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Depletion Characteristics of Silicon Microstrip Detectors

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Abstract

This note extends what is known about parallel-plate PIN diode depletion to the realm of silicon microstrip detectors. By solving the fields present in structures with strip detector geometry, the depletion voltages, strip-to-backplane capacitance, and C-V characteristics can be determined. These characteristics are valuable for evaluating and diagnosing silicon detectors.

Introduction

A silicon microstrip detector is basically a PIN diode, whose behavior is easily described. The center of the diode is lightly doped silicon, which under the influence of a light electric field becomes "depleted" meaning that the free electrons (holes) have been driven out, leaving a positively (negatively) charged insulator behind. On either side of the inert material are p doped and n doped regions which form the diode junction. When reverse biased, little current flows and a portion of the inert material will be depleted. For this paper, it is assumed that any nonzero electric field will deplete the silicon.

In order to determine this in more detail, the first step is to determine the solution to Poisson's equation for the geometries of interest. Next, we use this information to determine how the silicon depletes.

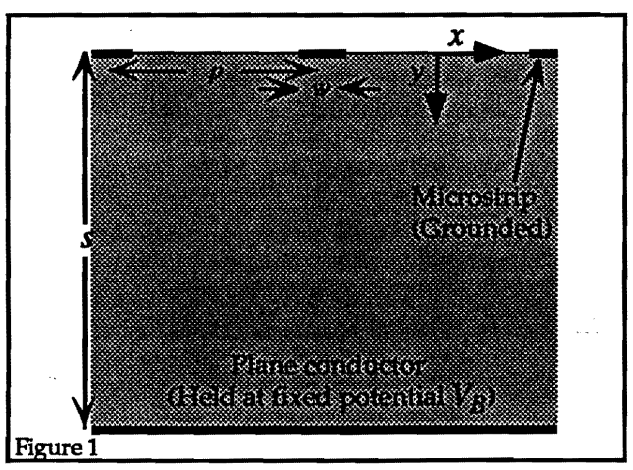

Figure 1

Solution to Poisson's Equation

Parallel Plate Geometry

For the case of a parallel-plate PIN diode, Poisson's equation reduces to the one-dimensional form $\frac{d^2\phi(y)}{dy^2} = -\frac{\rho}{\varepsilon_0\varepsilon}$. Here ϕ is the potential, ρ is the charge density of the depleted silicon, and ε is the dielectric constant of silicon. With one plane grounded and the other at fixed potential V_B, the boundary conditions are:

1. $\phi(0) = 0$
2. $\phi(s) = V_B$

Accordingly, the potential is:

$$\phi = \left(V_B + \frac{\rho s^2}{2\varepsilon_0\varepsilon}\right)\frac{y}{s} - \frac{\rho}{2\varepsilon_0\varepsilon}y^2 \qquad (1)$$

Strip Geometry

Consider the geometry of a microstrip detector, which has an infinite number of parallel strips running along the z-axis on one side, and a plane conductor on the other. A cross section in the x–y plane is shown in figure 1, which is periodic in x. The strips are grounded, while the backplane is held at potential V_B. By choosing the coordinate system to have its origin directly between two strips, the potential has the form

$$\phi(x,y) = \begin{cases} \sum_{n=0}^{\infty} F_n(y)\cos\frac{2\pi nx}{p} & 0 \le y \le s \\ \sum_{n=0}^{\infty} F'_n(y)\cos\frac{2\pi nx}{p} & y < 0 \end{cases}$$

since the potential must be periodic in x. For this to satisfy Poisson's equation in two dimensions $\frac{\partial^2\phi(x,y)}{\partial x^2} + \frac{\partial^2\phi(x,y)}{\partial y^2} = -\frac{\rho}{\varepsilon_0\varepsilon}$, this can be written

$\underline{0 \le y \le s}$

$$\phi(x,y) = -\frac{\rho y^2}{2\varepsilon_0 \varepsilon} + Ey +$$

$$\frac{pE\varepsilon}{2\pi(\varepsilon+1)} \sum_{n=0}^{\infty} \left[A_n e^{-2\pi n y/p} + B_n e^{2\pi n y/p} \right] \cos \frac{2\pi n x}{p}$$

$\underline{y < 0}$

$$\phi(x,y) = E'y +$$

$$\frac{pE\varepsilon}{2\pi(\varepsilon+1)} \sum_{n=0}^{\infty} \left[A'_n e^{2\pi n y/p} + B'_n e^{-2\pi n y/p} \right] \cos \frac{2\pi n x}{p}$$

where $E, E', A_0 \ldots A_n \ldots, A'_0 \ldots A'_n \ldots$ are constants to be determined from the boundary conditions:

1. ϕ must be finite as $y \to -\infty$.
2. ϕ has no x dependence once $y = d$.
3. ϕ is continuous at $y = 0$.
4. ϕ is zero when $y = 0$ and x is on a strip.
5. The electric displacement is continuous between the silicon and air boundary ($y = 0$ and x is *between* strips).
6. The potential at the plane conductor is given by $\phi(x,s) = V_B$.

Applying the first three boundary conditions simplifies the potential to the form:

$\underline{0 \le y \le s}$ (2a)

$$\phi(x,y) = \frac{pE\varepsilon}{2\pi(\varepsilon+1)} \sum_{n=0}^{\infty} A_n e^{-2\pi n y/p} \cos \frac{2\pi n x}{p} + Ey - \frac{\rho y^2}{2\varepsilon_0 \varepsilon}$$

$\underline{y < 0}$ (2b)

$$\phi(x,y) = \frac{pE\varepsilon}{2\pi(\varepsilon+1)} \sum_{n=0}^{\infty} A_n e^{2\pi n y/p} \cos \frac{2\pi n x}{p}$$

Boundary conditions 4 and 5 can be summarized, using the substitution $\xi = \dfrac{2\pi x}{p}$:

$$\sum_{n=1}^{\infty} n A_n \cos n\xi = 1 \quad 0 \le \xi < \pi \tfrac{w}{p}$$

$$\sum_{n=0}^{\infty} A_n \cos n\xi = 0 \quad \pi \tfrac{w}{p} < \xi \le \pi$$

These two equations constrain all A_n. The solution to this problem can be found in [1]. It takes the form:

$$A_0 = \sqrt{\tfrac{1}{2}} \int_0^{\pi\left(1-\frac{w}{p}\right)} h_1(t)\,dt$$

$$A_n = \sqrt{\tfrac{1}{2}} \int_0^{\pi\left(1-\frac{w}{p}\right)} h_1(t)\left(P_n(\cos t) + P_{n-1}(\cos t)\right) dt$$

with $h_1(t) = \dfrac{2}{\pi} \dfrac{d}{dt} \displaystyle\int_0^t \frac{x \sin \frac{1}{2} x}{\sqrt{\cos x - \cos t}}\,dx$. Substituting this yields:

Table 1.

$\frac{w}{p}$	A_0	A_1	A_2	A_3	A_4	A_5	A_6	A_7
0.02	6.8836	1.9968	-0.9961	0.6610	-0.4921	0.3903	-0.3216	0.2721
0.04	5.4913	1.9909	-0.9843	0.6435	-0.4689	0.3616	-0.2877	0.2331
0.06	4.6913	1.9811	-0.9648	0.6147	-0.4314	0.3161	-0.2351	0.1744
0.08	4.1240	1.9674	-0.9379	0.5756	-0.3815	0.2572	-0.1693	0.1041
0.10	3.6847	1.9498	-0.9039	0.5272	-0.3215	0.1893	-0.0973	0.0322
0.12	3.3267	1.9285	-0.8633	0.4708	-0.2543	0.1170	-0.0262	-0.0320
0.14	3.0251	1.9036	-0.8165	0.4079	-0.1829	0.0457	0.0371	-0.0805
0.16	2.7650	1.8751	-0.7641	0.3402	-0.1107	-0.0200	0.0868	-0.1079
0.18	2.5368	1.8431	-0.7068	0.2695	-0.0408	-0.0758	0.1189	-0.1127
0.20	2.3338	1.8078	-0.6454	0.1976	0.0238	-0.1184	0.1317	-0.0967
0.22	2.1514	1.7693	-0.5805	0.1264	0.0805	-0.1457	0.1256	-0.0648
0.24	1.9861	1.7277	-0.5130	0.0576	0.1269	-0.1568	0.1033	-0.0242
0.26	1.8354	1.6833	-0.4437	-0.0071	0.1617	-0.1522	0.0691	0.0173
0.28	1.6971	1.6362	-0.3735	-0.0661	0.1838	-0.1337	0.0284	0.0522

Values for the constants A_0 through A_7 for different width to pitch ratios.

$$A_0 = \frac{\sqrt{2}}{\pi} \int_0^{\pi\left(1-\frac{w}{p}\right)} \frac{x \sin \frac{1}{2} x}{\sqrt{\cos x + \cos \pi \frac{w}{p}}}\,dx \qquad (3)$$

$h_1(t)$ can be simplified slightly to the form:

$$h_1(t) = \tfrac{\sqrt{2}}{\pi} t \cdot \tan t$$

$$+ \tfrac{\sqrt{2}}{\pi} \sin t \int_0^{\frac{1}{2}t} \frac{dx}{\sqrt{\cos x - \cos \frac{1}{2} t}\left(\cos x + \sqrt{\cos x - \cos \frac{1}{2} t}\right)}$$

Apparently these can only be solved numerically. Fortunately, these are only functions of the ratio width to pitch; some values are shown in table 1. As A_0 is the most useful of these, it is shown plotted in figure 2 as a function of $\frac{w}{p}$.

Finally, E is constrained by boundary condition 6:

$$E = \frac{V_B + \dfrac{\rho s^2}{2\varepsilon_0 \varepsilon}}{\frac{p\varepsilon}{2\pi(\varepsilon+1)} A_0 + s} \qquad (6)$$

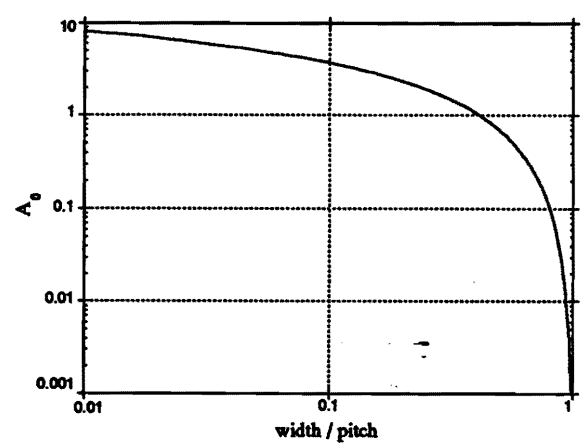

Figure 2.

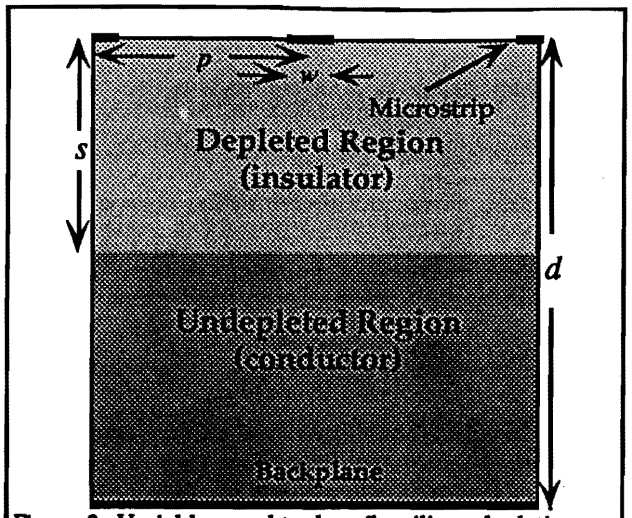

Figure 3. Variables used to describe silicon depletion.

Depletion of Silicon

Conditions for Depletion

Determining depleted and undepleted regions is only possible if some symmetry can be utilized, so we assume that the undepleted region has a planar boundary between conductive and insulating regions. In this case, the depletion will look like figure 3, where the depleted region is a distance s from one side of the silicon, which has a depth d. Thus, the condition $E=0$ determines the depletion depth s; the potential is constant for $s \le y \le d$.

Parallel Plate PIN Diode

Applying the condition $E=0$ at the depleted-undepleted boundary to equation 1 is equivalent to:

$$V_B = \frac{\rho s^2}{2\varepsilon_0 \varepsilon}$$

which allows us to define the depletion voltage as the voltage at which all silicon is first depleted, *i.e.* $s=d$:

$$V_{d0} = \frac{\rho d^2}{2\varepsilon_0 \varepsilon} \qquad (5)$$

The capacitance of such a partially depleted geometry is easily calculated, but in order to make the results appear similar to those later in the paper I divide the PIN diode into strips of width p, and consider the contribution to the capacitance per unit length (in the z-direction) of each strip to the total front-side to back-side capacitance.

$$c = \frac{C}{l} = \frac{\lambda}{V_B} = \frac{\sigma p}{Es} = \varepsilon_0 \varepsilon \frac{p}{s}$$

The relationship between bias voltage and capacitance is thus:

$$c = \sqrt{\frac{\varepsilon_0 \varepsilon p^2 \rho}{2V_B}} = \frac{\varepsilon_0 \varepsilon p}{d}\sqrt{\frac{V_{d0}}{V_B}} \qquad (6)$$

Note that once $V_B \ge V_{D0}$ this capacitance remains constant at $c = \varepsilon_0 \varepsilon_{si} \frac{p}{d}$.

Junction Side Microstrip

Junction type detectors will begin depleting in cylinders around the strips themselves. As the depleted region grows, one expects that it will eventually have a planar boundary with the undepleted region; this occurs once $s > p$. Similar to the PIN case, applying the condition $E=0$ at the back conductor in equation 2a yields:

$$V_B = V_{d0}\frac{s^2}{d^2} + V_{d0}\left(\frac{p}{\pi d}\frac{\varepsilon}{\varepsilon+1}A_0(\tfrac{w}{p})\right)\frac{s}{d} \qquad (7)$$

while the capacitance will have the form

$$c = \varepsilon_0 \varepsilon \frac{p}{s + \frac{p}{2\pi}\frac{\varepsilon}{\varepsilon+1}A_0(\tfrac{w}{p})} \qquad (8)$$

putting these two together:

$$c = \frac{\varepsilon_0 \varepsilon p}{d}\sqrt{\frac{V_{d0}}{V_B + \left(\frac{p}{2\pi d}\frac{\varepsilon}{\varepsilon+1}A_0(\tfrac{w}{p})\right)^2}} \qquad (9)$$

Notice that as $\frac{w}{p} \to 1$, $A_0(\frac{w}{p}) \to 0$, and equation 9 agrees with equation 6. The full depletion voltage for this geometry is:

$$V_d = V_{d0}\left(1 + \frac{p}{\pi d}\frac{\varepsilon}{\varepsilon+1}A_0(\tfrac{w}{p})\right) \qquad (10)$$

Ohmic Side Microstrip

For ohmic side detector silicon, the undepleted region will start near the (planar) back side and will therefore be identical to the diode case as long as $V_B < V_{d0}$. The geometry of the strips will begin to play a role once $V_B \approx V_{d0}$. At this point, the undepleted region is a very thin plane which spans the entire front side of the detector. The detector will fully deplete when the bias voltage reaches the value in equation 10, at which point the capacitance will be given by equation 8. Between these two points, the bias will slowly deplete the material

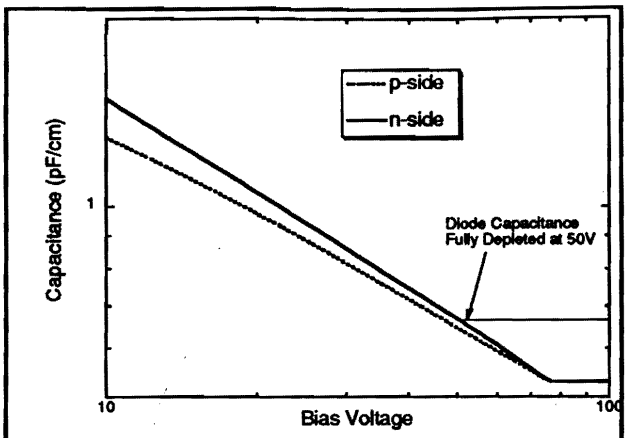

Figure 4. C-V prediction for a 100 μm pitch, 20 μm width, 150μm depth detector with $V_{d0} = 50V$ (eq. 6 and 7)

between the two strips, making an effective width of the strip w'. One can easily solve for the value of $A_0(\frac{w'}{p})$ using what we know so far of the relationship between bias voltage and geometry (equation 7):

$$\frac{d}{2}\left(\frac{V_B}{V_{d0}}-1\right)=\frac{p}{\pi d}\frac{\varepsilon}{\varepsilon+1}A_0(\tfrac{w}{p})$$

With this effective geometry the capacitance is given by:

$$c=\frac{2\varepsilon_0\varepsilon_{si}p}{d}\frac{V_{D0}}{V_{D0}+V_B} \qquad (11)$$

Notice that both c and $\frac{\partial c}{\partial V_B}$ are continuous at $V_B = V_{d0}$.

Using equations 9 and 11, the capacitances of the different types of detectors can be found as functions of bias voltage; one case is shown in figure 4. Both types of junction fully deplete to the same capacitance, given by equation 8. In most cases, the C-V curve will be indistinguishable from the PIN diode, except for a change in the full depletion voltage and final body capacitance.

Double-Sided Microstrip

For double sided detectors with width w on the junction side and w' on the ohmic side, the depletion characteristics are easily predicted. If

$$V_B \le V_{d0}\left(1+\frac{p}{\pi d}\frac{\varepsilon}{\varepsilon+1}A_0(\tfrac{w}{p})\right)$$

the double sided detector will behave exactly like a single-sided junction type detector. However, once the area near the ohmic side strips begins depleting, the body capacitance will change in a manner similar to equation 10. Full depletion voltage in this case is:

$$V_d = V_{d0}\left[1+\frac{p}{\pi d}\frac{\varepsilon}{\varepsilon+1}\left(A_0(\tfrac{w}{p})+A_0(\tfrac{w'}{p})\right)\right]$$

and capacitance at full depletion is:

$$c=\varepsilon_0\varepsilon_{si}\frac{p}{d+\frac{p}{2\pi}\frac{\varepsilon}{\varepsilon+1}\left(A_0(\tfrac{w}{p})+A_0(\tfrac{w'}{p})\right)}$$

Conclusion

This note introduces a simple method for calculating strip to backplane capacitances for microstrip detectors as a function of bias voltage. These solutions indicate slight differences between the diode depletion model and the strip detector. In particular, the voltage to bias a detector is increased while the final body capacitance is decreased. Slight differences in the shape of the C-V curve might also be measurable. All of the results in this note have been thoroughly checked using numerical solutions to Poisson's equation on a discrete mesh, using the method of [2].

References

[1] Sneddon, I.N., *Mixed Boundary Value Problems in Potential Theory*, North-Holland, Amsterdam and Wiley-Interscience, New York (1966).

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[3] C. LeVier, *Capacitance in Silicon Strip Detectors*, UC Santa Cruz Senior Thesis, SCIPP 92/26.

[4] E. Barberis *et al.* Capacitances in Silicon Microstrip Detectors, presented at the *International Symposium on Development and Application of Semiconductor Tracking Detectors* at Hiroshima, Japan, May 22-24, 1993.