SSC-SDE-20

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SOLENOIDAL DETECTOR NOTES

HADRONIC SHOWER SHAPES IN DEPTH FROM CCFR DATA
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Abstract

We find an easy, physical parameterization of the average depth development shape of π^--induced hadronic showers in the CCFR Fe-Scintillator calorimeter at 25, 50, 90, 140 and 250 GeV.

Data sample

The CCFR data [1] consist of measurements of pulse height in each of the 28 depth samples in this calorimeter exposed to a π^- beam at energies of 25, 50, 90, 140 and 250 GeV. The calorimeter mass consists of 28 modules, each module consists of an Fe-scintillator-Fe sandwich, and both the Fe plates and the liquid scintillator volume are 2-inches thick. This sums to about 19 λ_p, and is therefore very deep on the scale of any SSC calorimeter. Each liquid scintillator samples the debris from 4 inches of upstream Fe and 2 inches of scintillator, which is 0.669 λ_p or 5.90 X_0. This is uniform throughout the calorimeter except for the first scintillator layer, which samples the debris from only 2 inches of upstream Fe and 2 inches of scintillator. There is no transverse shower information, and essentially no transverse shower leakage in these 3 meter wide modules.

Analysis

We want a function which represents the average shower shape for hadrons interacting at $\lambda=0$. Starting from CCFR data, we must first estimate the interaction point of the π for each event, and then shift the data to $\lambda=0$, and accumulate for the average. The conversion point is estimated by assuming the shower population rises exponentially from the single minimum-ionizing particle level before conversion. Figure 1(a,b) shows an event with a pion interaction between gaps 3 and 4, while Figure 2(a,b) shows a (rare) event which interacts much later between gaps 11 and 12. The distribution of pulse heights per gap before the peak of the shower is shown in Figure 3, displaying a clear minimum ionizing peak at about E_{min} = 0.20 GeV per gap crossing. Then we can write the shower particle population at the start as

$$dE/d\lambda = E_{min} e^{+\lambda/a}, \tag{0.1}$$

where a is estimated from the maximum pulse height observed in a gap following a minimum ionizing gap. If the conversion point is λ_c, the absorption length depth of each Fe-Scint-Fe sandwich is $d=.669\lambda_p$, and the depth of the scintillator is $s=.064\lambda_p$, then the expected pulse height in the next gap is

$$E_{next\ gap} = E_{min} \int_{d-s}^{d} e^{(\lambda-\lambda_c)/a} d\lambda. \tag{0.2}$$

Solving this for λ_c gives the conversion point which, as expected, is approximately uniformly distributed between gaps. Using this conversion point, all showers are

[1] F.S. Merritt, et al., NIM **A245** (1986)27.

shifted to start at $\lambda=0$, each $d=.669\lambda$ data bin is subdivided into 10 smaller bins, the energy is apportioned among the 10 bins proportional to a crude shower shape from a previous pass over the data, and a direct average is taken.

Shower Shape Function

By inspection, these average shower shapes at E = 25, 50, 90, 140 and 250 GeV all rise exponentially, turn over at a height proportional to E, then slowly fall exponentially. This is just what simple shower theory suggests. So the rising exponential can be written as

$$f_1 = E_{min} e^{+\lambda/\lambda_1} \tag{0.3}$$

and the falling exponential as

$$f_2 = [\frac{E}{n_\pi m_\pi c^2}] e^{-\lambda/\lambda_2}, \tag{0.4}$$

where the factor of $m_\pi c^2$ is included so that n_π will come out to be of order 10, or so. E_{min} is 0.20 GeV as before, E is the incident π^- energy, n_π is to be fitted, and λ_1 and λ_2 are the rising and falling e-folding lengths, also to be fitted. The factor $n_\pi m_\pi$ is like the hadronic equivalent of the "critical energy" in an electromagnetic shower, i.e., the energy at which energy loss becomes more important than further pion production. Note that the linear factor E in the more slowly falling exponential, when matched to the rising exponential, guarantees that the depth position of shower maximum increases logarithmically with E. The above functions are in units of energy lost per gap, which is $d=.669\ \lambda_p$ in this device, whereas a general function ought to be in genuine units of GeV per absorption length. Therefore, both pieces should be divided by d.

One way to match these two functions at shower maximum is to take their product over their sum (like adding resistors in parallel), resulting in the shower shape function we use here

$$\frac{dE}{d\lambda} = \frac{1}{d} \frac{f_1 f_2}{f_1 + f_2}, \tag{0.5}$$

where f_1 and f_2 are given above. The parameters λ_1, λ_2, and n_π are all, as expected, nearly energy-independent. In order to avoid numerical overflow in computing e^{λ/λ_1} for large λ, I have added $\lambda/50$ to λ_1 in the denominator. There is a small positive curvature at small λ at all energies. This may be due to a rising average total cross section for hadrons as the average particle energy in the shower decreases. It may also have a contribution from early diffractive scatters. These effects lead to a non-exponential shower build-up. To cure this (small) discrepancy, I have added a quadratic term to the rising exponential which becomes linear after a distance of λ_3. The results of these fits are shown in Figures 4(a-e) for all five energies. The energy dependence of the parameters in the fitted function is shown in Figure 5. Thus, the final hadronic shower shape function can be written as

$$\frac{dE}{d\lambda} = \frac{[E_{min}/d][E/n_\pi m_\pi] e^{+\lambda\xi/\lambda_1} e^{-\lambda/\lambda_2}}{E_{min} e^{+\lambda\xi/\lambda_1} + [E/n_\pi m_\pi] e^{-\lambda/\lambda_2}} \tag{0.6}$$

where the factor ξ in the rising exponential takes care of the round-off problem and the non-exponential early rise, and is given by

$$\xi = \frac{\lambda_1}{(\lambda_1 + \lambda/50)} \frac{\lambda}{(\lambda + \lambda_3)} \approx 1. \tag{0.7}$$

The other parameters are easily represented by expressions which are nearly constant in incident hadron energy from 25 to 250 GeV,

$$\lambda_1 = .035[ln(E) - E/150.] \quad \approx .13 \tag{0.8}$$

$$\lambda_2 = .24ln(E/.1) \quad \approx 1.57 \tag{0.9}$$

$$\lambda_3 = .03ln(E/.1) + 250./E^2 \quad \approx .25 \tag{0.10}$$

$$n_\pi = 1.3ln(E/.1) \quad \approx 9.5, \tag{0.11}$$

and where, finally, E_{min}=.20 GeV and d=.669 λ_p are constants which convert measurements in this particular device to units of GeV per proton absorption length. Average values of λ_1, λ_2, λ_3, and n_π can be used, instead of these weakly energy-dependent forms. (These forms are just arbitrary functions cooked up to take out the last bit of energy variation in the shape function.)

I have compared the shape obtained at 140 GeV here with a very similar measurement by Holder, et al., for their 140 GeV π shape in an Fe calorimeter. [2] The shapes are very close, *except* that the estimated starting point for my showers are earlier than Holder, et al. by about 10 cm of Fe, or about 0.6 λ_p. This comparison is shown in Figure 6. I think that my estimate of the shower starting point is OK, and that Holder, et al., have shifted their zero point. This distance of 0.6 λ_p is about the quadratic start-up distance, so hopefully I haven't made a blunder here.

FORTRAN Code

The following code computes the shower shape in depth as a function of λ, in unit of proton absorption lengths, and as a function of incident hadron energy E, in units of GeV. The resulting function dE_dλ(E,λ) is in units of GeV of energy loss per proton absorption length of Fe.

```
Function dE_dλ(E,λ)          ! Equivalent shower energy per λp
Real E, λ, nπ
Parameter d=.669, m=.140, Emin=.20
λ1 = .035 * ( alog(E) - E/150.)
λ2 = .24 * alog(E/.1)
λ3 = .03 * alog(E/.1) + 250./E**2
nπ = 1.3 * alog(E/.1)
ξ = λ1 / (λ1 + λ/50) * λ / (λ + λ3)
f1 = Emin * exp( + λ*ξ/λ1 )
f2 = E / (nπ * m) * exp( - λ/λ2 )
dE_dλ = (f1 * f2 / d) / (f1 + f2)
return
end
```

[2] Holder, M., et al., NIM 108 (1973) 541.

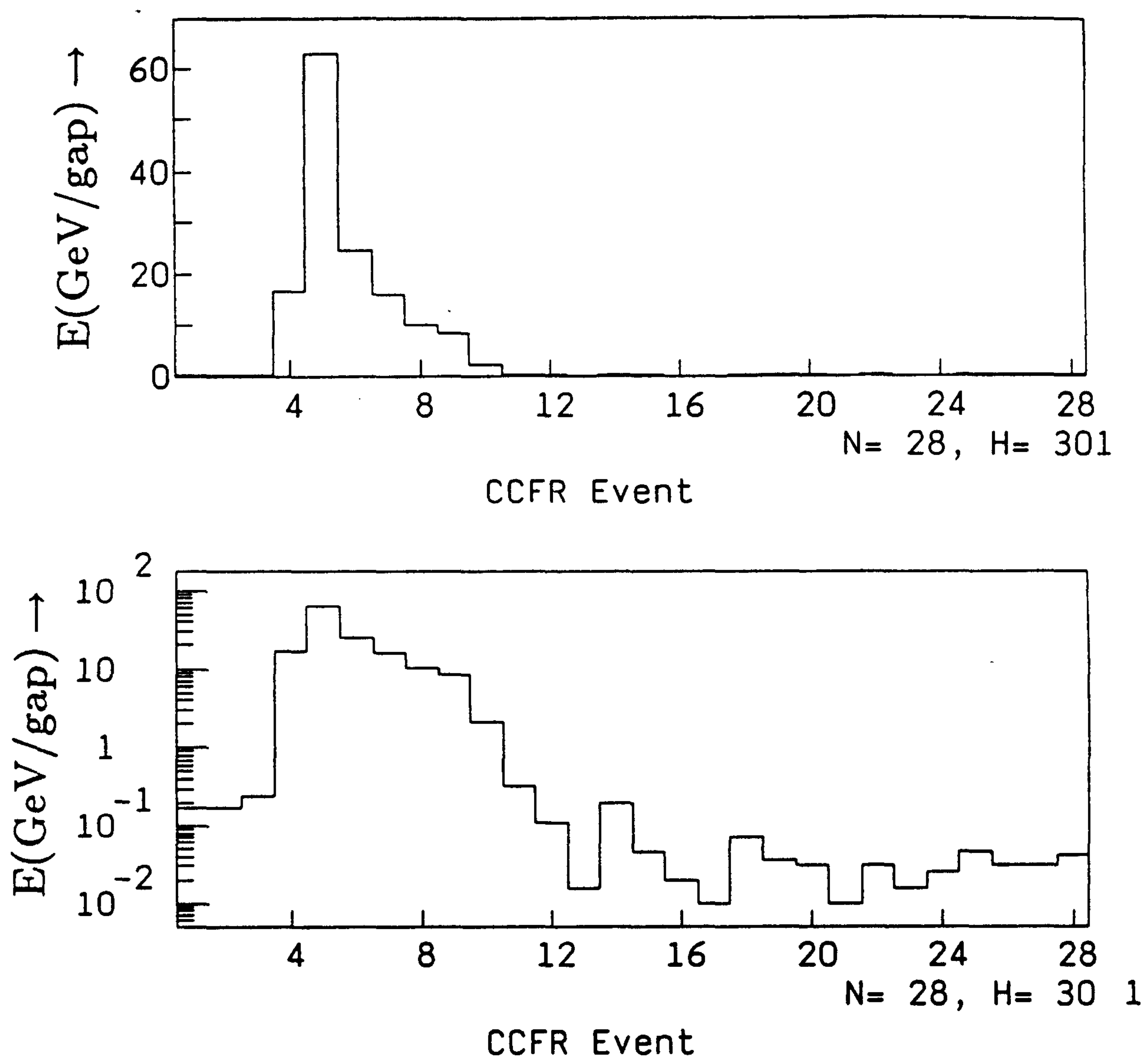


Figure 1. (a) The individual pulse heights in each of the 28 gaps for a 140 GeV π^- interacting between gaps 3 and 4. (b) The same data, but on a log scale to make the very low, minimum ionizing pulse heights at 0.2 GeV/gap visible.

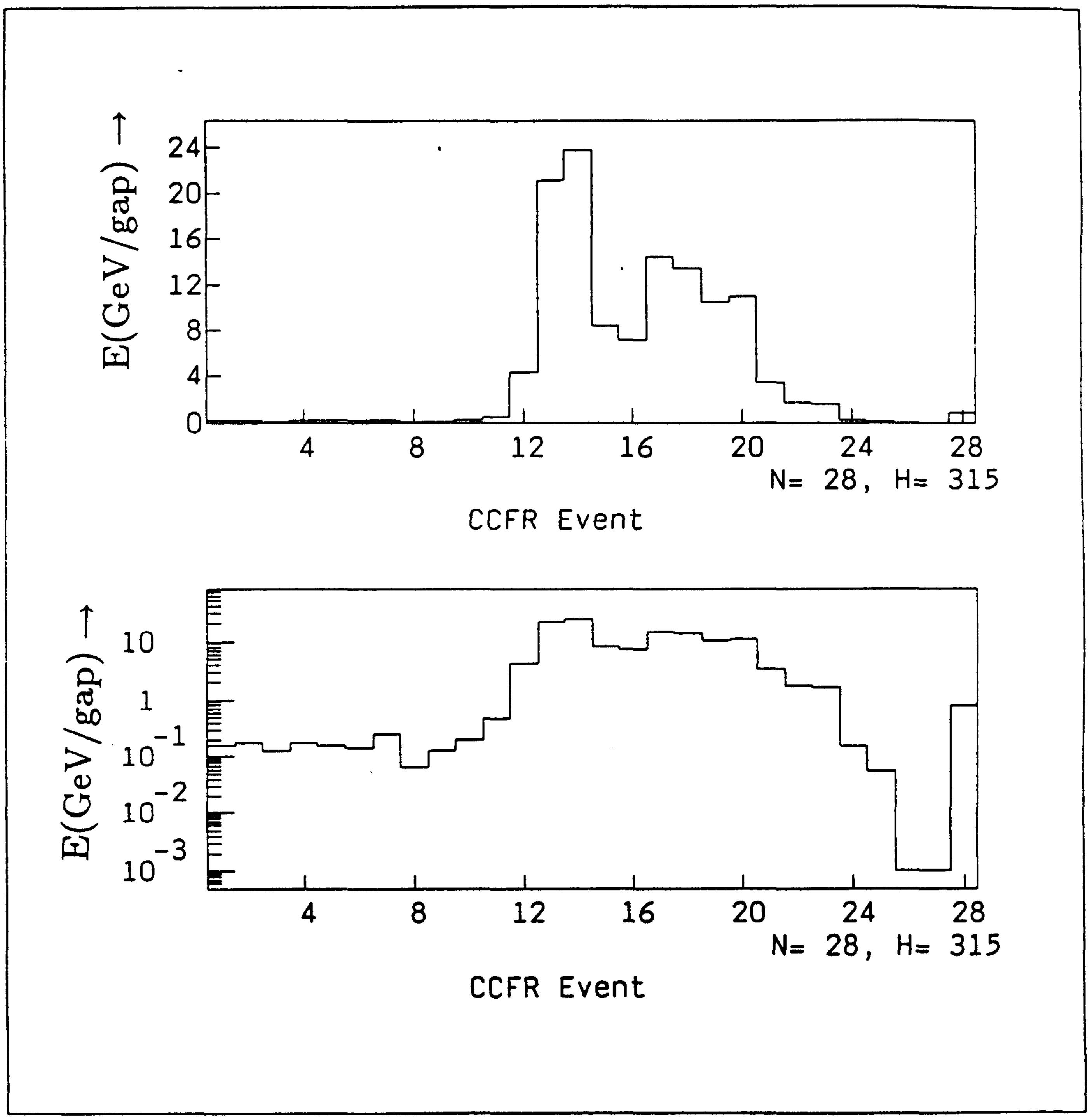


Figure 2. (a) The individual pulse heights in each of the 28 gaps for a 140 GeV π^- interacting late in the calorimeter, between gaps 11 and 12. (b) The same event on a log scale.

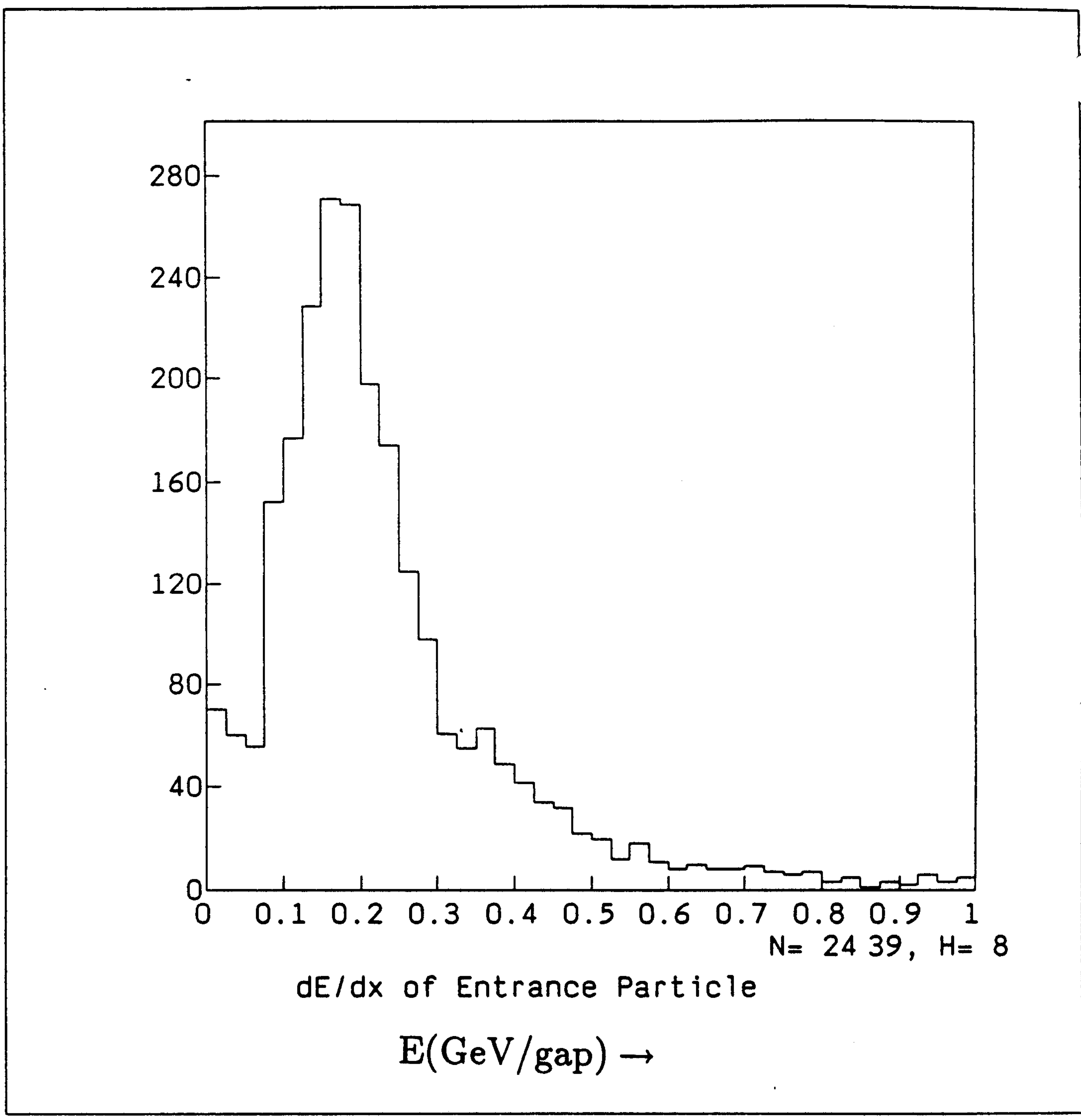


Figure 3. The distribution of gap pulse heights at 140 GeV for all gaps before the estimated starting point, λ_c, showing a clear minimum ionizing peak at about E_{min}=.2 GeV, in equivalent shower energy units.

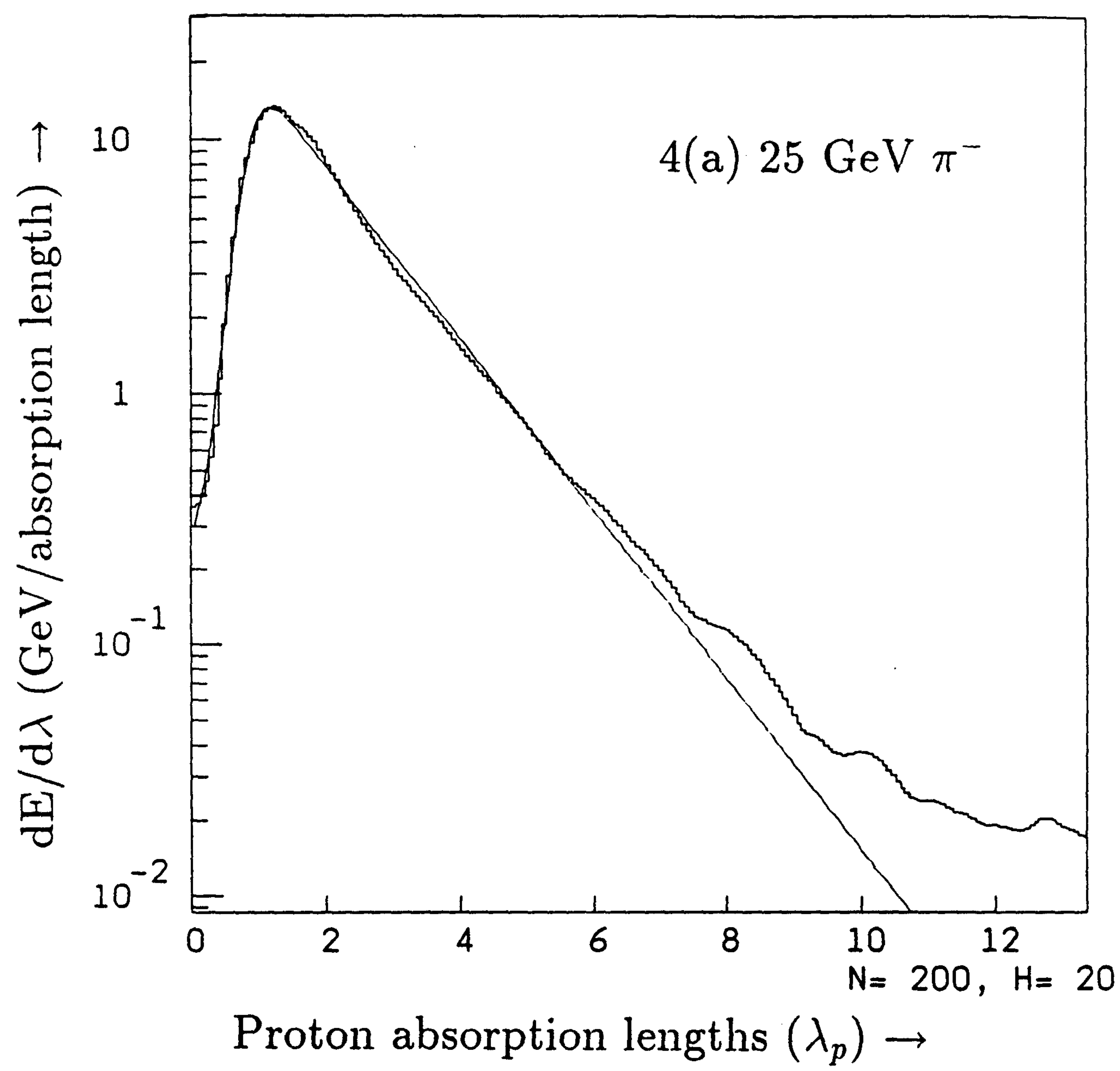
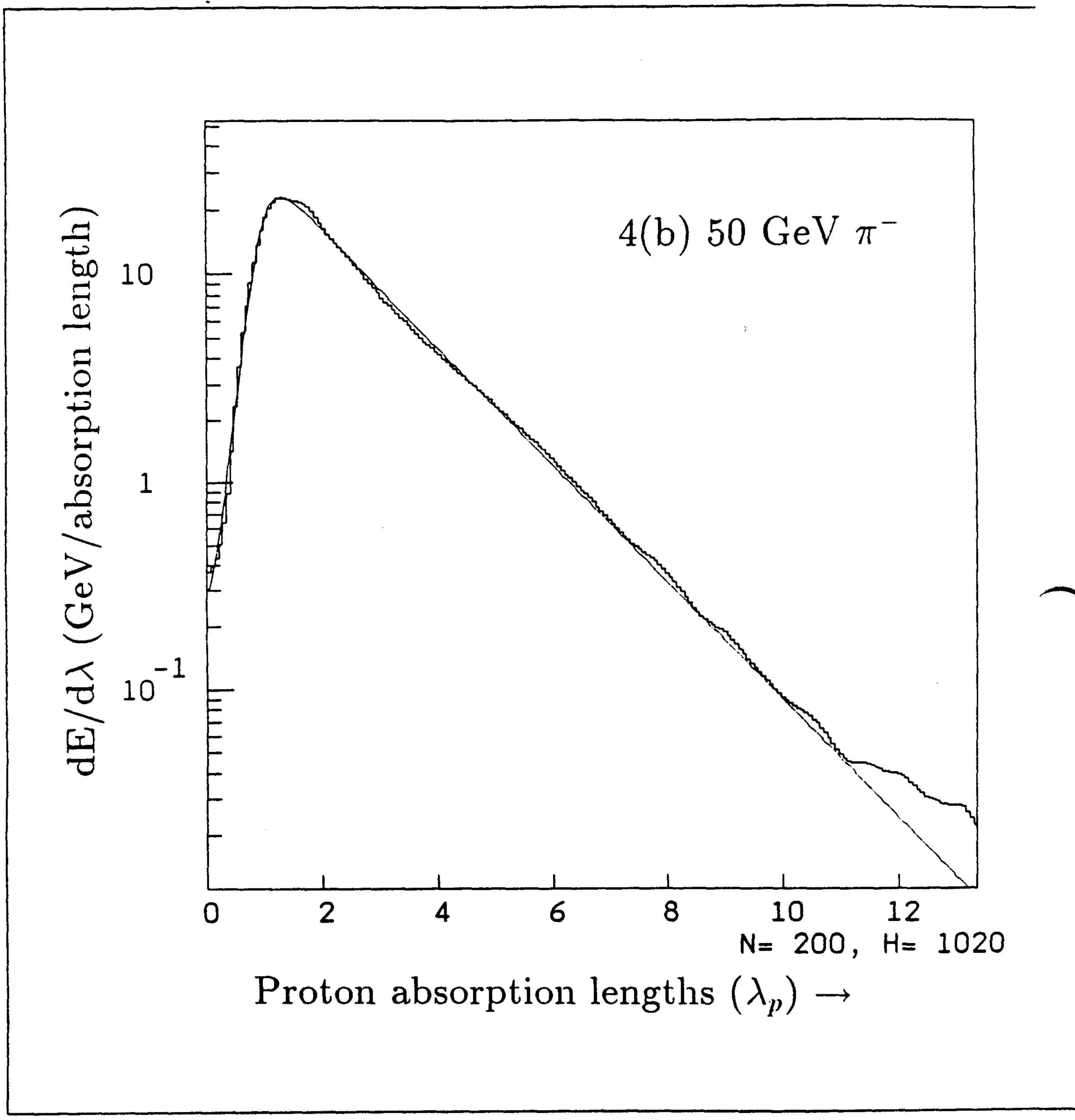
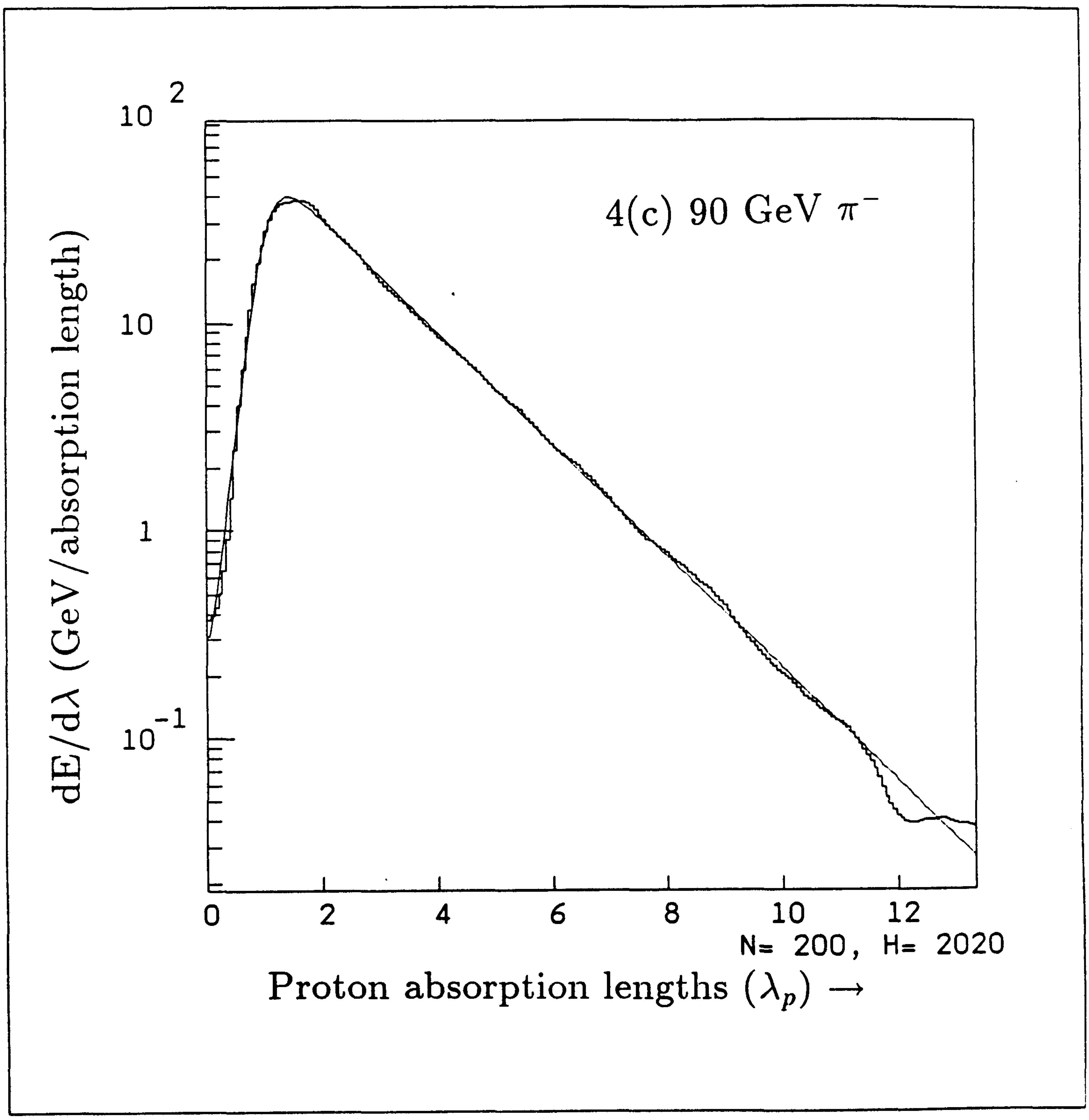
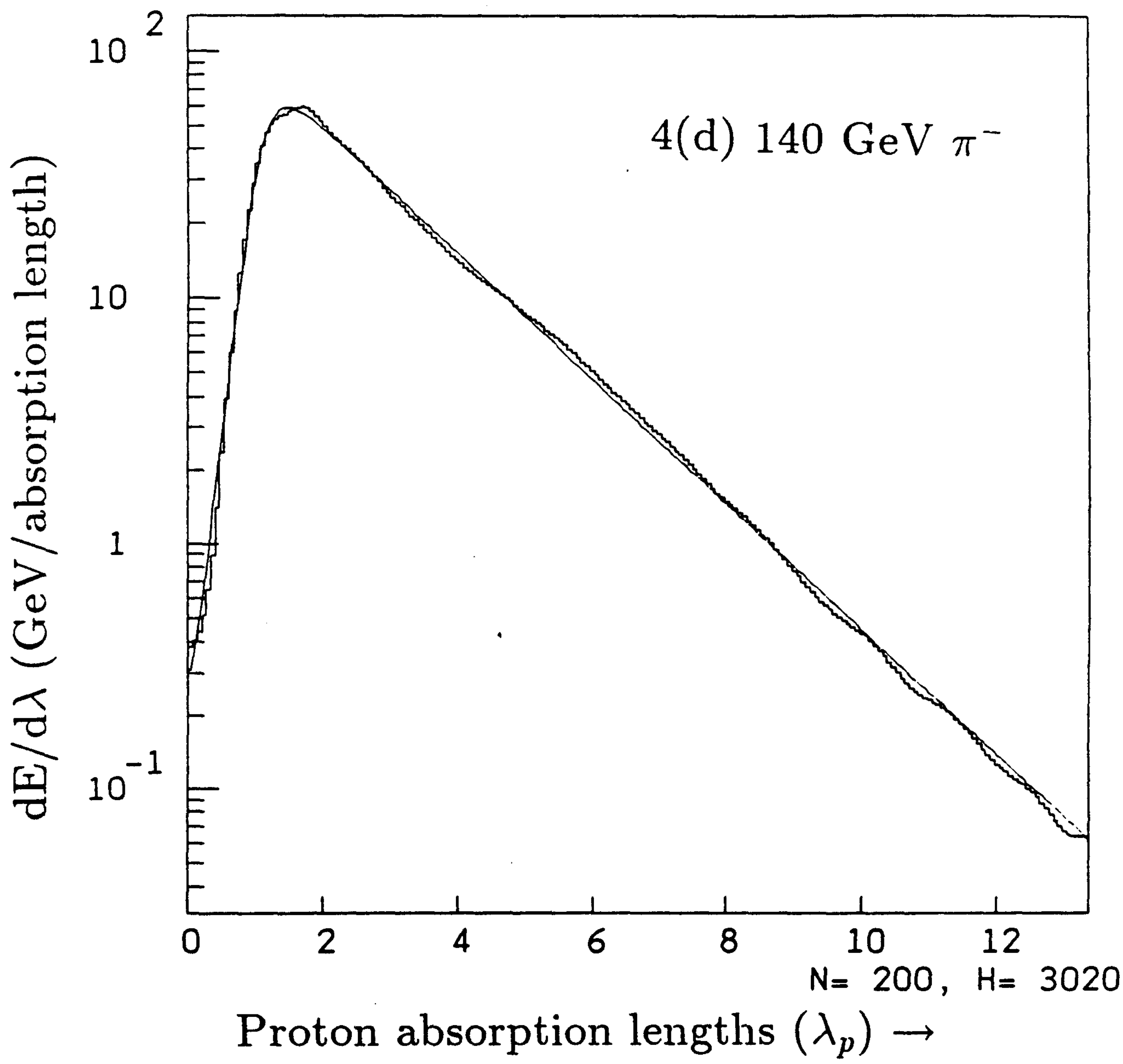


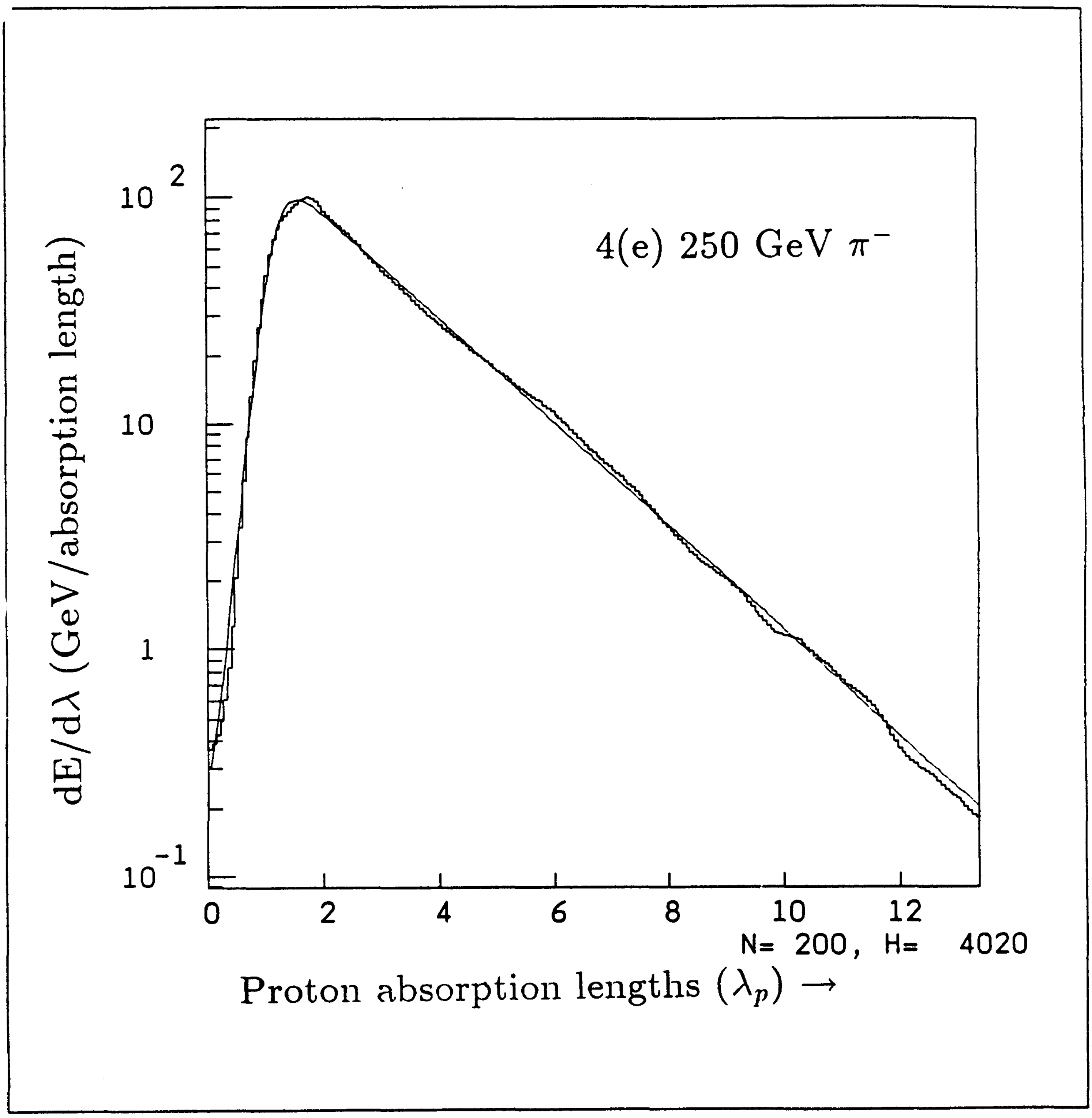
Figure 4. Results of the fits to the shower shape distributions at (a) 25 GeV, (b) 50 GeV, (c) 90 GeV, (d) 140 GeV, and (e) 250 GeV. The χ^2 values are fairly good for all fits.


4(b) 50 GeV π^-
dE/dλ (GeV/absorption length)
10
1
10^{-1}
0
2
4
6
8
10
12
N= 200, H= 1020
Proton absorption lengths (λ_p) →



4(c) 90 GeV π^-
10^2
10
1
10^{-1}
dE/dλ (GeV/absorption length)
0
2
4
6
8
10
12
N= 200, H= 2020
Proton absorption lengths (λ_p) →


4(d) 140 GeV π^-
dE/dλ (GeV/absorption length)
10 2
10
1
10 -1
0
2
4
6
8
10
12
N= 200, H= 3020
Proton absorption lengths (λ_p) →


4(e) 250 GeV π^-
dE/dλ (GeV/absorption length)
10^2
10
1
10^{-1}
0
2
4
6
8
10
12
N= 200, H= 4020
Proton absorption lengths (λ_p) →

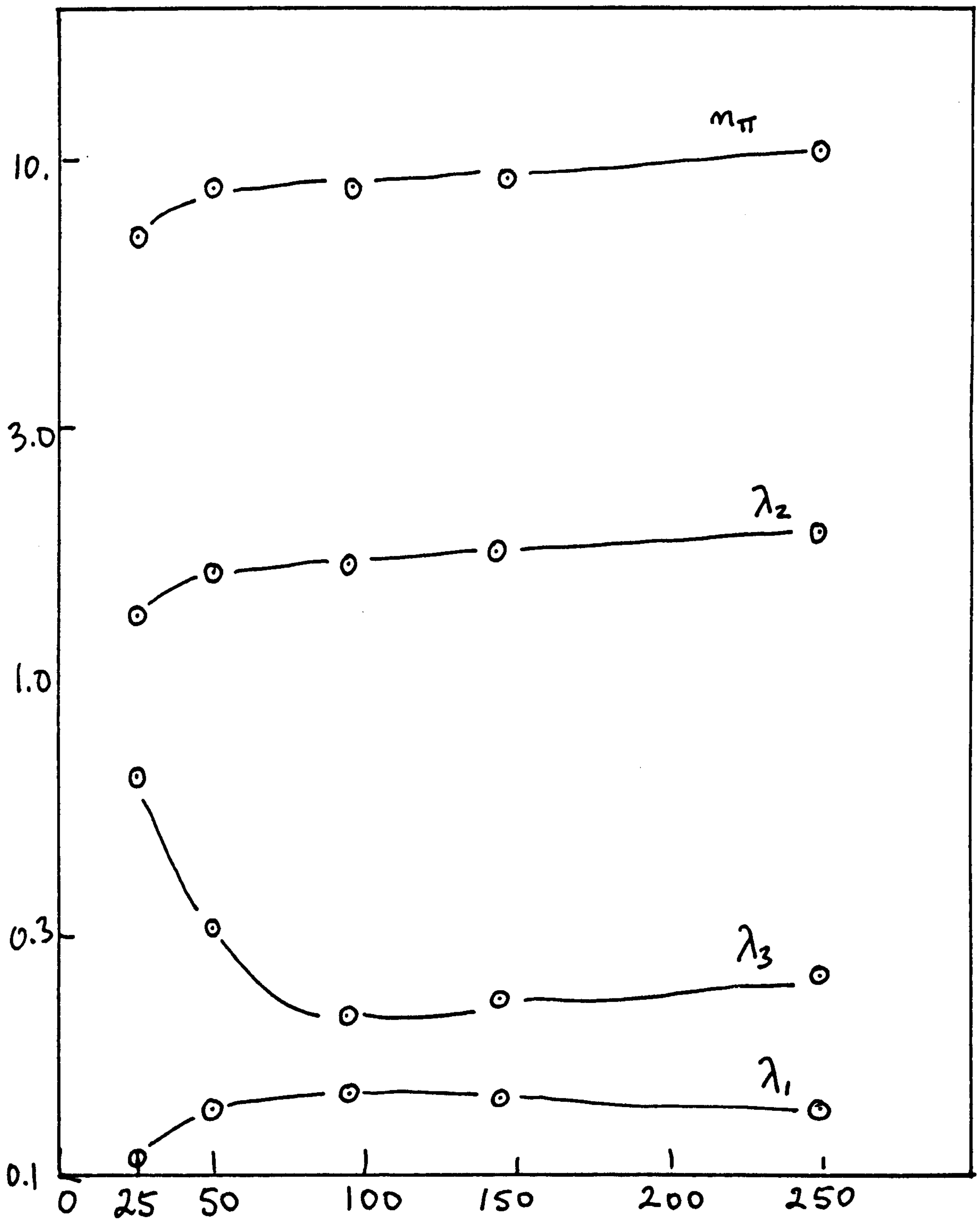


Figure 5. Energy dependence of the fitted parameters, λ_1, λ_2, λ_3, and n_π.

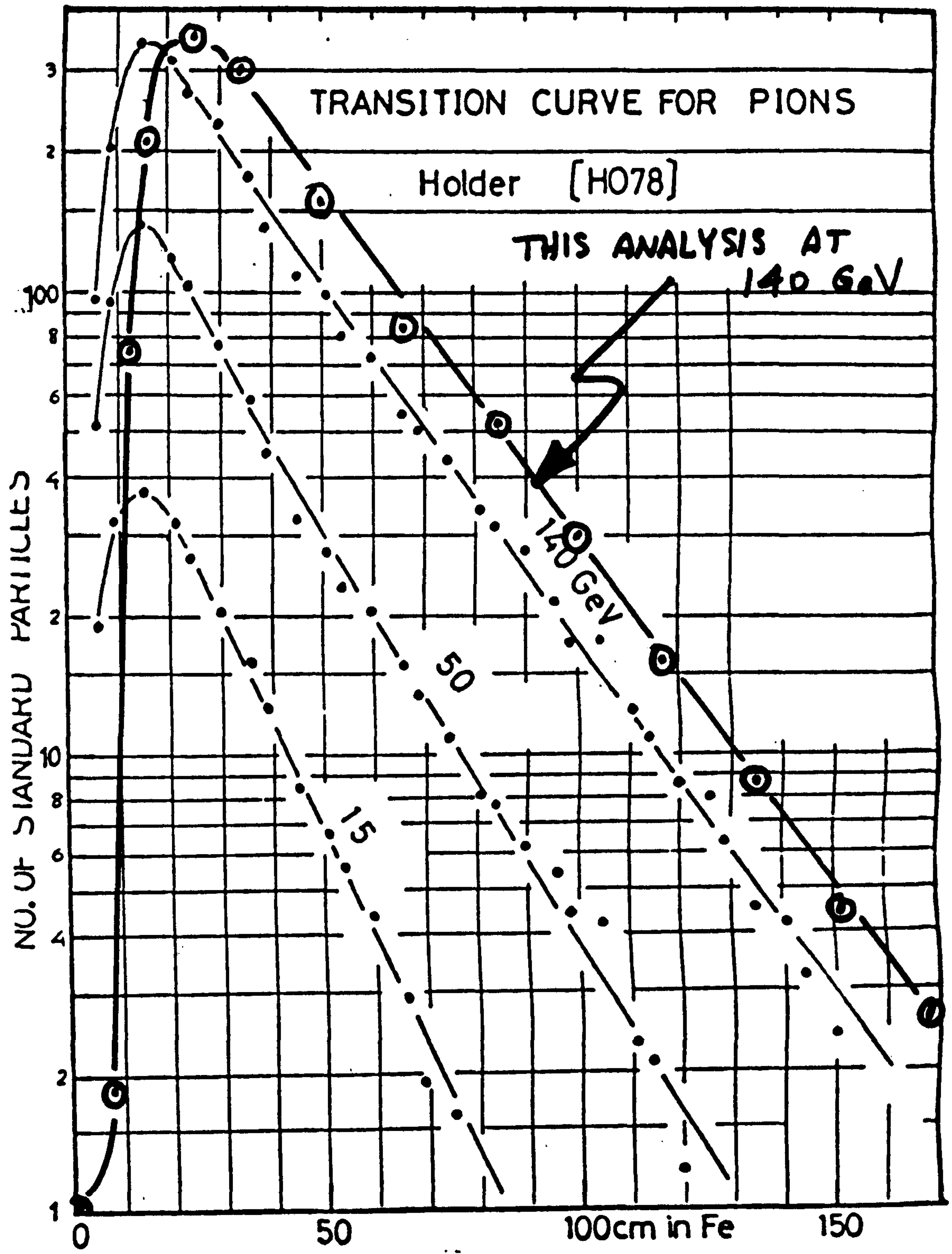


Figure 6. A comparison of our shower shape at 140 GeV with Holder, et al., also at 140 GeV and also in an Fe calorimeter.