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SOLENOIDAL DETECTOR NOTES

NOTE ON MUON MOMENTUM RESOLUTION

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Abstract

Momentum resolution for muons is analyzed for a system consisting of 100 layers of tracking equally spaced between 0.5 m and 1.6 m from the beam. With 200 μm resolution the result is $\sigma(p_\perp)/p_\perp = 0.74 p_\perp(\mathrm{TeV})$. Constraining the vertex to 20 μm reduces this by a factor 0.31. If the calorimeter is inside the coil further improvement can be achieved by making an additional measurement beyond the calorimeter. The improvement is dramatic only for transverse momenta beyond 1.5 TeV and thus does not furnish a compelling reason for a large radius coil.

The analysis of Gluckstern (NIM **24**, 381 (1963)) is adequate for exploring the muon momentum resolution of a system consisting of a central tracker supplemented by a vertex constraint and an additional measurement made just after the calorimeter and just before the coil (in the instance of the LSD design). The specific geometry studied has an inner tracking radius of 0.5 m and an outer tracking radius of 1.6m. The coil begins at 4.0 m and between the tracker and the coil is a calorimeter with 300 radiation lengths of material. The tracker is assumed to have 100 wires, each giving a position measurement with rms error $\epsilon_0 = 200\mu m$. We shall suppose the vertex is known to $20\mu m$. The measurement made after the calorimeter suffers from multiple coulomb scattering with an rms displacement

$$\delta = \frac{1}{\sqrt{3}} \frac{14 \times 10^{-6}}{p(TeV)} \sqrt{\mathcal{N}} L = \frac{336\ \mu m}{p(TeV)}$$

where $\mathcal{N} = 300$ is the number of radiation lengths of material and $L = 2.4$ m is the depth of the calorimeter. The effective resolution is

$$\epsilon_{coil} = \sqrt{\epsilon_0^2 + \delta^2}.$$

At 90^0 to the beam we have

$$p(GeV) = 0.3B(T)R(m)$$

or

$$\frac{1}{R(m)} = \frac{0.3B(T)}{p(GeV)}.$$

Now Gluckstern considers a series of measurements in the xy plane, perpendicular to a putative magnetic field. In the absence of the magnetic field a fit

$$y = \alpha + \theta x + \frac{1}{2} c x^2$$

will yield an expectation for $c^2 = 1/R^2$ that determines the uncertainty in the measurement of p according to

$$\begin{aligned} \delta \langle \frac{1}{R^2} \rangle^{\frac{1}{2}} &= \langle c^2 \rangle^{\frac{1}{2}} \\ &= \delta \frac{0.3B}{p} \end{aligned}$$

$$= \frac{\delta p}{p^2} 0.3B$$
$$= \frac{\delta p_\perp}{p_\perp^2} 0.3B$$

where $p_\perp$ is in GeV, B is in tesla, and c is in m^{-1}.

From a set of measurements (x_n, y_n) $n = 0, \cdots N$ and a set of weights f_n we determine α, θ, and c by minimizing

$$\chi^2 = \sum_{n=0}^{N} [y(x_n) - y_n]^2 f_n$$

with the result

$$\frac{1}{2}c = \frac{\sum_{n=0}^{N} \begin{vmatrix} F_0 & F_1 & 1 \\ F_1 & F_2 & x_n \\ F_2 & F_3 & x_n^2 \end{vmatrix} y_n f_n}{\begin{vmatrix} F_0 & F_1 & F_2 \\ F_1 & F_2 & F_3 \\ F_2 & F_3 & F_4 \end{vmatrix}}$$

with

$$F_i = \sum_{n=0}^{N} f_n x_n^i$$

or with

$$Q_n = f_n \begin{vmatrix} F_0 & F_1 & 1 \\ F_1 & F_2 & x_n \\ F_2 & F_3 & x_n^2 \end{vmatrix},$$
$$\frac{1}{2}c = \frac{\sum Q_n y_n}{\sum Q_n x_n^2}.$$

Thus

$$\left\langle \frac{1}{4}c^2 \right\rangle = \frac{\langle \sum Q_n y_n \sum Q_m y_m \rangle}{(\sum Q_n x_n^2)^2}.$$

Because the $f_n s$ can be scaled by a constant, $\sum Q_n x_n^2$ can be fixed to some value. If the errors are uncorrelated

$$\langle y_m y_n \rangle = \epsilon_n^2 \delta_{mn}$$

and

$$\langle \frac{1}{4}c^2 \rangle = \frac{\sum \epsilon_n^2 Q_n^2}{(\sum x_n^2 Q_n)^2}.$$

We now choose the f_i's to minimize $\langle c^2 \rangle$ using Lagrange multipliers, following Gluckstern. From the definition of Q_n clearly $\sum Q_n = 0, \sum Q_n x_n = 0$ so we minimize

$$\sum \epsilon_n^2 Q_n^2 + \lambda_0 \sum Q_n + \lambda_1 \sum Q_n x_n + \lambda_2 \sum Q_n x_n^2$$

varying the f_n

$$\epsilon_n^2 Q_n + \lambda_0 + \lambda_1 x_n + \lambda_2 x_n^2 = 0.$$

Comparing with the definition of Q_n we see that $f_n = 1/\epsilon_n^2$ times a constant independent of n.

Equal spacing and weights

Suppose $\epsilon_n = \epsilon$ independent of n and further suppose $x_n = \frac{nL}{N}$ with $N >> 1$. Then if $f_n = 1$

$$\begin{aligned} F_j &= \sum x_n^j = \sum (\frac{n}{N} L)^j \\ &\approx \int_0^1 dx N x^j L^j = \frac{NL^j}{j+1} \end{aligned}$$

and

$$\begin{aligned} Q_n &= \begin{vmatrix} N & LN/2 & 1 \\ LN/2 & L^2N/3 & nL/N \\ L^2N/3 & L^3N/4 & n^2L^2/N^2 \end{vmatrix} \\ &= \frac{L^4N^2}{12}(\frac{n^2}{N^2} - \frac{n}{N} + \frac{1}{6}) \end{aligned}$$

and

$$\begin{aligned} \left\langle \frac{1}{4}c^2 \right\rangle &= \epsilon^2 \frac{\sum(\frac{n^2}{N^2} - \frac{n}{N} + \frac{1}{6})^2}{\left[\sum(\frac{n^2}{N^2} - \frac{n}{N} + \frac{1}{6})\frac{n^2L^2}{N^2}\right]^2} \\ &\approx \frac{\epsilon^2}{L^4} \frac{N\int_0^1 dx(x^2 - x + \frac{1}{6})^2}{\left[N\int_0^1 dx(x^2 - x + \frac{1}{6})x^2\right]^2} \\ &= \frac{\epsilon^2}{NL^4} 180 \end{aligned}$$

$$\langle c^2 \rangle = \frac{720\epsilon^2}{NL^4}$$

This leads to the result for a tracking chamber with equally spaced wires

$$\frac{\delta p_\perp}{p_\perp^2} = \sqrt{\frac{720}{N}}\frac{\epsilon(\mathrm{m})}{0.3B(\mathrm{Tesla})L^2(\mathrm{m})}$$

as quoted on p. 350 of the 1987 Berkeley Workshop Proceedings.

The result from the tracking without the vertex constraint gives

$$\frac{\delta p_\perp}{p_\perp} = 0.74 p_\perp(TeV)$$

for $\epsilon = 200\mu$m and 0.55 $p_\perp(TeV)$ for $\epsilon = 150\mu$m as stated in the '87 Workshop Proceedings.

Vertex Constraint

Let us add an additional point at $x = -L' = -L\lambda$ with an uncertainty ϵ' so

$$F_j = \frac{NL^j}{j+1} + (-\lambda)^j L^j \eta$$

where the new point has the weight $\eta = \epsilon^2/\epsilon'^2$.

The addition of the vertex constraint improves the resolution by a factor depending on η and λ. The result is shown in Figure 1. In the limiting case $\lambda = 0$, F_1, F_2, F_3, and F_4 are unaffected while

$$F_0 = N + \eta$$

$$Q_n = \frac{L^4N^2}{12}\left[\left(\frac{n^2}{N^2} - \frac{n}{N} + \frac{1}{6}\right) + \frac{\eta}{N}\left(\frac{4n^2}{N^2} - \frac{3n}{N}\right)\right]$$

$$Q_v = \frac{L^4N^2}{12}\frac{\eta}{6}.$$

In this way we find for $\lambda = 0$

$$\left\langle \frac{1}{4}c^2 \right\rangle = \frac{\epsilon^2}{NL^4}\cdot\left[\frac{\frac{80\eta^2}{N^2} + \frac{260}{9}\frac{\eta}{N} + \frac{20}{9}}{(\frac{\eta}{N} + \frac{1}{9})^2}\right]$$

so that for $\eta >> N$

$$\left\langle \frac{1}{4}c^2 \right\rangle = \frac{\epsilon^2}{NL^4}80$$

a reduction of $4/9 = (2/3)^2$. Of course increasing λ improves the result. In fact in the extreme case $\lambda \to \infty$ as well as $\eta \to \infty$ we find

$$
\begin{aligned}
Q_n &= N^2 L^4 \frac{\eta}{N} \lambda^3 \Big(-\frac{1}{2} + \frac{n}{N} \Big) \\
Q_v &= N^2 L^4 \eta \frac{\lambda^2}{12}
\end{aligned}
$$

and

$$\left\langle \frac{1}{4} c^2 \right\rangle = \frac{\epsilon^2}{NL^4} \frac{12}{\lambda^2},$$

an improvement in $\langle c^2 \rangle^{\frac{1}{2}}$ by

$$\frac{1}{\lambda} \sqrt{\frac{1}{15}}.$$

For the particular geometry specified above, $\lambda = 0.5/1.1 = 0.45$. If $\epsilon' = 20\mu\text{m}, \epsilon = 200\mu\text{m}$ then $\eta = 100$. The reduction in $\langle c^2 \rangle^{\frac{1}{2}}$ is, from Fig. 1, 0.31, so we determine

$$
\begin{aligned}
\frac{\sigma(p_\perp)}{p_\perp} &= 0.31 \times 0.76 \; p_\perp (TeV) \\
&= 0.24 p_\perp (TeV)
\end{aligned}
$$

Momentum Resolution Enhancement for LSD

It is straightforward to include a single measurement subsequent to the calorimeter for the LSD design. Of course for low $p_\perp$ the Coulomb scattering renders this measurement useless, but at high $p_\perp$ it is significant. Figure 2 shows results for the factor by which the resolution is improved from that with tracking only by inclusion of an extra point at 4.0 m. The solid curve includes the vertex constraint while the dotted one does not. The results are shown in Table 1 as well.

Comments

An evaluation of the results of Table I requires some idea of the physics demands. Two important requirements are furnished by the search for the Higgs boson and the search for a new Z.

$p_\perp$(GeV)	Tracking	Tracking + Vertex +Outer	Tracking+Outer	Tracking+Vertex
200	0.15	0.042	0.10	0.047
500	0.38	0.081	0.14	0.12
1000	0.76	0.11	0.17	0.24
2000	1.52	0.16	0.25	0.47

Table 1: The value of $\sigma(p_\perp)/p_\perp$ for various values of $p_\perp$. The tracking is from 0.5 m to 1.6 m. The outer measurement is made at 4.0 m. The Tracking + Vertex column is given by 0.31 times the Tracking column. The factors relating the Tracking + Outer column and the Tracking + Vertex + Outer column to the Tracking column may be read off Fig. 2.

For the Higgs search the values of $p_\perp$ are generally less than 200 GeV even for $m_H = 800$ GeV so tracking plus the vertex constraint would be adequate, given the conventional standard that 10% resolution suffices for the purpose.

For the search for a new Z the $p_\perp$s might be very much greater. However, assuming $e - \mu$ universality the Z' mass would be determined from the e^+e^- signal. The role of the muons would be to provide additional statistics confirming the result, and possibly to measure the forward–backward asymmetry. This requires a sign determination and thus a resolution of about 30% or so. From Table I we see that the addition of the outer measurement would raise the maximum $p_\perp$ for which this was possible from about 1.2 TeV to well beyond 2 TeV. However the large number of Z's required for the measurement means that the asymmetry could not be obtained any for a Z' near the maximum observable value of about 6 TeV. As a result the $p_\perp$'s would generally be less than 1.2 TeV. It appears that the extra lever arm provided by the calorimeter is not crucial to any anticipated muon physics provided a vertex constraint is used.

There might be concern that if the resolution is very poor the Z' signal might get lost in the Drell-Yan background. This is not a problem. It suffices to compare the Z' and continuum contributions to $q\bar{q} \to \mu^+\mu^-$. The resonant cross section can be approximated by

$$\sigma_{q\bar{q}\to\mu^+\mu^-} = 12\pi^2 \frac{\Gamma_q \Gamma_\mu}{\Gamma M} \delta(s - M^2) \times \frac{1}{3}$$

where $\Gamma_q = \Gamma(Z \to q\bar{q})$, $\Gamma_\mu = \Gamma(Z \to \mu^+\mu^-)$, $\Gamma = \Gamma_{tot}$, and the factor of 1/3 comes from requiring the q and $\bar{q}$ to have the same color.

The continuum cross section through the γ^* is

$$\sigma_{q\bar{q}\to\gamma^*\to\mu^+\mu^-} = \frac{4\pi\alpha^2}{3s} e_Q^2 \times \frac{1}{3}$$

By analogy the cross section through the Z (assuming $M_{Z'} >> M_Z$) is

$$\sigma_{q\bar{q}\to Z\to\mu^+\mu^-} = \frac{4\pi\alpha^2}{3s} \frac{(T_{3Q} - e_Q \sin^2\theta_W)^2(-1/2 + \sin^2\theta_W)^2}{\sin^4\theta_W \cos^4\theta_W} \times \frac{1}{3}$$

The total continuum (Drell-Yan) cross section is the sum of the γ^* and Z contributions,

$$\sigma_{DY} = \frac{4\pi\alpha^2}{3s} K \times \frac{1}{3}$$

where K is 0.72 for u quarks and 0.53 for d quarks.

If the signal and background are integrated over an interval, ΔM, much broader than the width of the Z' (or the resolution of the lepton pair), the ratio of signal to background is

$$\frac{\sigma_{Z'}}{\sigma_{continuum}} = \frac{9\pi}{2\alpha^2} B_q B_\mu \frac{\Gamma}{M} \frac{1}{(\Delta M/M)K}$$

where B_q is the branching ratio for $Z' \to q\bar{q}$ and B_μ is the branching ratio for $Z' \to \mu^+\mu^-$. If we simply use the values from the Z, $B_q \approx 0.1$, $B_\mu \approx 0.03$, $\Gamma/M \approx 0.03$, we find

$$\frac{\sigma_{Z'}}{\sigma_{continuum}} = \frac{24}{(\Delta M/M)K}$$

This shows that the Drell-Yan background will not be a problem even if ΔM is a large fraction of M.

Figure Captions

1. The factor by which $\sigma(p_\perp)/p_\perp$ is reduced by the addition of a vertex constraint. Tracking is assumed to occur over a distance L and the vertex is located a distance $L' = \lambda L$ from the tracking. The result is shown for $\eta = 1, 10, 100, 1000$, where $\eta = \epsilon^2/\epsilon'^2$ and ϵ is the spatial resolution of the tracker (assumed to have 100 layers) and ϵ' is the uncertainty in the location of the vertex.

2. The factor by which $\sigma(p_\perp)/p_\perp$ is reduced by the addition of a point beyond the tracking, with multiple scattering included as described in the text. The solid curve shows the result with the vertex constraint and the dashed line shows the result without the vertex constraint.

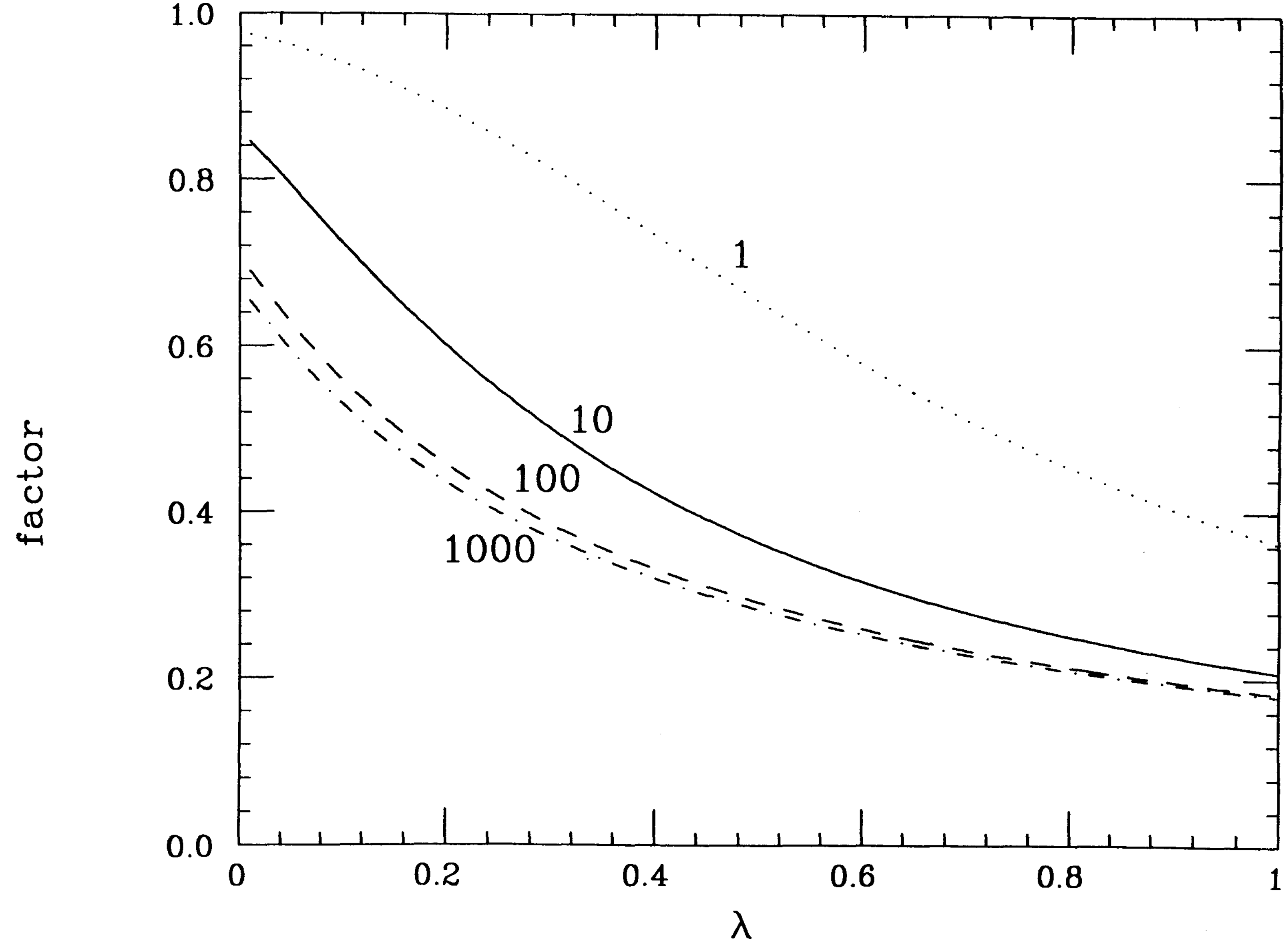

Fig. 1

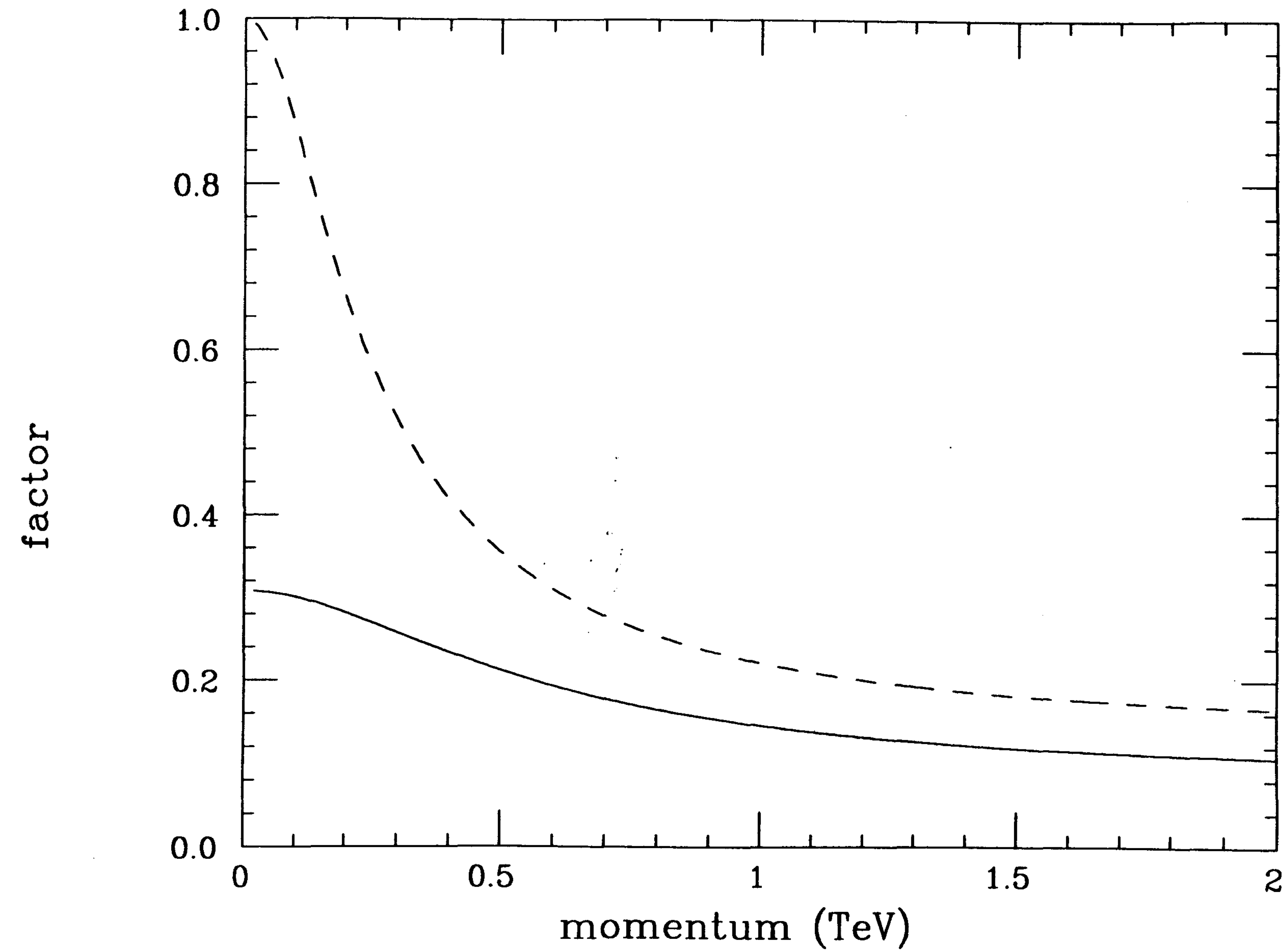

Fig. 2