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The Higgs Sector in the Minimal Supersymmetric Model: Radiative Corrections and Their Implications*

HOWARD E. HABER
Santa Cruz Institute for Particle Physics
University of California, Santa Cruz, CA 95064

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Abstract

The Higgs sector of the Minimal Supersymmetric Model (MSSM) is a CP-conserving two-Higgs doublet model which depends (at tree-level) on two Higgs parameters. As a result, this model is very predictive and testable in future experiments. When radiative corrections are included, some of the tree-level predictions of the model are substantially altered if m_t is large. Various implications of the radiatively corrected MSSM Higgs sector are explored. The theoretical upper bound to the lightest Higgs mass is determined and the influence of radiative corrections on Higgs masses and couplings is exhibited. Implications for Higgs phenomenology are briefly discussed.

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Abstract

The Higgs sector of the Minimal Supersymmetric Model (MSSM) is a CP-conserving two-Higgs doublet model which depends (at tree-level) on two Higgs parameters. As a result, this model is very predictive and testable in future experiments. When radiative corrections are included, some of the tree-level predictions of the model are substantially altered if m_t is large. Various implications of the radiatively corrected MSSM Higgs sector are explored. The theoretical upper bound to the lightest Higgs mass is determined and the influence of radiative corrections on Higgs masses and couplings is exhibited. Implications for Higgs phenomenology are briefly discussed.

1. The Two-Higgs Doublet Model

I shall begin with a brief review of the general (non-supersymmetric) two-Higgs doublet extension of the Standard Model.[1] Let Φ_1 and Φ_2 denote two complex $Y = 1$, $SU(2)_L$ doublet scalar fields. The most general gauge invariant scalar potential is given by

$$\begin{aligned}\mathcal{V} = {} & m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - [m_{12}^2\Phi_1^\dagger\Phi_2 + \text{h.c.}] \\ & + \tfrac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1)^2 + \tfrac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\ & + \left\{\tfrac{1}{2}\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \left[\lambda_6(\Phi_1^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)\right]\Phi_1^\dagger\Phi_2 + \text{h.c.}\right\}.\end{aligned} \tag{1}$$

In most discussions of two-Higgs-doublet models, the terms proportional to λ_6 and λ_7 are absent. This can be achieved by imposing a discrete symmetry $\Phi_1 \to -\Phi_1$ on the model. Such a symmetry would also require $m_{12} = 0$ unless we allow a soft violation of this discrete symmetry by dimension-two terms.* For the moment, I will refrain from setting any of the coefficients in eq. (1) to zero. In principle, m_{12}^2, λ_5, λ_6 and λ_7 can be complex. In this paper, I shall ignore the possibility of CP-violating effects in the Higgs sector by choosing all coefficients in eq. (1) to be real. The scalar fields will develop non-zero vacuum expectation values if the mass matrix m_{ij}^2 has at least one negative eigenvalue. Imposing CP invariance and $U(1)_{EM}$ gauge symmetry, the minimum of the potential is

$$\langle\Phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v_1\end{pmatrix}, \qquad \langle\Phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v_2\end{pmatrix}, \tag{2}$$

where the v_i can be chosen to be real. It is convenient to introduce the following notation:

$$v^2 \equiv v_1^2 + v_2^2, \qquad t_\beta \equiv \tan\beta \equiv \frac{v_2}{v_1}. \tag{3}$$

Of the original eight scalar degrees of freedom, three Goldstone bosons are absorbed ("eaten") by the $W^\pm$ and Z. The remaining five physical Higgs particles are: two CP-even scalars (H^0 and h^0, with $m_{h^0} \le m_{H^0}$), one CP-odd scalar (A^0) and a charged Higgs pair ($H^\pm$). The mass parameters m_{11} and m_{22} can be eliminated by minimizing the scalar potential. The resulting squared masses for the CP-odd and charged Higgs states are

$$\begin{aligned} m_{A^0}^2 &= \frac{m_{12}^2}{s_\beta c_\beta} - \tfrac{1}{2}v^2\left(2\lambda_5 + \lambda_6 t_\beta^{-1} + \lambda_7 t_\beta\right), \\ m_{H^\pm}^2 &= m_{A^0}^2 + \tfrac{1}{2}v^2(\lambda_5 - \lambda_4). \end{aligned} \tag{4}$$

The two CP-even Higgs states mix according to the following squared mass matrix:

$$\begin{aligned}\mathcal{M}^2 = {} & m_{A^0}^2\begin{pmatrix} s_\beta^2 & -s_\beta c_\beta \\ -s_\beta c_\beta & c_\beta^2\end{pmatrix} \\ & + v^2\begin{pmatrix} \lambda_1 c_\beta^2 + 2\lambda_6 s_\beta c_\beta + \lambda_5 s_\beta^2 & (\lambda_3+\lambda_4)s_\beta c_\beta + \lambda_6 c_\beta^2 + \lambda_7 s_\beta^2 \\ (\lambda_3+\lambda_4)s_\beta c_\beta + \lambda_6 c_\beta^2 + \lambda_7 s_\beta^2 & \lambda_2 s_\beta^2 + 2\lambda_7 s_\beta c_\beta + \lambda_5 c_\beta^2\end{pmatrix},\end{aligned} \tag{5}$$

where $s_\beta \equiv \sin\beta$ and $c_\beta \equiv \cos\beta$. The physical mass eigenstates are

$$\begin{aligned} H^0 &= (\sqrt{2}\,\mathrm{Re}\,\Phi_1^0 - v_1)\cos\alpha + (\sqrt{2}\,\mathrm{Re}\,\Phi_2^0 - v_2)\sin\alpha, \\ h^0 &= -(\sqrt{2}\,\mathrm{Re}\,\Phi_1^0 - v_1)\sin\alpha + (\sqrt{2}\,\mathrm{Re}\,\Phi_2^0 - v_2)\cos\alpha. \end{aligned} \tag{6}$$

The corresponding masses are

$$m_{H^0,h^0}^2 = \tfrac{1}{2}\left[\mathcal{M}_{11}^2 + \mathcal{M}_{22}^2 \pm \sqrt{(\mathcal{M}_{11}^2 - \mathcal{M}_{22}^2)^2 + 4(\mathcal{M}_{12}^2)^2}\,\right], \tag{7}$$

* This latter requirement is sufficient to guarantee the absence of Higgs-mediated tree-level flavor changing neutral currents.

and the mixing angle α is obtained from

$$\sin 2\alpha = \frac{2\mathcal{M}_{12}^2}{\sqrt{(\mathcal{M}_{11}^2 - \mathcal{M}_{22}^2)^2 + 4(\mathcal{M}_{12}^2)^2}}\,,$$
$$\cos 2\alpha = \frac{\mathcal{M}_{11}^2 - \mathcal{M}_{22}^2}{\sqrt{(\mathcal{M}_{11}^2 - \mathcal{M}_{22}^2)^2 + 4(\mathcal{M}_{12}^2)^2}}\,. \tag{8}$$

The phenomenology of the two-Higgs doublet model depends in detail on the various couplings of the Higgs bosons to gauge bosons, Higgs bosons and fermions. The Higgs couplings to gauge bosons follow from gauge invariance and are thus model independent. For example, the coupling of the two CP-even Higgs bosons to W and Z pairs is given in terms of the angles α and β by

$$g_{h^0VV} = g_V m_V \sin(\beta - \alpha)$$
$$g_{H^0VV} = g_V m_V \cos(\beta - \alpha)\,, \tag{9}$$

where $g_V \equiv g\ [g/\cos\theta_W]$ for $V = W\ [Z]$. There are no tree-level couplings of A^0 or $H^\pm$ to VV. Gauge invariance also determines the strength of the trilinear couplings of one gauge boson to two Higgs bosons. For example,

$$g_{h^0A^0Z} = \frac{g\cos(\beta-\alpha)}{2\cos\theta_W}\,,$$
$$g_{H^0A^0Z} = \frac{-g\sin(\beta-\alpha)}{2\cos\theta_W}\,. \tag{10}$$

The pattern of couplings of h^0 and H^0 to $W^\pm H^\mp$ is similar. In summary, I record below the "angle factor" that appears in the various Higgs boson–gauge boson couplings

$\cos(\beta-\alpha)$	$\sin(\beta-\alpha)$
$H^0W^+W^-$	$h^0W^+W^-$
H^0ZZ	h^0ZZ
ZA^0h^0	ZA^0H^0
$W^\pm H^\mp h^0$	$W^\pm H^\mp H^0$

(11)

The Higgs couplings to fermions are model dependent, although their form is often constrained by discrete symmetries that are imposed in order to avoid tree-level flavor changing neutral currents mediated by Higgs exchange.[2] An example of a model that respects this constraint is one in which one Higgs doublet (before symmetry breaking) couples exclusively to down-type fermions and the other Higgs doublet couples exclusively to up-type fermions. This is the pattern of couplings found in the minimal supersymmetric model (MSSM). The results in this case are as follows. The couplings of the neutral Higgs bosons to $f\bar{f}$ relative to the Standard Model value, $gm_f/2m_W$, are given by (using 3rd family notation)

$$\begin{array}{llll} H^0t\bar{t}: & \dfrac{\sin\alpha}{\sin\beta} & H^0b\bar{b}: & \dfrac{\cos\alpha}{\cos\beta} \\ h^0t\bar{t}: & \dfrac{\cos\alpha}{\sin\beta} & h^0b\bar{b}: & \dfrac{-\sin\alpha}{\cos\beta} \\ A^0t\bar{t}: & \gamma_5\cot\beta & A^0b\bar{b}: & \gamma_5\tan\beta, \end{array} \tag{12}$$

(the γ_5 indicates a pseudoscalar coupling), and the charged Higgs boson coupling is given by

$$g_{H^-t\bar{b}} = \frac{g}{2\sqrt{2}m_W}\,[m_t\cot\beta(1+\gamma_5) + m_b\tan\beta(1-\gamma_5)]. \tag{13}$$

Finally, the 3-point and 4-point Higgs self-couplings depend on the two-Higgs-doublet potential [eq. (1)]. The Feynman rules for the most important trilinear Higgs vertices are listed below:

$$\begin{aligned} g_{h^0A^0A^0} &= \frac{2m_W}{g}\big[\lambda_1 s_\beta^2 c_\beta s_\alpha - \lambda_2 c_\beta^2 s_\beta c_\alpha - \tilde{\lambda}_3(s_\beta^3 c_\alpha - c_\beta^3 s_\alpha) + 2\lambda_5 s_{\beta-\alpha} \\ &\quad - \lambda_6 s_\beta(c_\beta s_{\alpha+\beta} + s_\alpha c_{2\beta}) - \lambda_7 c_\beta(c_\alpha c_{2\beta} - s_\beta s_{\alpha+\beta})\big]\,, \\ g_{H^0A^0A^0} &= \frac{-2m_W}{g}\big[\lambda_1 s_\beta^2 c_\beta c_\alpha + \lambda_2 c_\beta^2 s_\beta s_\alpha + \tilde{\lambda}_3(s_\beta^3 s_\alpha + c_\beta^3 c_\alpha) - 2\lambda_5 c_{\beta-\alpha} \\ &\quad - \lambda_6 s_\beta(c_\beta c_{\alpha+\beta} + c_\alpha c_{2\beta}) + \lambda_7 c_\beta(s_\beta c_{\alpha+\beta} + s_\alpha c_{2\beta})\big]\,, \\ g_{H^0h^0h^0} &= \frac{-6m_W}{g}\big[\lambda_1 s_\alpha^2 c_\beta c_\alpha + \lambda_2 c_\alpha^2 s_\beta s_\alpha + \tilde{\lambda}_3(s_\alpha^3 s_\beta + c_\alpha^3 c_\beta - \tfrac{2}{3}c_{\beta-\alpha}) \\ &\quad - \lambda_6 s_\alpha(c_\beta c_{2\alpha} + c_\alpha c_{\alpha+\beta}) + \lambda_7 c_\alpha(s_\beta c_{2\alpha} + s_\alpha c_{\alpha+\beta})\big]\,, \\ g_{H^0H^+H^-} &= g_{H^0A^0A^0} - \frac{2m_W}{g}(\lambda_5 - \lambda_4)c_{\beta-\alpha}\,, \\ g_{h^0H^+H^-} &= g_{h^0A^0A^0} - \frac{2m_W}{g}(\lambda_5 - \lambda_4)s_{\beta-\alpha}\,, \end{aligned} \tag{14}$$

where I have used the notation

$$\tilde{\lambda}_3 \equiv \lambda_3 + \lambda_4 + \lambda_5\,. \tag{15}$$

It is interesting to note that couplings of the charged Higgs bosons satisfy relations analogous to that of $m_{H^\pm}$ given in eq. (4).

Finally, consider the experimental constraints on the parameters of the two-Higgs doublet model. Limits on the charged and neutral Higgs masses have been obtained at LEP. The LEP limits on the charged Higgs mass depend only on $m_{H^\pm}$ and the ratio of branching fractions $BR(H^+ \to \tau^+\nu)/BR(H^+ \to c\bar{s})$. All four LEP detector collaborations quote similar limits on the charged Higgs mass.[3] The most conservative limit is $m_{H^\pm} > 36.5$ GeV (at 95% confidence level) if the hadronic decay modes dominate. The limit improves if some fraction of the charged Higgs bosons decay to $\tau\nu$, reaching $m_{H^\pm} > 43$ GeV if $BR(H^+ \to \tau^+\nu) = 100\%$. The LEP limits on the masses of h^0 and A^0 are obtained by searching simultaneously for $Z \to h^0 f\bar{f}$ and $Z \to h^0 A^0$.[4,5] The ZZh^0 and Zh^0A^0 couplings which govern these two decay rates depend on $\sin(\beta - \alpha)$ [see eqs. (9) and (10)]. Thus, one can use the LEP data to deduce limits on m_{h^0} and m_{A^0} as a function of $\sin(\beta - \alpha)$.[5] Stronger limits can be obtained in the MSSM where $\sin(\beta - \alpha)$ is fixed by other model parameters. The most complete analysis published to date is by the ALEPH Collaboration.[5] Limits of $m_{h^0} > 41$ GeV and $m_{A^0} > 20$ GeV (at 95% CL) are obtained when other MSSM model parameters are varied within their allowed ranges.

The experimental information on the parameter $\tan\beta$ is quite meager. For definiteness, I shall assume that the Higgs-fermion couplings are specified as shown in eqs. (12) and (13). In the Standard Model, the Higgs coupling to top quarks is proportional to $gm_t/2m_W$, and is therefore the strongest of all Higgs-fermion couplings. For $\tan\beta < 1$, the Higgs couplings to top-quarks are further enhanced by a factor of $1/\tan\beta$. As a result, some weak experimental limits on $\tan\beta$ exist based on the non-observation of virtual effects involving the $H^-t\bar{b}$ coupling. Clearly, such limits depend both on $m_{H^\pm}$ and $\tan\beta$. For example, for $m_{H^\pm} \simeq m_W$, limits from the analysis of B^0–$\overline{B^0}$ mixing imply that $\tan\beta \gtrsim 0.5$.[6] No comparable limits exist based on top-quark couplings to neutral Higgs bosons.

Theoretical constraints on $\tan\beta$ are also useful. If $\tan\beta$ becomes too small, then the Higgs coupling to top quarks becomes strong. In this case, the tree-unitarity of processes involving the Higgs-top quark Yukawa coupling is violated. Perhaps this should not be regarded as a theoretical defect, although it does render any perturbative analysis unreliable. A rough lower bound advocated by ref. 6, $\tan\beta \gtrsim m_t/600$ GeV, corresponds to a Higgs-top quark coupling in the perturbative region. A similar argument involving the Higgs-bottom quark coupling would yield $\tan\beta \lesssim 120$. A more solid theoretical constraint is based on the requirement that Higgs-fermion couplings remain finite when running from the electroweak scale to some large energy scale Λ. (Beyond Λ, one assumes that new physics enters.) The limits on $\tan\beta$ depend on m_t and the choice of the high energy scale Λ. Using the renormalization group equations given in the Appendix, we integrate from the

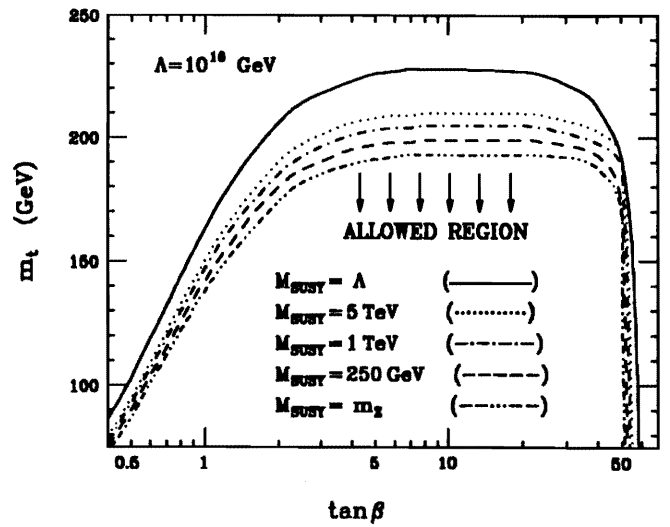

Fig. 1. The region of $\tan\beta$–m_t parameter space in which all running Higgs-fermion Yukawa couplings remain finite at all energy scales, μ, from m_Z to $\Lambda = 10^{16}$ GeV.[7] Non-supersymmetric two-Higgs-doublet (one-loop) renormalization group equations (RGEs) are used for $m_Z \le \mu \le M_{\rm SUSY}$ and the RGEs of the minimal supersymmetric model are used for $M_{\rm SUSY} \le \mu \le \Lambda$ (see the Appendix). Five different values of $M_{\rm SUSY}$ are shown; the allowed parameter space lies below the respective curves.

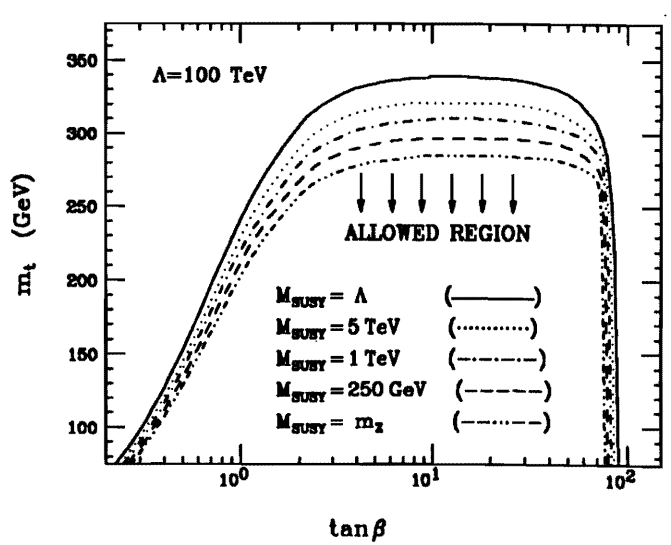

Fig. 2. The region of $\tan\beta$–m_t parameter space in which all running Higgs-fermion Yukawa couplings remain finite at all energy scales from m_Z to $\Lambda = 100$ TeV. See caption to fig. 1.

electroweak scale to Λ (allowing for the possible existence of a supersymmetry-breaking scale, $m_Z \leq M_{\rm SUSY} \leq \Lambda$), and determine the region of $\tan\beta$–m_t parameter space in which the Higgs-fermion Yukawa couplings remain finite.[7,8] (The t, b and τ are all included in the analysis.) The results are shown in figs. 1 and 2 for two different choices of Λ.[7] The allowed region of parameter space lies below the curves shown. For example, if there is no new physics (other than perhaps minimal supersymmetry) below the grand unification scale of 10^{16} GeV, then based on the CDF limit[9] of $m_t > 91$ GeV, one would conclude that $0.5 \lesssim \tan\beta \lesssim 50$. The lower limit on $\tan\beta$ becomes even sharper if the top-quark mass is heavier. Remarkably, the limits on $\tan\beta$ do not get substantially weaker for Λ as low as 100 TeV. Finally, it is interesting to note that the limits on $\tan\beta$ shown in fig. 2 are not very different from those that emerge from models of low-energy supersymmetry based on supergravity which strongly favor $\tan\beta > 1$.[10]

2. The Higgs Sector of the MSSM at Tree Level

The Higgs sector of the MSSM is a CP-conserving two-Higgs-doublet model, with a Higgs potential whose dimension-four terms respect supersymmetry and with restricted Higgs-fermion couplings in which the $Y = -1$ [$Y = +1$] Higgs doublet couples only to down-type [up-type] fermions.[1] Using the notation of eq. (1), the quartic couplings λ_i are given by

$$\begin{aligned} \lambda_1 &= \lambda_2 = \tfrac{1}{4}(g^2 + g'^2)\,, \\ \lambda_3 &= \tfrac{1}{4}(g^2 - g'^2)\,, \\ \lambda_4 &= -\tfrac{1}{2}g^2\,, \\ \lambda_5 &= \lambda_6 = \lambda_7 = 0\,. \end{aligned} \tag{16}$$

Inserting these results into eqs. (4) and (5), it follows that

$$\begin{aligned} m^2_{A^0} &= m^2_{12}(\tan\beta + \cot\beta)\,, \\ m^2_{H^\pm} &= m^2_{A^0} + m^2_W\,, \end{aligned} \tag{17}$$

and the neutral CP-even mass matrix is given by

$$\mathcal{M}^2 = \begin{pmatrix} m^2_{A^0}\sin^2\beta + m^2_Z\cos^2\beta & -(m^2_{A^0} + m^2_Z)\sin\beta\cos\beta \\ -(m^2_{A^0} + m^2_Z)\sin\beta\cos\beta & m^2_{A^0}\cos^2\beta + m^2_Z\sin^2\beta \end{pmatrix}. \tag{18}$$

The eigenvalues of $\mathcal{M}^2$ are the squared masses of the two CP-even Higgs scalars

$$m^2_{H^0,h^0} = \tfrac{1}{2}\left(m^2_{A^0} + m^2_Z \pm \sqrt{(m^2_{A^0} + m^2_Z)^2 - 4m^2_Z m^2_{A^0}\cos^2 2\beta}\right). \tag{19}$$

and the diagonalizing angle is α, with

$$\cos 2\alpha = -\cos 2\beta\left(\frac{m^2_{A^0} - m^2_Z}{m^2_{H^0} - m^2_{h^0}}\right), \qquad \sin 2\alpha = -\sin 2\beta\left(\frac{m^2_{H^0} + m^2_{h^0}}{m^2_{H^0} - m^2_{h^0}}\right). \tag{20}$$

From the expressions for the Higgs masses obtained above, the following inequalities are easily established

$$\begin{aligned} m_{h^0} &\leq m_{A^0} \\ m_{h^0} &\leq m|\cos 2\beta| \leq m_Z\,, \qquad \text{with } m \equiv min(m_Z, m_{A^0}) \\ m_{H^0} &\geq m_Z\,, \\ m_{H^\pm} &\geq m_W\,. \end{aligned} \tag{21}$$

Thus, in the MSSM, two parameters (conveniently chosen to be m_{A^0} and $\tan\beta$) suffice to fix all other tree-level Higgs sector parameters.

3. A Theoretical Upper Limit on the Lightest MSSM Higgs Mass

The tree-level predictions of section 2 have important phenomenological consequences. For example, the bound $m_{h^0} \leq m_Z$, if reliable, would have significant implications for future experiments at LEP-II. In principle, experiments running at LEP-II operating at $\sqrt{s} = 200$ GeV and design luminosity would either discover the Higgs boson (via $e^+e^- \to h^0 Z$) or rule out the MSSM. (Whether this is possible to do in practice depends on whether Higgs bosons with $m_{h^0} \approx m_Z$ can be detected.[11]) However, $m_{h^0} \leq m_Z$ need not be respected when radiative corrections are incorporated. In the radiative corrections to the neutral CP-even Higgs squared-mass matrix, the 22-element is shifted by a term proportional to $(g^2 m_t^4/m_W^2)\,\ln(M_{\tilde{t}}^2/m_t^2)$.[12–14] Such a term arises from an incomplete cancellation between top-quark and top-squark loop contributions to the neutral Higgs boson self-energy. If m_t is large, this term significantly alters the tree-level predictions.

Hempfling and I computed the exact one-loop expression for the light Higgs mass bound, as a function of all the relevant supersymmetric parameters.[12] This bound is saturated in the formal limit where $\tan\beta \to \infty$ (with all down-type fermions masses set equal to zero) and $m_{A^0} \geq m_Z$, m_{h^0}. The expression we obtained is quite cumbersome, although straightforward to evaluate numerically. However, it is useful to display an approximate expression, valid for a certain range of supersymmetric parameters. If all supersymmetric mass parameters are roughly

of order $M_{\rm SUSY}$ and if $m_Z < m_t \ll M_{\rm SUSY}$, then

$$m_{h^0}^2 - m_Z^2 \lesssim \frac{3g^2 m_Z^2}{16\pi^2 c_W^2}\left\{ \ln\left(\frac{M_{\tilde{Q}}^2}{m_t^2}\right)\left[\frac{2m_t^4 - m_t^2 m_Z^2}{m_Z^4} + \tfrac{1}{6}\left(1 - \tfrac{8}{3}s_W^2 + \tfrac{32}{9}s_W^4\right)\right] \right.$$
$$+\ln\left(\frac{M_{\tilde{Q}}^2}{m_Z^2}\right)\left[\tfrac{1}{3}\left(1 - \tfrac{8}{3}s_W^2 + \tfrac{32}{9}s_W^4\right) + \tfrac{1}{2}\left(1 - \tfrac{4}{3}s_W^2 + \tfrac{8}{9}s_W^4\right)\right.$$
$$\left.\left. +\tfrac{1}{3}\left(1 - 2s_W^2 + 4s_W^4\right)\right] + \frac{m_t^2}{3m_Z^2}\right\}$$
$$-\frac{g^2 m_Z^2}{48\pi^2 c_W^2}\left[(27 - 54s_W^2 + 32s_W^4)\ln\left(\frac{M_{\tilde{\chi}}^2}{m_Z^2}\right) - (1 - 2s_W^2 + 2s_W^4)\ln\left(\frac{m_{A^0}^2}{m_Z^2}\right)\right], \tag{22}$$

where $s_W \equiv \sin\theta_W$, $c_W \equiv \cos\theta_W$, $M_{\tilde{Q}}$ is a common soft-supersymmetry-breaking diagonal squark mass term, and $M_{\tilde{\chi}}$ is a common neutralino/chargino mass. The radiatively corrected light Higgs mass bound will be written as

$$m_{h^0} \leq m_Z + \Delta m_h\,, \tag{23}$$

which defines the quantity Δm_h. A numerical calculation of Δm_h is displayed in fig. 3. As advertised, the dominant correction to the tree-level formula increases as the fourth power of m_t, and therefore can be quite large. Nevertheless, for values of $m_t \lesssim 250$ GeV, the perturbative one-loop calculation is reliable. This can be verified by estimating the largest two-loop contributions to Δm_h and showing that the one-loop result is stable.[15]

It is also evident from eq. (22) that the dependence of the squared Higgs mass shift on $M_{\tilde{Q}}^2$ is logarithmic. Thus, even if $M_{\tilde{Q}}$ is significantly smaller than 1 TeV, Δm_h can be appreciable if m_t is sufficiently large. This is illustrated in fig. 4, where Δm_h is plotted as a function of $M_{\tilde{Q}}$ for $m_t = 100$, 150 and 200 GeV. These results are based on an exact numerical one-loop computation; the approximate formula given in eq. (22) is unreliable for values of $M_{\tilde{Q}}$ approaching m_t.

4. Radiative Corrections to the MSSM Higgs Masses

One can also compute radiative corrections to the full CP-even Higgs mass-squared matrix. Various calculations employing various approximations have appeared in the literature.[14,16–21] Here, I will present the results based on a calculation of the mass-squared matrix in which all leading logarithmic terms are included. Details can be found in ref. 17. The method goes as follows. We take the supersymmetry breaking scale ($M_{\rm SUSY}$) to be somewhat larger than the electroweak scale.

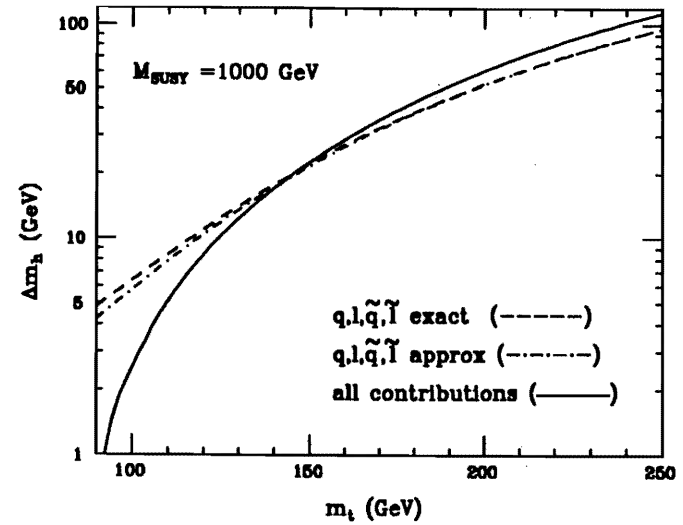

Fig. 3. The light Higgs mass bound ($m_{h^0} \leq m_Z + \Delta m_h$) including one-loop radiative corrections. The dashed line denotes the contribution to Δm_h due to three generations of quarks, leptons, and their supersymmetric scalar partners (assumed to have a common soft-supersymmetry breaking mass of $M_{\tilde{Q}} = 1$ TeV); $\tilde{t}_L$-$\tilde{t}_R$ mixing is neglected. The dot-dashed line is a plot of the corresponding contribution to eq. (22). The solid line includes *all* contributions to the exact one-loop calculation of Δm_h where all supersymmetric mass parameters (including the A parameter that controls top-squark mixing) are equal to $M_{\rm SUSY} = 1$ TeV.

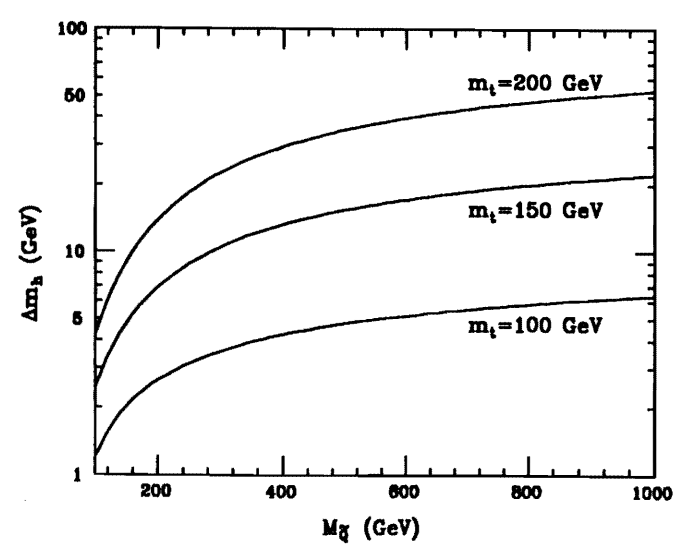

Fig. 4. The contribution to Δm_h of three generations of quarks, leptons, and their supersymmetric scalar partners as a function of the common soft-supersymmetry breaking scalar mass, $M_{\tilde{Q}}$, for three different values of the top quark mass. Squark mixing is neglected.

For simplicity, we assume that the masses of all supersymmetric particles (squarks, sleptons, neutralinos and charginos) are roughly degenerate and of order $M_{\rm SUSY}$. This means that various soft-supersymmetry breaking parameters such as the diagonal squark mass parameter, $M_{\tilde{Q}}$, and the gaugino Majorana mass terms, as well as the supersymmetric Higgs mass parameter are all roughly equal to $M_{\rm SUSY}$. Admittedly, this is a crude approximation. However, deviations from this assumption will lead to non-leading logarithmic corrections which tend to be small if the supersymmetric particles are not widely split in mass.

The leading logarithmic expressions for Higgs masses are obtained from eqs. (4) and (5) by treating the λ_i as running parameters evaluated at the electroweak scale, $M_{\rm weak}$. In addition, we identify the W and Z masses by

$$\begin{aligned} m_W^2 &= \tfrac{1}{4}g^2(v_1^2+v_2^2)\,, \\ m_Z^2 &= \tfrac{1}{4}(g^2+g'^2)(v_1^2+v_2^2)\,, \end{aligned} \tag{24}$$

where the running gauge couplings are evaluated at $M_{\rm weak}$. Of course, the gauge couplings, g and g' are known from experimental measurements which are performed at the scale $M_{\rm weak}$. The $\lambda_i(M_{\rm weak}^2)$ are determined from supersymmetry. Namely, if supersymmetry were unbroken, then the λ_i would be fixed according to eq. (16). Since supersymmetry is broken, we regard eq. (16) as boundary conditions for the running parameters, valid at (and above) the energy scale $M_{\rm SUSY}$. That is, we take

$$\begin{aligned} \lambda_1(M_{\rm SUSY}^2) &= \lambda_2(M_{\rm SUSY}^2) = \tfrac{1}{4}[g^2(M_{\rm SUSY}^2)+g'^2(M_{\rm SUSY}^2)], \\ \lambda_3(M_{\rm SUSY}^2) &= \tfrac{1}{4}[g^2(M_{\rm SUSY}^2)-g'^2(M_{\rm SUSY}^2)], \\ \lambda_4(M_{\rm SUSY}^2) &= -\tfrac{1}{2}g^2(M_{\rm SUSY}^2), \\ \lambda_5(M_{\rm SUSY}^2) &= \lambda_6(M_{\rm SUSY}^2) = \lambda_7(M_{\rm SUSY}^2) = 0\,, \end{aligned} \tag{25}$$

in accordance with the tree-level relations of the MSSM. At scales below $M_{\rm SUSY}$, the gauge and quartic couplings evolve according to the renormalization group equations (RGEs) of the non-supersymmetric two-Higgs-doublet model given in eqs. (A.5)–(A.7). These equations are of the form:

$$\frac{dp_i}{dt} = \beta_i(p_1,p_2,\ldots) \qquad \text{with } t \equiv \ln\mu^2\,, \tag{26}$$

where μ is the energy scale, and the p_i are the parameters of the theory ($p_i = g_j^2, \lambda_k, \ldots$). The relevant β-functions can be found in the Appendix. The boundary conditions together with the RGEs imply that, at the leading-log level, λ_5, λ_6 and λ_7 are zero at all energy scales. Solving the RGEs with the supersymmetric boundary conditions at $M_{\rm SUSY}$, one can determine the λ_i at the weak scale. The resulting values for $\lambda_i(M_{\rm weak})$ are then inserted into eqs. (4) and (5) to obtain the radiatively corrected Higgs masses. Having solved the one-loop RGEs, the Higgs masses that result correctly include the leading logarithmic radiative corrections summed to all orders in perturbation theory.

The RGEs can be solved by numerical analysis on the computer. But it is instructive to solve the RGEs iteratively. In first approximation, we can take the right hand side of eq. (26) to be independent of μ^2. That is, we compute the β_i by evaluating the parameters p_i at the scale $\mu = M_{\rm SUSY}$. Then, integration of the RGEs is trivial, and we obtain

$$p_i(M_{\rm weak}^2) = p_i(M_{\rm SUSY}^2) - \beta_i \ln\left(\frac{M_{\rm SUSY}^2}{M_{\rm weak}^2}\right). \tag{27}$$

Note that this iterative solution corresponds to computing the one-loop radiative corrections in which only terms proportional to $\ln M_{\rm SUSY}^2$ are kept. It is straightforward to work out the one-loop leading logarithmic expressions for the λ_i and the Higgs masses. First consider the charged Higgs mass. Since $\lambda_5(\mu^2) = 0$ at all scales, we need only consider λ_4. Evaluating β_{λ_4} at $\mu = M_{\rm SUSY}$, we compute

$$\begin{aligned} \lambda_4(m_W^2) = -\tfrac{1}{2}g^2 - \frac{1}{32\pi^2}\Bigg[&\left(\tfrac{4}{3}N_g + \tfrac{1}{6}N_H - \tfrac{10}{3}\right)g^4 + 5g^2g'^2 \\ &- \frac{3g^4}{2m_W^2}\left(\frac{m_t^2}{s_\beta^2}+\frac{m_b^2}{c_\beta^2}\right) + \frac{3g^2m_t^2m_b^2}{s_\beta^2c_\beta^2m_W^4}\Bigg]\ln\frac{M_{\rm SUSY}^2}{m_W^2}\,. \end{aligned} \tag{28}$$

The terms proportional to the number of generations $N_g = 3$ and the number of Higgs doublets $N_H = 2$ that remain in the low-energy effective theory at the scale $\mu = m_W$ have their origin in the running of g^2 from $M_{\rm SUSY}$ down to m_W. In deriving this expression, I have taken $M_{\rm weak} = m_W$. This is a somewhat arbitrary decision, since another reasonable choice would yield a result that differs from eq. (28) by a non-leading logarithmic term. Comparisons with a more complete calculation show that one should choose $M_{\rm weak} = m_W$ in computations involving the charged Higgs (and gauge) sector, and $M_{\rm weak} = m_Z$ in computations involving the neutral sector.

The above analysis also assumes that $m_t \sim \mathcal{O}(m_W)$. Although this is a good assumption, we can improve the above result somewhat when $m_t > m_W$ by decoupling the (t,b) weak doublet from the low-energy theory for scales below m_t. The terms in eq. (28) that are proportional to m_t^2 and/or m_b^2 arise from self-energy diagrams containing a tb loop. Thus, such a term should not be present for

$m_W \leq \mu \leq m_t$. In addition, we recognize the term in eq. (28) proportional to the number of generations N_g as arising from the contributions to the self-energy diagrams containing either quark or lepton loops (and their supersymmetric partners). To identify the contribution of the tb loop to this term, simply write

$$N_g = \tfrac{1}{4}N_g(N_c+1) = \tfrac{1}{4}N_c + \tfrac{1}{4}[N_c(N_g-1)+N_g]\,, \tag{29}$$

where $N_c = 3$ colors. Thus, we identify $\frac{1}{4}N_c$ as the piece of the term proportional to N_g that is due to the tb loop. The rest of this term is then attributed to the lighter quarks and leptons. Finally, the remaining terms in eq. (28) are due to the contributions from the gauge and Higgs boson sector. The final result is[22]

$$\begin{aligned}\lambda_4(m_W^2) = &-\tfrac{1}{2}g^2 - \frac{N_c g^4}{32\pi^2}\left[\frac{1}{3} - \frac{1}{2m_W^2}\left(\frac{m_t^2}{s_\beta^2}+\frac{m_b^2}{c_\beta^2}\right) + \frac{m_t^2 m_b^2}{s_\beta^2 c_\beta^2 m_W^4}\right]\ln\frac{M_{\rm SUSY}^2}{m_t^2}\\ &- \frac{1}{96\pi^2}\left\{\left[N_c(N_g-1)+N_g+\tfrac{1}{2}N_H-10\right]g^4 + 15g^2g'^2\right\}\ln\frac{M_{\rm SUSY}^2}{m_W^2}\,.\end{aligned} \tag{30}$$

Inserting this result (and $\lambda_5 = 0$) into eq. (4), we obtain the one-loop leading-log formula for the charged Higgs mass

$$\begin{aligned}m_{H^\pm}^2 = m_A^2 + m_W^2 &+ \frac{N_c g^2}{32\pi^2 m_W^2}\left[\frac{2m_t^2 m_b^2}{s_\beta^2 c_\beta^2} - m_W^2\left(\frac{m_t^2}{s_\beta^2}+\frac{m_b^2}{c_\beta^2}\right) + \tfrac{2}{3}m_W^4\right]\ln\frac{M_{\rm SUSY}^2}{m_t^2}\\ &+ \frac{m_W^2}{48\pi^2}\left\{\left[N_c(N_g-1)+N_g+\tfrac{1}{2}N_H-10\right]g^2 + 15g'^2\right\}\ln\frac{M_{\rm SUSY}^2}{m_W^2}\,.\end{aligned} \tag{31}$$

Since this derivation makes use of the two-Higgs-doublet RGEs for the λ_i, there is an implicit assumption that the full two-doublet Higgs spectrum survives in the low-energy effective theory at $\mu = m_W$. This means that we *must* take $N_H = 2$ in the formulae above. It also means that m_{A^0} cannot be much larger than m_W.* Of course, eq. (31) is only a one-loop result. This result is improved by using the full RGE solution to $\lambda_4(m_W^2)$

$$m_{H^\pm}^2 = m_{A^0}^2 - \tfrac{1}{2}\lambda_4(m_W^2)(v_1^2+v_2^2)\,. \tag{32}$$

Although the leading-log formula for $m_{H^\pm}$ [eq. (31)] gives a useful indication as to the size of the radiative corrections, non-leading logarithmic contributions can also be important in certain regions of parameter space. A more complete set of radiative corrections can be found in the literature.[16,22–25] In the numerical results to be exhibited below, important non-leading corrections to the charged Higgs mass are also included (as described in ref. 22). However, it should be emphasized that the radiative corrections to the charged Higgs mass are significant only for $\tan\beta < 1$, a region of MSSM parameter space not favored in supersymmetric models.

The computation of the neutral CP-even Higgs masses follows a similar procedure. The results are summarized below.[17] From eq. (5), we see that we only need results for λ_1, λ_2 and $\widetilde{\lambda}_3 \equiv \lambda_3+\lambda_4+\lambda_5$. (Recall that $\lambda_5 = \lambda_6 = \lambda_7 = 0$ at all energy scales.) By iterating the corresponding RGEs as before, we end up with

$$\begin{aligned}\lambda_1(m_Z^2) = &\ \tfrac{1}{4}[g^2+g'^2](m_Z^2) + \frac{g^4}{384\pi^2 c_W^4}\left[P_t\ln\left(\frac{M_{\rm SUSY}^2}{m_t^2}\right)\right.\\ &\left.+\left(12N_c\frac{m_b^4}{m_Z^4 c_\beta^4} - 6N_c\frac{m_b^2}{m_Z^2 c_\beta^2} + P_f + P_g + P_{2H}\right)\ln\left(\frac{M_{\rm SUSY}^2}{m_Z^2}\right)\right],\\ \lambda_2(m_Z^2) = &\ \tfrac{1}{4}[g^2+g'^2](m_Z^2) + \frac{g^4}{384\pi^2 c_W^4}\left[\left(P_f+P_g+P_{2H}\right)\ln\left(\frac{M_{\rm SUSY}^2}{m_Z^2}\right)\right.\\ &\left.+\left(12N_c\frac{m_t^4}{m_Z^4 s_\beta^4} - 6N_c\frac{m_t^2}{m_Z^2 s_\beta^2} + P_t\right)\ln\left(\frac{M_{\rm SUSY}^2}{m_t^2}\right)\right],\\ \widetilde{\lambda}_3(m_Z^2) = &-\tfrac{1}{4}[g^2+g'^2](m_Z^2) - \frac{g^4}{384\pi^2 c_W^4}\left[\left(-3N_c\frac{m_t^2}{m_Z^2 s_\beta^2} + P_t\right)\ln\left(\frac{M_{\rm SUSY}^2}{m_t^2}\right)\right.\\ &\left.+\left(-3N_c\frac{m_b^2}{m_Z^2 c_\beta^2} + P_f + P_g' + P_{2H}'\right)\ln\left(\frac{M_{\rm SUSY}^2}{m_Z^2}\right)\right],\end{aligned} \tag{33}$$

where

$$\begin{aligned}P_t &\equiv N_c(1-4e_u s_W^2 + 8e_u^2 s_W^4)\,,\\ P_f &\equiv N_g\{N_c[2-4s_W^2+8(e_d^2+e_u^2)s_W^4] + [2-4s_W^2+8s_W^4]\} - P_t\,,\\ P_g &\equiv -44+106s_W^2-62s_W^4\,,\\ P_g' &\equiv 10+34s_W^2-26s_W^4\,,\\ P_{2H} &\equiv -10+2s_W^2-2s_W^4\,,\\ P_{2H}' &\equiv 8-22s_W^2+10s_W^4\,.\end{aligned} \tag{34}$$

In the above formulae, the electric charges of the quarks are $e_u = 2/3$, $e_d = -1/3$, and the subscripts t, f, g and $2H$ indicate that these are the contributions from the

* If $m_{A^0} \sim \mathcal{O}(M_{\rm SUSY})$, then $H^\pm$, H^0 and A^0 would all have masses of order $M_{\rm SUSY}$, and the effective low-energy theory below $M_{\rm SUSY}$ would be that of the minimal Standard Model. Clearly, the above computation would not be appropriate in this case.

top quark, the fermions (leptons and quarks excluding the top quark), the gauge bosons and the two Higgs doublets (and corresponding supersymmetric partners), respectively. As in the derivation of $\lambda_4(m_W^2)$ above, we have improved our analysis by removing the effects of top-quark loops below $\mu = m_t$. The procedure to do this is subtle and is discussed in more detail in ref. 17. However, the following pedestrian technique works: consider the RGE for $g^2 + g'^2$ valid for $\mu < M_{\rm SUSY}$

$$\frac{d}{dt}(g^2+g'^2) = \frac{1}{96\pi^2}\left[\left(8g^4+\tfrac{40}{3}g'^4\right)N_g + (g^4+g'^4)N_H - 44g^4\right]. \tag{35}$$

This equation is used to run $g^2+g'^2$, which appears in eq. (25), from $M_{\rm SUSY}$ down to m_Z. As before, we identify the term proportional to N_g as corresponding to the fermion loops. We can explicitly extract the t-quark contribution by noting that

$$\begin{aligned} N_g\left(8g^4+\tfrac{40}{3}g'^4\right) &= \frac{g^4 N_g}{c_W^4}\left[\tfrac{64}{3}s_W^4 - 16s_W^2 + 8\right] \\ &= \frac{g^4}{c_W^4}\Big\{N_c[1+(N_g-1)](1-4e_u s_W^2 + 8e_u^2 s_W^4) \\ &\quad + N_c N_g(1+4e_d s_W^2 + 8e_d^2 s_W^4) + N_g(2-4s_W^2+8s_W^4)\Big\}, \end{aligned} \tag{36}$$

where in the first line of the last expression, the term proportional to 1 corresponds to the t-quark contribution while the term proportional to $N_g - 1$ accounts for the u and c-quarks; the second line contains the contributions from the down-type quarks and leptons respectively. Thus, iterating to one-loop,

$$\begin{aligned} [g^2+g'^2](M_{\rm SUSY}^2) &= [g^2+g'^2](m_Z^2) + \frac{g^4}{96\pi^2 c_W^4}\left[P_t \ln\left(\frac{M_{\rm SUSY}^2}{m_t^2}\right)\right. \\ &\quad \left. + \left[P_f + (s_W^4+c_W^4)N_H - 44c_W^4\right]\ln\left(\frac{M_{\rm SUSY}^2}{m_Z^2}\right)\right]. \end{aligned} \tag{37}$$

This result and terms that are proportional to m_t^2 and m_t^4 yield the terms in eq. (33) that contain $\ln(M_{\rm SUSY}^2/m_t^2)$.

The final step is to insert the expressions obtained in eq. (33) into eq. (5). The resulting matrix elements for the mass-squared matrix to one-loop leading logarithmic accuracy are given by

$$\begin{aligned} \mathcal{M}_{11}^2 &= m_{A^0}^2 s_\beta^2 + m_Z^2 c_\beta^2 + \frac{g^2 m_Z^2 c_\beta^2}{96\pi^2 c_W^2}\left[P_t \ln\left(\frac{M_{\rm SUSY}^2}{m_t^2}\right)\right. \\ &\quad \left. + \left(12N_c\frac{m_b^4}{m_Z^4 c_\beta^4} - 6N_c\frac{m_b^2}{m_Z^2 c_\beta^2} + P_f + P_g + P_{2H}\right)\ln\left(\frac{M_{\rm SUSY}^2}{m_Z^2}\right)\right] \\ \mathcal{M}_{22}^2 &= m_{A^0}^2 c_\beta^2 + m_Z^2 s_\beta^2 + \frac{g^2 m_Z^2 s_\beta^2}{96\pi^2 c_W^2}\left[\left(P_f + P_g + P_{2H}\right)\ln\left(\frac{M_{\rm SUSY}^2}{m_Z^2}\right)\right. \\ &\quad \left. + \left(12N_c\frac{m_t^4}{m_Z^4 s_\beta^4} - 6N_c\frac{m_t^2}{m_Z^2 s_\beta^2} + P_t\right)\ln\left(\frac{M_{\rm SUSY}^2}{m_t^2}\right)\right] \\ \mathcal{M}_{12}^2 &= -s_\beta c_\beta\Big\{m_{A^0}^2 + m_Z^2 + \frac{g^2 m_Z^2}{96\pi^2 c_W^2}\left[\left(P_t - 3N_c\frac{m_t^2}{m_Z^2 s_\beta^2}\right)\ln\left(\frac{M_{\rm SUSY}^2}{m_t^2}\right)\right. \\ &\quad \left. + \left(-3N_c\frac{m_b^2}{m_Z^2 c_\beta^2} + P_f + P_g' + P_{2H}'\right)\ln\left(\frac{M_{\rm SUSY}^2}{m_Z^2}\right)\right]\Big\}, \end{aligned} \tag{38}$$

Diagonalizing this matrix [eq. (38)] yields the radiatively corrected Higgs masses and mixing angle α. As above, we have implicitly assumed that m_{A^0} cannot be much larger than m_Z, since the effective low-energy theory contains the full two-Higgs-doublet spectrum. One can check that if $m_b = 0$ and $\sin\beta = 1$, then $m_{h^0}^2 = \mathcal{M}_{22}^2$ reproduces the leading logarithmic terms given in eq. (22) (after putting $M_{\tilde{Q}} = M_{\tilde{\chi}} = M_{\rm SUSY}$ and $m_{A^0} = m_Z$). The leading-log formulae presented above are expected to be accurate as long as: (i) the scale characterizing supersymmetric masses, $M_{\rm SUSY}$, is large and sufficiently separated from m_Z (say, $M_{\rm SUSY} \gtrsim 500$ GeV), (ii) m_t is somewhat above m_Z (say, $m_t \gtrsim 125$ GeV) while still being small compared to $M_{\rm SUSY}$, and (iii) the squark mixing parameters are not unduly large. In particular, (ii) is an important condition—it is the dominance of the leading $m_t^4 \ln(M_{\rm SUSY}^2/m_t^2)$ term that guarantees that the non-leading logarithmic terms are unimportant. I have checked the reliability of these results by comparing the predictions derived from eq. (38) with those of ref. 16 which includes non-leading logarithmic contributions to the scalar mass-squared matrix.

Results for the radiatively corrected Higgs masses are shown in figs. 5 and 6, for $M_{\rm SUSY} = 1$ TeV. A number of features are noteworthy. There exists a range of parameters in which the tree-level bound, $m_{h^0} \le m_{A^0}$ is violated. In fact, the results of fig. 5 indicate that in the region of small $\tan\beta$ and small m_{A^0}, it is possible to have $m_{h^0} > 2m_{A^0}$, which would permit the decay $h^0 \to A^0A^0$. The tree-level bound $m_{H^\pm} \ge m_W$ can also be violated, but only if $\tan\beta \lesssim 0.5$ and m_{A^0} is small

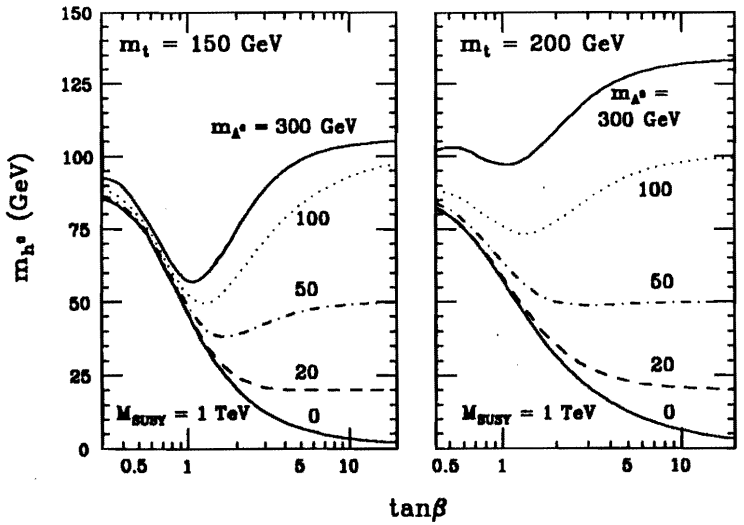

Fig. 5. The complete RGE-improved h^0 mass as a function of $\tan\beta$ for $m_t = 150$ and 200 GeV. The various curves shown correspond to $m_{A^0} = 0, 20, 50, 100$ and 300 GeV as indicated. For values of m_{A^0} above 300 GeV, the value of m_{h^0} depends very weakly on m_{A^0} and lies nearly on top of the upper curve shown in each figure.

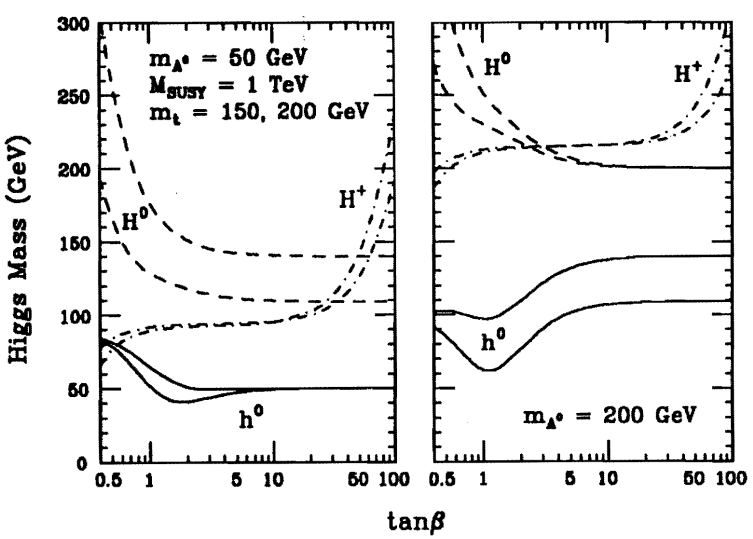

Fig. 6. The masses of h^0, H^0 and $H^\pm$ in the MSSM for $m_{A^0} = 50$ and 200 GeV. The neutral CP-even Higgs masses are obtained from a calculation that includes the leading-log one-loop radiative corrections. The charged Higgs mass is obtained from a similar calculation, but important non-leading logarithmic effects have also been included.[22] All supersymmetric masses are assumed to be roughly degenerate of order $M_{\rm SUSY} = 1$ TeV. The two curves for each Higgs mass shown correspond to $m_t = 150$ and 200 GeV. The larger neutral Higgs mass corresponds to the larger m_t choice. In the case of $H^\pm$, $m_{H^\pm}$ increases [decreases] with m_t for large [small] $\tan\beta$.

(see fig. 6). The small $\tan\beta$ region corresponds to an enhanced Higgs-top quark Yukawa coupling. This also explains the increase of m_{H^0} in this region, which is being controlled by the m_t^4/s_β^2 factor in $\mathcal{M}_{22}^2$ [eq. (38)]. Finally, this same factor is responsible for the violation of the bound $m_{h^0} \leq m_Z$ as described in section 3. Indeed, for $M_{\rm SUSY} = 1$ TeV, $m_t = 200$ GeV, and $m_{A^0} \gtrsim 200$ GeV, one sees that $m_{h^0} > m_Z$ independent of the value of $\tan\beta$. Thus, there is a non-negligible region of parameter space in which the h^0 is kinematically inaccessible to LEP-II.

5. Implications of the Radiatively Corrected Higgs Sector

Using the results of the previous section, one can obtain the leading logarithmic corrections to the various Higgs couplings, and proceed to investigate Higgs phenomenology in detail.[26] Here, I shall describe the procedure we use to obtain the Higgs couplings and briefly indicate some of the consequences. To obtain radiatively corrected couplings which are accurate in the leading logarithmic approximation, it is sufficient to use the tree-level couplings in which the parameters are taken to be running parameters evaluated at the electroweak scale. First, I remind the reader that $\tan\beta$ and m_{A^0} are input parameters. Next, we obtain the CP-even Higgs mixing angle α by diagonalizing the radiatively corrected CP-even Higgs mass matrix [eq. (38)]. With the angle α in hand one may compute, for example, $\cos(\beta-\alpha)$ and $\sin\alpha$. These results can be used to obtain the Higgs couplings to gauge bosons [eq. (11)] and fermions [eq. (12)]. Finally, the Higgs self-couplings [eq. (14)] are obtained by making use of eqs. (30) and (33) (with $\lambda_5 = \lambda_6 = \lambda_7 = 0$). The end result is a complete set of Higgs boson decay widths and branching ratios which include leading-log radiative corrections. For example, inserting the one-loop leading-log formulae for the λ_i into eq. (14), we find[27]

$$\begin{aligned}
\frac{g_{h^0A^0A^0}}{gm_Z/2c_W} &= -c_{2\beta}s_{\beta+\alpha}\left\{1+\frac{g^2}{96\pi^2c_W^2}\left[P_t\ln\left(\frac{M_{\rm SUSY}^2}{m_t^2}\right)+P_f\ln\left(\frac{M_{\rm SUSY}^2}{m_Z^2}\right)\right]\right\} \\
&+\frac{g^2N_c}{16\pi^2m_W^2m_Z^2}\left\{\left[\frac{s_\alpha s_\beta^2}{c_\beta^3}(2m_b^4-m_b^2m_Z^2c_\beta^2)-\frac{(c_\alpha s_\beta^3-s_\alpha c_\beta^3)}{2c_\beta^2}m_b^2m_Z^2\right]\ln\left(\frac{M_{\rm SUSY}^2}{m_Z^2}\right)\right. \\
&\left.-\left[\frac{c_\alpha c_\beta^2}{s_\beta^3}(2m_t^4-m_t^2m_Z^2s_\beta^2)+\frac{(c_\alpha s_\beta^3-s_\alpha c_\beta^3)}{2s_\beta^2}m_t^2m_Z^2\right]\ln\left(\frac{M_{\rm SUSY}^2}{m_t^2}\right)\right\} \\
&-\frac{g^2}{192\pi^2c_W^2}\left[s_{2\beta}c_{\beta+\alpha}(P_{2H}+P_g)-2(c_\alpha s_\beta^3-s_\alpha c_\beta^3)(P'_{2H}+P'_g)\right]\ln\left(\frac{M_{\rm SUSY}^2}{m_Z^2}\right).
\end{aligned} \tag{39}$$

When radiative corrections have been incorporated, new possibilities arise

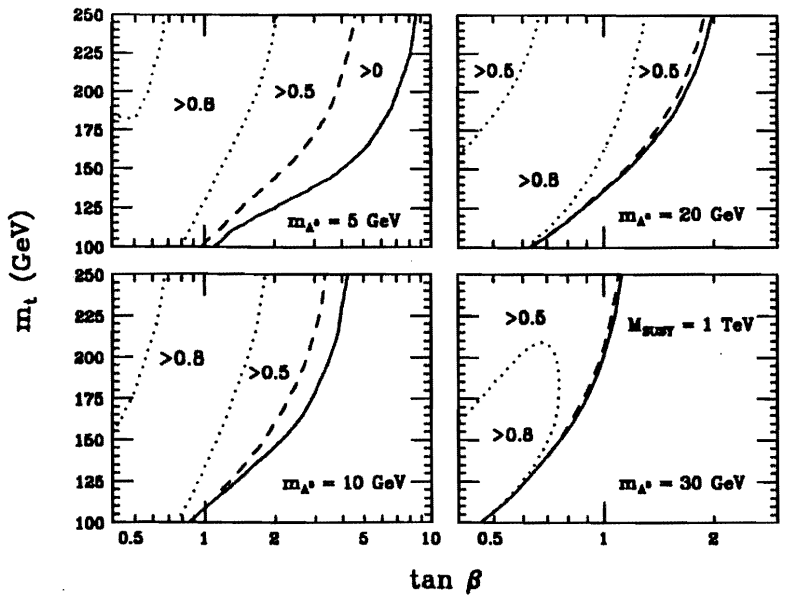

Fig. 7. Regions of nonvanishing $BR(h^0 \to A^0A^0)$ for $m_{A^0} = 5, 10, 20$ and 30 GeV. To the right of the solid curves, $m_{h^0} < 2m_{A^0}$, and the decay $h^0 \to A^0A^0$ is kinematically forbidden. To the left of the dashed curve, $BR(h^0 \to A^0A^0) \geq 0.5$ and between the dotted curves, $BR(h^0 \to A^0A^0) \geq 0.8$. $M_{\rm SUSY} = 1$ TeV in all four graphs. Taken from ref. 27.

which did not exist at tree-level. One example, mentioned at the end of section 4, is the possibility of the decay $h^0 \to A^0A^0$, which is kinematically forbidden at tree-level but allowed for some range of MSSM parameters.[24,27] Once allowed, $h^0 \to A^0A^0$ is almost certainly the dominant decay mode as shown in fig. 7 taken from ref. 27. These results indicate the importance of the search for $h^0 \to A^0A^0$ at LEP. As m_{A^0} increases beyond 30 GeV, the region of parameter space quickly shrinks where this decay is permitted.

For the heavier Higgs states, there are many possible final state decay modes. The various branching ratios are complicated functions of the MSSM parameter space.[28] This indicates a rich phenomenology for Higgs searches at future colliders.[28,29] Although the possibility of a Higgs discovery at LEP still remains, the effects of the radiative corrections (particularly if m_t is near the upper end of its expected range) suggest that the success of the Higgs Hunt must await the supercollider era. Presumably, the SSC and LHC will uncover direct evidence for supersymmetric particles, if "low-energy" supersymmetry exists. In this case, the details of the Higgs sector will contain crucial information regarding the structure of the theory—the mechanism of electroweak symmetry breaking and the nature of the TeV scale physics that lies beyond the Standard Model.

APPENDIX A: Renormalization Group Equations

In this Appendix, I have collected the one-loop renormalization group equations (RGEs) that are needed in the analysis presented in this paper.[17,30] Schematically, the RGEs at one-loop take the form

$$\frac{dp_i}{dt} = \beta_i(p_1, p_2, ..), \qquad \text{where } t \equiv \ln \mu^2 , \tag{A.1}$$

where μ is the energy scale, and the parameters p_i stand for the Higgs boson self-couplings λ_i ($i = 1 \ldots 7$), the squared Yukawa couplings h_f^2 ($f = t$, b and τ; the two lighter generations can be neglected), and the squared gauge couplings g_j^2 (j =3, 2, 1) corresponding to SU(3)×SU(2)×U(1) respectively. The g_j are normalized such that they are equal at the grand unification scale. It is also convenient to define

$$g \equiv g_2 , \qquad g' \equiv \sqrt{\tfrac{3}{5}} g_1 , \tag{A.2}$$

where g and g' are normalized in the usual way for low-energy electroweak physics, *i.e.* $\tan\theta_W = g'/g$.

I now list the β-functions required for the analysis presented in this paper. Two cases will be given, depending on whether μ is above or below the scale of supersymmetry breaking, $M_{\rm SUSY}$.

1. $\mu > M_{\rm SUSY}$

$$\begin{aligned}
\beta_{h_t^2} &= \frac{h_t^2}{16\pi^2}\left[6h_t^2 + h_b^2 - \tfrac{16}{3}g_3^2 - 3g^2 - \tfrac{13}{9}g'^2\right] \\
\beta_{h_b^2} &= \frac{h_b^2}{16\pi^2}\left[6h_b^2 + h_t^2 + h_\tau^2 - \tfrac{16}{3}g_3^2 - 3g^2 - \tfrac{7}{9}g'^2\right] \\
\beta_{h_\tau^2} &= \frac{h_\tau^2}{16\pi^2}\left[4h_\tau^2 + 3h_b^2 - 3g^2 - 3g'^2\right] \\
\beta_{g'^2} &= \frac{g'^4}{48\pi^2}\left[10N_g + \tfrac{3}{2}N_H\right] \\
\beta_{g^2} &= \frac{g^4}{48\pi^2}\left[6N_g + \tfrac{3}{2}N_H - 18\right] \\
\beta_{g_3^2} &= \frac{g_3^4}{48\pi^2}\left[6N_g - 27\right] .
\end{aligned} \tag{A.3}$$

Here $N_g = 3$ is the number of generations, $N_H = 2$ is the number of scalar doublets,

and the Higgs-fermion Yukawa couplings are given by

$$h_t = \frac{g m_t}{\sqrt{2} m_W \sin\beta}, \qquad h_{d_i} = \frac{g m_d}{\sqrt{2} m_W \cos\beta}, \qquad (d_i = b, \tau). \tag{A.4}$$

2. $\mu < M_{\rm SUSY}$

$$\begin{aligned}
\beta_{h_t^2} &= \frac{h_t^2}{16\pi^2}\left[\tfrac{9}{2}h_t^2 + \tfrac{1}{2}h_b^2 - 8g_3^2 - \tfrac{9}{4}g^2 - \tfrac{17}{12}g'^2\right] \\
\beta_{h_b^2} &= \frac{h_b^2}{16\pi^2}\left[\tfrac{9}{2}h_b^2 + \tfrac{1}{2}h_t^2 + h_\tau^2 - 8g_3^2 - \tfrac{9}{4}g^2 - \tfrac{5}{12}g'^2\right] \\
\beta_{h_\tau^2} &= \frac{h_\tau^2}{16\pi^2}\left[\tfrac{5}{2}h_\tau^2 + 3h_b^2 - \tfrac{9}{4}g^2 - \tfrac{15}{4}g'^2\right] \\
\beta_{g'^2} &= \frac{g'^4}{48\pi^2}\left[\tfrac{20}{3}N_g + \tfrac{1}{2}N_H\right] \\
\beta_{g^2} &= \frac{g^4}{48\pi^2}\left[4N_g + \tfrac{1}{2}N_H - 22\right] \\
\beta_{g_3^2} &= \frac{g_3^4}{48\pi^2}\left[4N_g - 33\right].
\end{aligned} \tag{A.5}$$

The notation is the same as in the previous case. Moreover, in writing down the RGEs for the Higgs-fermion Yukawa couplings, I have assumed that the Higgs-fermion interaction is the same as in the MSSM; namely, the $Y = -1$ [$Y = 1$] Higgs doublet couples exclusively to down-type [up-type] fermions.

Finally, I list the RGEs for the Higgs self-couplings of the general two-Higgs doublet model (with the Higgs-fermion couplings as specified above). First, I need to define the anomalous dimensions of the two Higgs fields:

$$\begin{aligned}
\gamma_1 &= \frac{1}{64\pi^2}\left[9g^2 + 3g'^2 - 4\sum_i N_{ci} h_{d_i}^2\right], \\
\gamma_2 &= \frac{1}{64\pi^2}\left[9g^2 + 3g'^2 - 4\sum_i N_{ci} h_{u_i}^2\right],
\end{aligned} \tag{A.6}$$

where the sum over i is taken over three generations of quarks (with $N_c = 3$) and

leptons (with $N_c = 1$). The β-functions for the Higgs self-couplings in the general CP-conserving non-supersymmetric two-Higgs-doublet model (with the Higgs-fermion couplings as specified above) are given by

$$\begin{aligned}
\beta_{\lambda_1} &= \frac{1}{16\pi^2}\Big\{6\lambda_2^2 + 2\lambda_3^2 + 2\lambda_3\lambda_4 + \lambda_4^2 + \lambda_5^2 + 12\lambda_6^2 \\
&\qquad + \tfrac{3}{8}\left[2g^4 + (g^2 + g'^2)^2\right] - 2\sum_i N_{ci} h_{d_i}^4\Big\} - 2\lambda_1\gamma_1 \\
\beta_{\lambda_2} &= \frac{1}{16\pi^2}\Big\{6\lambda_2^2 + 2\lambda_3^2 + 2\lambda_3\lambda_4 + \lambda_4^2 + \lambda_5^2 + 12\lambda_7^2 \\
&\qquad + \tfrac{3}{8}\left[2g^4 + (g^2 + g'^2)^2\right] - 2\sum_i N_{ci} h_{u_i}^4\Big\} - 2\lambda_2\gamma_2 \\
\beta_{\lambda_3} &= \frac{1}{16\pi^2}\Big\{(\lambda_1 + \lambda_2)(3\lambda_3 + \lambda_4) + 2\lambda_3^2 + \lambda_4^2 + \lambda_5^2 + 2\lambda_6^2 + 2\lambda_7^2 + 8\lambda_6\lambda_7 \\
&\qquad + \tfrac{3}{8}\left[2g^4 + (g^2 - g'^2)^2\right] - 2\sum_i N_{ci} h_{u_i}^2 h_{d_i}^2\Big\} - \lambda_3(\gamma_1 + \gamma_2) \\
\beta_{\lambda_4} &= \frac{1}{16\pi^2}\Big[\lambda_4(\lambda_1 + \lambda_2 + 4\lambda_3 + 2\lambda_4) + 4\lambda_5^2 + 5\lambda_6^2 + 5\lambda_7^2 + 2\lambda_6\lambda_7 \\
&\qquad + \tfrac{3}{2}g^2 g'^2 + 2\sum_i N_{ci} h_{u_i}^2 h_{d_i}^2\Big] - \lambda_4(\gamma_1 + \gamma_2) \\
\beta_{\lambda_5} &= \frac{1}{16\pi^2}\Big[\lambda_5(\lambda_1 + \lambda_2 + 4\lambda_3 + 6\lambda_4) + 5(\lambda_6^2 + \lambda_7^2) + 2\lambda_6\lambda_7\Big] - \lambda_5(\gamma_1 + \gamma_2) \\
\beta_{\lambda_6} &= \frac{1}{16\pi^2}\Big[\lambda_6(6\lambda_1 + 3\lambda_3 + 4\lambda_4 + 5\lambda_5) + \lambda_7(3\lambda_3 + 2\lambda_4 + \lambda_5)\Big] - \tfrac{1}{2}\lambda_6(3\gamma_1 + \gamma_2) \\
\beta_{\lambda_7} &= \frac{1}{16\pi^2}\Big[\lambda_7(6\lambda_2 + 3\lambda_3 + 4\lambda_4 + 5\lambda_5) + \lambda_6(3\lambda_3 + 2\lambda_4 + \lambda_5)\Big] - \tfrac{1}{2}\lambda_7(\gamma_1 + 3\gamma_2).
\end{aligned} \tag{A.7}$$

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