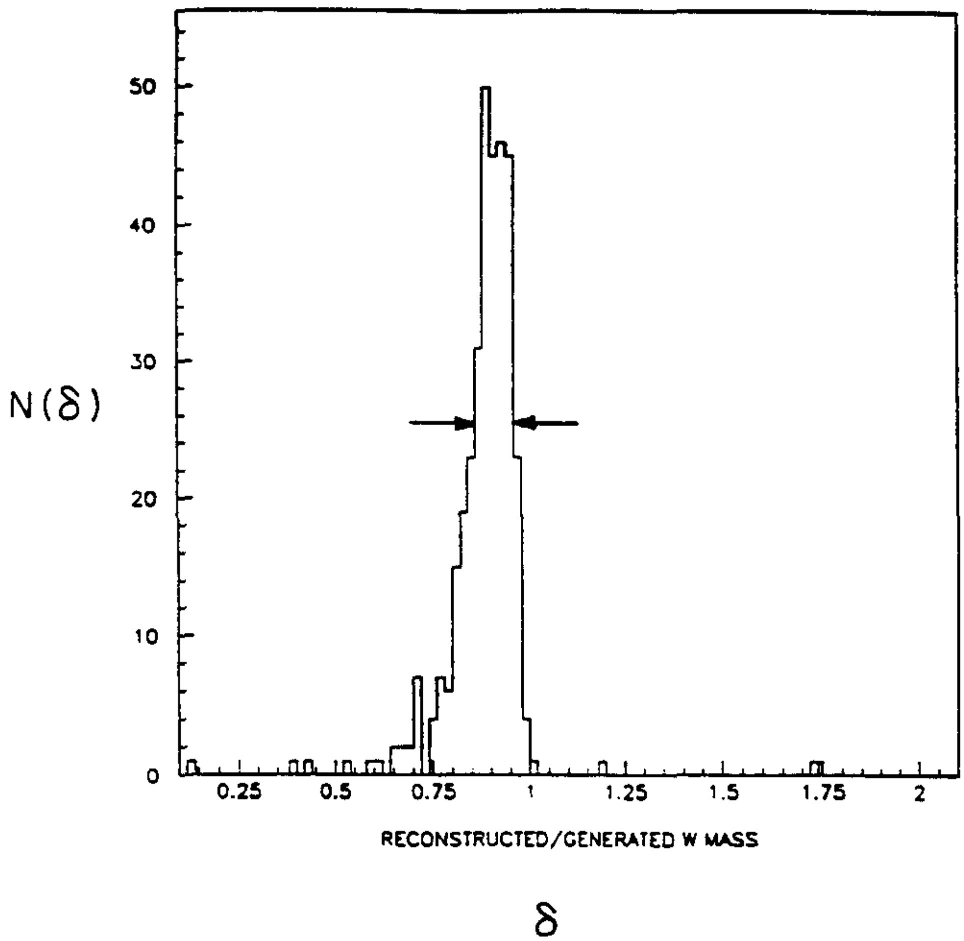

Fig. 3. Histogram of δ for low pt W events, R = 0.5, and only W fragments are used.

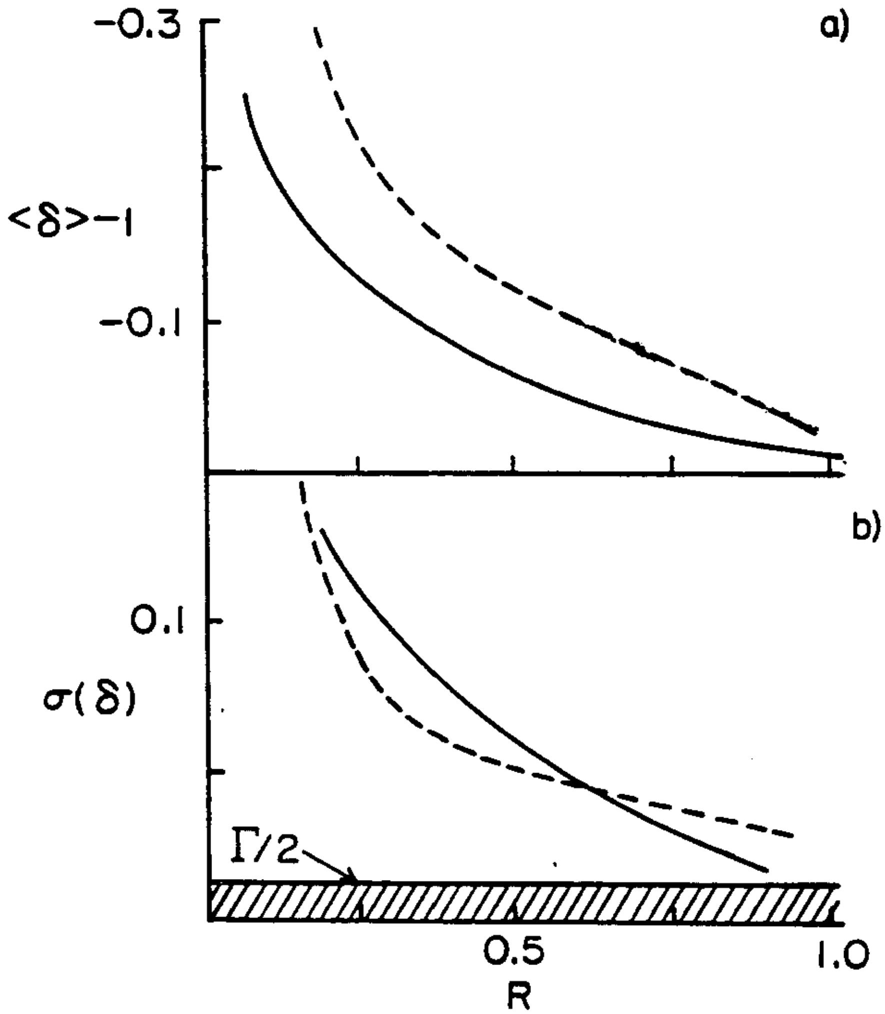

Fig. 4a. Plot of $<\delta> - 1$ as a function of R. The curves are a hand calculation (solid) and the SSCSIM results (dashed). Only W fragments are used.

Fig. 4b. As in Fig. 4a except the standard deviation of δ is plotted as a function of R. Also shown is the scale set by the natural width Γ.

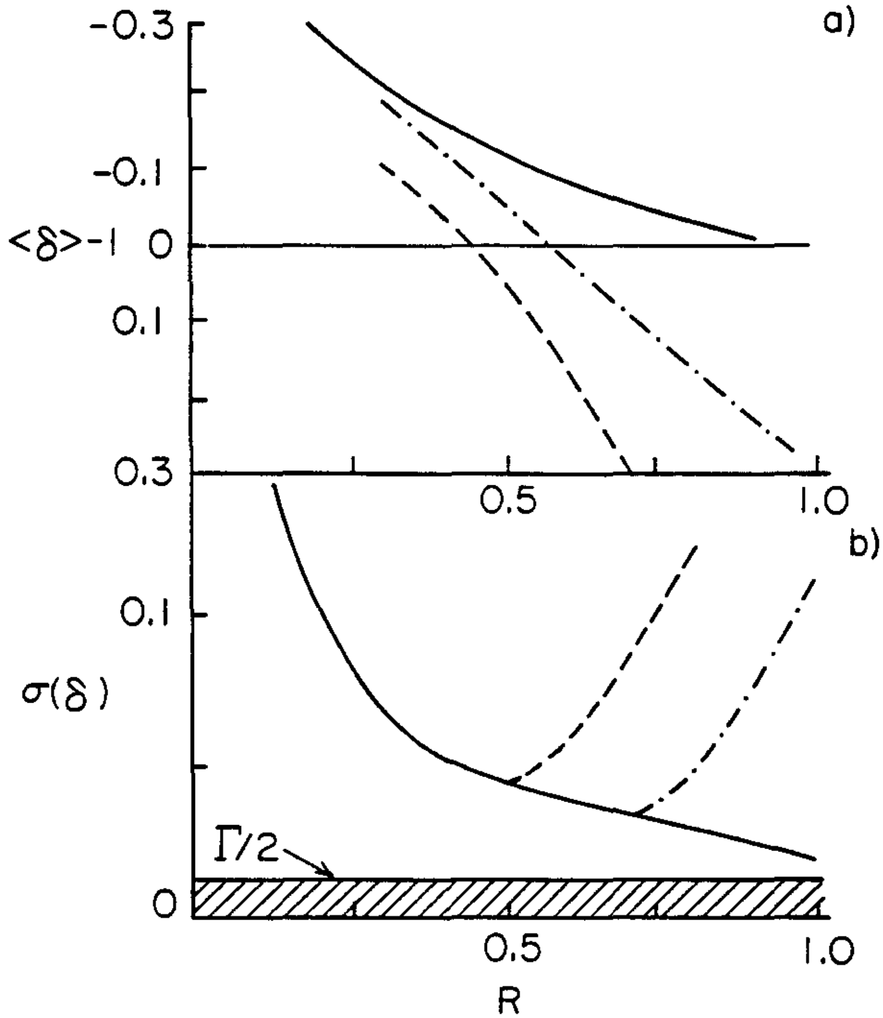

Fig. 5a. Plot of $<\delta>$ - 1 as a function of R using only W fragments (solid), all tracks in the event (dashed), or all tracks with kt > 1 GeV (dot-dashed).

Fig. 5b. As in Fig. 5a except the standard deviation of δ is plotted as a function of R.

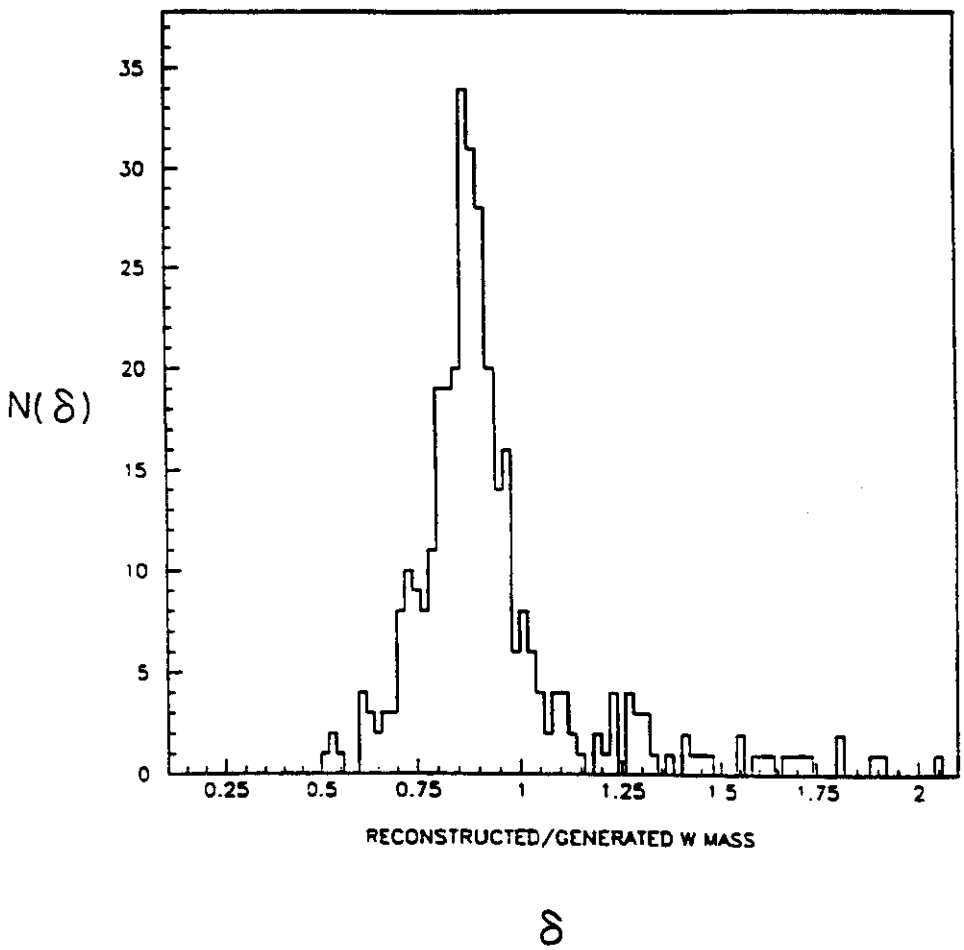

Fig. 6. Histogram of δ for R = 0.5 and low pt W events when only tracks with kt > 1.0 GeV are used to calculate M_{jj}.

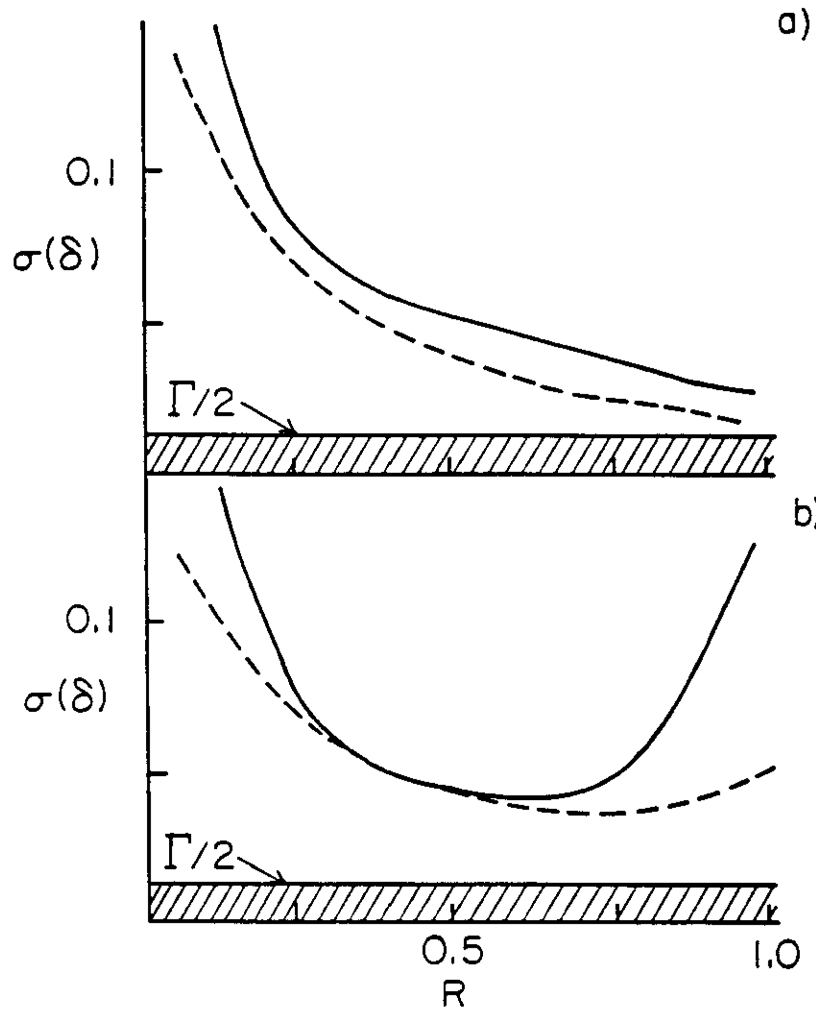

Fig. 7a. Plot of the standard deviation of δ as a function of R for low pt W events (solid) and for the H(800) source of Ws (dashed). Only fragments of the W are used.

Fig. 7b. As in Fig. 7a. except that all tracks in the event are used if they have kt > 1.0 GeV.

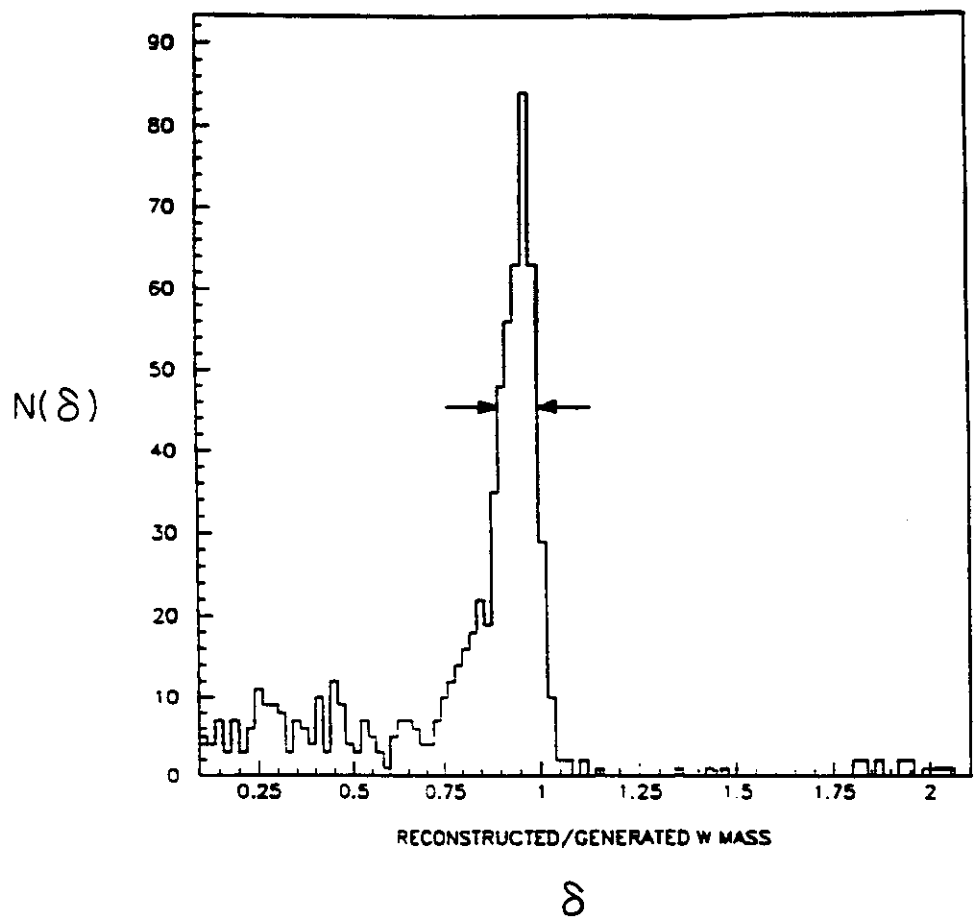

Fig. 8. Histogram of δ for $R = 0.5$ and high pt events from H(800). All tracks in the event are used if they have kt > 1.0 GeV.

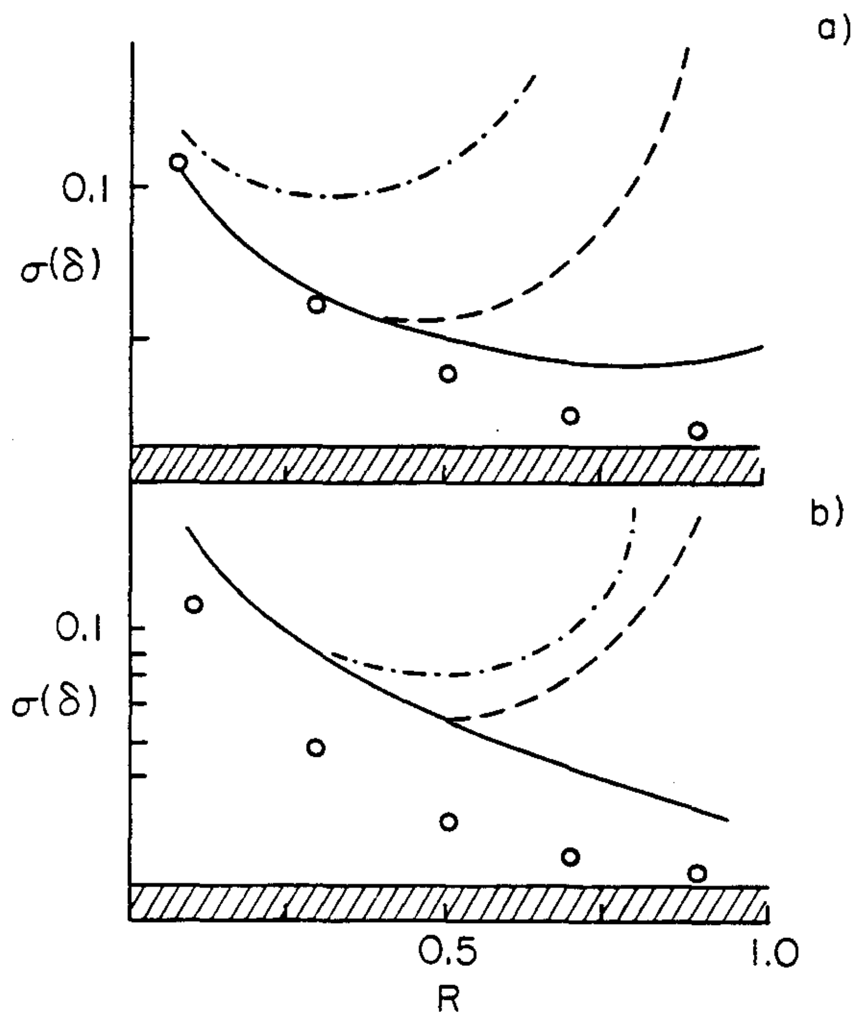

Fig. 9a. Plot of standard deviation of δ as a function of R for H(800) events keeping only tracks with kt > 1.0 GeV with no additional overlap events (solid), 8 overlap events (dashed), and 16 overlap events (dot-dashed). The results using only W fragments are shown as o symbols.

Fig. 9b. As in Fig. 9a except that only tracks with kt > 2.0 GeV are allowed.

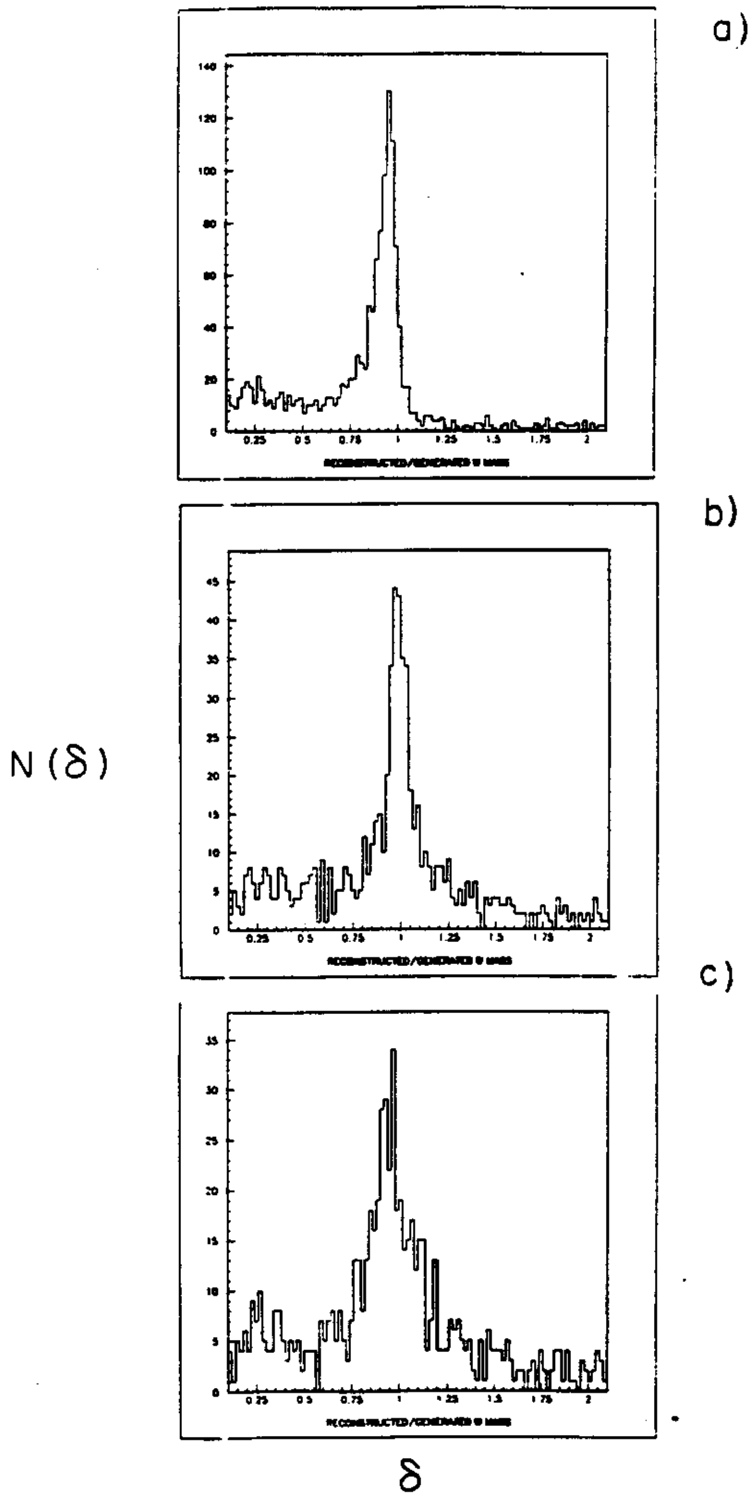

Fig. 10a. Histogram of δ for R = 0.5. The number of overlap events is zero, and only tracks with kt > 1.0 GeV are used.

Fig. 10b. As in Fig. 10a except there are 8 overlap events.

Fig. 10c. As in Fig. 10b except there are 16 overlap events and only tracks with kt > 2.0 GeV are used.