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NONPERTURBATIVE APPROACH TO GREEN AND VERTEX FUNCTIONS *

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Abstract

A path-integral approach to fermionic Green and vertex functions is presented, which has a systematic expansion in terms of contours, thereby enabling the evaluation of all-order results in the coupling constant while preserving the full γ-matrix structure of spinorial fields. Within an Abelian infrared setup, two applications are considered: the renormalized fermion propagator and an universal vertex function which effectively simulates the Isgur-Wise form factor—the basic component of strong-interaction dynamics in the weak decays of heavy mesons. The agreement between our theoretical predictions and the recent CLEO II data is excellent.

*Invited talk presented by the first author at the International Conference *Hadron Structure '94*, Košice, Slovakia, September 19-23, 1994.

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ABSTRACT

A path-integral approach to fermionic Green and vertex functions is presented, which has a systematic expansion in terms of contours, thereby enabling the evaluation of all-order results in the coupling constant while preserving the full γ-matrix structure of spinorial fields. Within an Abelian infrared setup, two applications are considered: the renormalized fermion propagator and an universal vertex function which effectively simulates the Isgur-Wise form factor—the basic component of strong-interaction dynamics in the weak decays of heavy mesons. The agreement between our theoretical predictions and the recent CLEO II data is excellent.

1. Introduction

The calculation of Green and vertex functions in a path-integral formulation of quantum field theory is quite well advanced, although most results are obtained within perturbation theory. In cases of exactly solvable models, like the Bloch-Nordsieck model [1],the calculation can be done without recourse to series expansion, and closed-form solutions can be obtained [2]. In order to be able to compute such quantities in a nonperturbative way, resummation techniques of the perturbation series in the coupling constant are needed. This is particularly important for the strong interactions because calculations of infrared (IR)-sensitive hadronic amplitudes based on Quantum Chromodynamics (QCD) demand the application of techniques beyond those of the standard renormalization group to account for the exponentiation of soft gluons.

Thus, we are motivated to ask if a Green or vertex function calculated within an Abelian setup can sufficiently *simulate* the characteristic features of non-Abelian quantities—at least in some limiting cases. If this attempt should prove tenable, one may ask if there is any practical advantage in phenomenological applications. The present approach adopts this philosophy and is characterized by three major conceptual features: (i) the conversion of fermionic *field* systems in d-dimensional Euclidean spacetime into a *particle-based* path-integral description in the sense of Polyakov [3], (ii) the incorporation of a *built-in* resolution scale in the initial casting of the field system, and (iii) the use of *geometry* rather than the expansion in powers of the coupling constant as a guiding principle to identify the leading contributions. Expanding in terms of contours makes it possible to obtain all-order expressions in the coupling constant, emphasizing the nonperturbative character of the approach. Furthermore, since our regularized casting of the field system takes place within a *continuous* spacetime background, meaning

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that the resolution scale can be sent to zero at every stage of the calculations, typical problems of lattice formulations, like fermion doubling, etc., are not encountered.

In the present exposition we concentrate on the IR structure of the fermion propagator and a suitably constructed *universal* vertex function, the latter with the purpose of simulating the dynamical features of the Isgur-Wise form factor [4]. In fact, using geometry to identify the IR-dominant contours, enables us to *effectively* describe the weak transition of a heavy meson into another heavy meson in terms of "infraparticle" [5] dynamics. Within our Abelian framework, the heavy quark is represented by a "heavy" electron, whereas the light degrees of freedom correspond to the soft-photon cloud. The justification of this treatment is provided by heavy-quark symmetry, which implies that the strong interactions of the heavy quark depend only on its four-velocity, so that its coupling to the gluon degrees of freedom is independent of its mass and appropriately described by a Wilson line[6]. As we will see below, this condition naturally complies with our IR specifications of dominant contours in the path integral.

2. Concepts of the particle-based description of fermionic field systems

Consider a quantum field theoretical system of Dirac fields in descrete formulation described by the action functional [7]

$$S^D = -\sum_{n=-\infty}^{\infty} \alpha^d \, \bar{\psi}(n\alpha) \left(\gamma_\mu \partial_\mu^D + m^D\right) \psi(n\alpha), \tag{1}$$

where α denotes the spacetime cutoff. The discretization scheme is specified by the action of the differential operator ∂^D on the spinor field assigned to the site $n\alpha$ and is given by an appropriately weighted average of regular derivatives within a volume of $\mathcal{O}(\alpha)$:

$$\left(\partial_\mu^D + m^D\right) \psi(n\alpha) = -\int d^d y \, [(\partial_\mu - m) \, f(|y|)] \, \psi(n\alpha + y). \tag{2}$$

Here $f(|y|)$ is a distribution decreasing rapidly to zero within the discretization size α. Moreover, $f(|y|)$ has the property $f(|y|) \stackrel{\alpha \to 0}{\longrightarrow} \delta(y)$, which is, e.g., satisfied by a Gaussian distribution. We may view S^D as resulting from the non-local action

$$S^{NL} = \int d^d x \int d^d y \, \bar{\psi}(x) \, [(\gamma_\mu \partial_\mu - m) f(|y|)] \, \psi(x + y) \tag{3}$$

by averaging within cells of size α^d. It thus appears that discretization emerges as the result of averaging over cells rather than as a replacement of the spacetime continuum by a lattice of points. Accordingly, in coordinate representation we have

$$\langle x | \hat{\partial}_\mu | x' \rangle = \frac{\partial}{\partial x} f(|x - x'|), \tag{4}$$

which takes the discretized form

$$\begin{aligned} \langle n^{i-1}\alpha | \partial_\mu^D | n^i \alpha \rangle &= \frac{1}{\alpha} \frac{\partial}{\partial n_\mu^{i-1}} \int_{-\pi/\alpha}^{\pi/\alpha} \frac{d^d p_i}{(2\pi)^d} \tilde{f}\left((p^i)^2 \alpha^2\right) \mathrm{e}^{-ip^i \cdot (n^i - n^{i-1})\alpha} \\ &= \frac{1}{\alpha} \frac{\partial}{\partial n_\mu^{i-1}} f\left(|n^i - n^{i-1}|\right), \end{aligned} \tag{5}$$

in which the discrete set of coordinate (particle) states obey the completeness and orthogonality conditions $\sum_n \alpha^d \; |n\alpha >< n\alpha| = 1$, $< n\alpha | n'\alpha > = \frac{\delta_{nn'}}{\alpha^d}$, and the second

equation in (5) follows by virtue of the property $\tilde{f}(p^2\alpha^2) \to 0$ in the first Brillouin zone $(-\frac{\pi}{\alpha} \leq p \leq \frac{\pi}{\alpha})$. Eq. (5) generalizes to a form which includes coupling to an external Abelian gauge field:

$$\left\langle n^{i-1}\alpha \left| \left(1 - \epsilon\gamma_\mu D^D_\mu\right) \right| n^i\alpha \right\rangle \simeq \frac{1}{\alpha^d} U\left(L_{n^{i-1}\alpha, n^i\alpha}\right) \left[1 - \frac{\alpha}{\epsilon}\gamma_\mu \left(n^i - n^{i-1}\right)_\mu\right] \times F\left(|n^i - n^{i-1}|\right), \tag{6}$$

where we have introduced

$$F\left(|n^i - n^{i-1}|\right) \equiv -\frac{(\epsilon/\alpha)^2\alpha^d}{|n^i - n^{i-1}|} \frac{\partial}{\partial|n^i - n^{i-1}|} f\left(|n^i - n^{i-1}|\right). \tag{7}$$

A convenient choice for $F\left(|n^i - n^{i-1}|\right)$, consistent with our requirements, is

$$F\left(|n^i - n^{i-1}|\right) \equiv \exp\left\{-\frac{|n^i - n^{i-1}|^2}{2}\right\}. \tag{8}$$

This distribution furnishes a relative weight that relates the discretized copies of the system at i and $i-1$, "sliding" by traversing a contour.

Similarly to the free case, the covariant differential operator is defined via its action on the spinor field at site $n\alpha$:

$$D^D_\mu \psi(n\alpha) = -\int d^dy\ [\partial_\mu f(|y|)]\, U\left(L_{n\alpha, n\alpha+y}\right) \psi(n\alpha + y) \tag{9}$$

where

$$U(L_{x,x+y}) \equiv \mathcal{P}\exp\left(-ig\int_{L_{x,x+y}} A_\mu(z)dz_\mu\right) \tag{10}$$

and $\mathcal{P}$ denotes a path-ordering prescription. For the purpose of explicit calculations, we display the expression for the effective action (neglecting the mass term):

$$-\ln Z_A = \mathrm{tr}\int_c^\infty \frac{dT}{T}\left(\prod_{i=0}^N \sum_{n^i} \alpha^d\right) \frac{\delta_{n^0,n^N}}{\alpha^d} \mathcal{P}\exp\left\{-\frac{\alpha}{\epsilon}\sum_{i=1}^N \left(n^i - n^{i-1}\right)_\mu \gamma_\mu\right\} \times \prod_{i=1}^N \frac{1}{\alpha^d} F\left(|n^i - n^{i-1}|\right) \exp\left\{-ig\sum_{i=1}^n \alpha\left(n^i - n^{i-1}\right)_\mu A_\mu(n^i\alpha)\right\}. \tag{11}$$

The corresponding continuous expression reads

$$-\ln Z_A \overset{c\to 0}{=} \mathrm{tr}\int_c^\infty \frac{dT}{T}\mathcal{N}\int_{x(0)=x(T)} [dx(\tau)]\mathcal{P}\exp\left\{\frac{1}{\epsilon}\int_0^T d\tau\gamma_\mu\dot{x}_\mu(\tau)\right\} \times \exp\left\{-\frac{1}{2\epsilon}\int_0^T d\tau\dot{x}^2(\tau)\right\} \exp\left\{ig\int_0^T d\tau\dot{x}_\mu(\tau)A_\mu[x(\tau)]\right\}, \tag{12}$$

where $\mathcal{N} = (2\pi)^{-Nd/2}\epsilon^{-Nd}$ and Schwinger's proper-time representation has been used. Note, however, that the parameter τ cannot be identified with physical time [3]; it only serves to describe in a continuous way dynamical development along a given contour which connects individual discretization arrangements ($\mathbb{R}^d$ "slices") labeled by i.

Before proceeding with applications, let us discuss the various scales entering the formalism. According to our discretization, it is clear that α accounts for *localization* in spacetime. Thus one may associate it with "particle size". The scale ϵ sets the pace by which successive spacetime "slices" enter the splitting of a given (closed) contour of total length T. Both scales characterize the spacetime resolution and one is well advised to send them to zero (in the local limit) at unit ratio, i.e., $\alpha,\ \epsilon \to 0$ with $\frac{\alpha}{\epsilon} = 1$. Turning our attention to c, we observe that it sets a lower bound on the length of closed contours entering the calculation of the tr in (12). Obviously, $c \gg \epsilon$, so that c should be the last scale to be sent to zero. As it turns out, c will assume the role of an ultraviolet (UV) cutoff for matter-field fluctuations, once the spacetime resolution scales have been safely set equal to zero.

3. IR structure of the 2-point fermion Green function

The full two-point Green function can be expressed in the form

$$G(x,y) = \int [dA_\mu(x)] \mathrm{e}^{-\frac{1}{2}\int d^d z A_\mu(z) \mathcal{D}^{-1}_{\mu\nu}(\lambda) A_\nu(z)} \det[G_D^{-1}(x,y|A)] G^D(x,y|A), \tag{13}$$

where a covariant gauge restriction has been imposed on the integration over A_μ, denoted by the gauge parameter λ, and the Green function $G^D(x,y|A)$ describes the first-quantized motion of spinor particles in the presence of an external gauge potential A_μ. It is defined in discretized form by

$$G^D(n\alpha, m\alpha|A) = \langle n\alpha| \left(\not{p}^D + m^D\right)^{-1} |m\alpha\rangle, \tag{14}$$

which, upon Schwinger's parametrization, becomes

$$\langle n\alpha| \left(\not{p}^D + m^D\right)^{-1}) |m\alpha\rangle = \lim_{c\to 0} \int_c^\infty dT\, \langle n\alpha| \mathrm{e}^{-T(\not{p}^D + m^D)} |m\alpha\rangle. \tag{15}$$

Following the procedure described in the previous section, we convert Eq. (13) into a particle-based path-integral to obtain

$$\begin{aligned} G(x,y) \overset{c\to 0}{=} \int_0^\infty dT \int_{\substack{x(0)=x \\ x(T)=y}} [dx(\tau)][dp(\tau)] \exp\left\{ i \int_0^T d\tau\, p(\tau)\cdot \dot{x}(\tau) \right\} \\ \times \mathcal{P} \exp\left\{ -\int_0^T d\tau [i\gamma \cdot p(\tau) + m] \right\} \left\langle \exp\left\{ ig \int_0^T d\tau \dot{x}(\tau)\cdot A[x(\tau)] \right\} \right\rangle_A , \end{aligned} \tag{16}$$

where

$$\begin{aligned} \left\langle \exp\left\{ ig \int_0^T d\tau\, \dot{x}(\tau)\cdot A[x(\tau)] \right\} \right\rangle_A = \int [dA_\mu(x)] \det[G^{-1}(x,y|A)] \\ \times \exp\left\{ ig \int_0^T d\tau\, \dot{x}(\tau)\cdot A[x(\tau)] \right\} \\ \times \exp\left\{ -\frac{1}{2} \int d^d z\, A_\mu(z) \mathcal{D}^{-1}_{\mu\nu}(\lambda) A_\nu(z) \right\}. \end{aligned} \tag{17}$$

Within our framework, the characterization of the IR domain involves two requirements [8]: First, gauge-field fluctuations are evaluated according to the quenched approximation which implies that there is no pair creation and hence the fermion determinant is set equal to 1. This is considered justifiable for this kinematic domain because the

Figure 1: (a) example of an IR-dominant contour. (b) example of a contour for which $x(\tau) \approx x(\tau')$ does not imply that $\tau \approx \tau'$. Such contours are excluded in the calculation of the two-point Green function.

active modes of the gauge field have energies much lower than the electron mass m. Second, we assume that the electron is almost on-shell and its momentum fluctuates around the mass-shell by an amount of order $k_0 \to 0$. The corresponding change in its four-velocity vanishes as $k_0/m \to 0$ and the electron acquires a "heaviness", which makes it reasonable to assume that the electron's motion is associated with reasonably smooth contours, meaning that if $x(\tau) \approx x(\tau')$, then $\tau \approx \tau'$, as illustrated in Fig. 1. Utilizing our first IR requirement, the contribution from the gauge-field sector reads

$$\left\langle \exp\left\{ ig \int_0^T d\tau\, \dot{x}(\tau) \cdot A[x(\tau)] \right\} \right\rangle_A = \exp\left\{ -g^2 \int_0^T d\tau \int_\tau^T d\tau' \dot{x}_\mu(\tau) \dot{x}_\nu(\tau') \right.$$
$$\left. \times \langle 0 | A_\mu[x(\tau)] A_\nu[x(\tau')] | 0 \rangle \right\}. \tag{18}$$

Covariant decomposition and use of a Fourier representation yields

$$\langle 0 | A_\mu[x(\tau)] A_\nu[x(\tau')] | 0 \rangle = A[x(\tau), x(\tau')]\, \delta_{\mu\nu} - (1-\lambda)\, B_{\mu\nu}[x(\tau), x(\tau')], \tag{19}$$

where for $d = 4$

$$A[x(\tau), x(\tau')] = \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2}\, e^{ik\cdot(x(\tau')-x(\tau))} = \frac{1}{4\pi^2} \frac{1}{|x(\tau') - x(\tau))|^2}, \tag{20}$$

$$B_{\mu\nu}[x(\tau), x(\tau')] = \int \frac{d^4k}{(2\pi)^4} \frac{k_\mu k_\nu}{k^4}\, e^{ik\cdot(x(\tau')-x(\tau))}$$
$$= \frac{1}{8\pi^2} \frac{\delta_{\mu\nu}}{|x(\tau')-x(\tau)|^2} - \frac{1}{4\pi^2} \frac{(x_\mu(\tau')-x_\mu(\tau))(x_\nu(\tau')-x_\nu(\tau))}{|x(\tau') - x(\tau)|^4}. \tag{21}$$

It is obvious that the remaining integrals in (20) and (21) contain infinities owing to their denominators which become singular when the contours $x(\tau)$ and $x(\tau')$ approach each other. This suggests a point-splitting regularization technique, and as such we use Witten's "ribbon regularization" [9]. Then, one may think of the contours as comprising the opposite edges of an untwisted and unwrinkled ribbon of infinitesimal width, the latter serving as an UV regulator. This treatment, in connection with the smoothness requirement imposed on contours in the IR region, enables a systematic expansion in terms of contours to identify the dominant contributions. In particular, quasi-free propagation emerges naturally within this context and there is no need to explicitly impose asymptotic conditions.

To achieve a uniform parametrization of contours along the ribbon, we make the replacement $x_\mu(\tau) \to x_\mu(\tau) + b\, n_\mu(\tau)$, where $n_\mu(\tau)$ is a unit vector normal to the contour's

direction and lying on the ribbon's surface, whereas b represents the width of the ribbon. Then we obtain the following regularized expressions:

$$A^{reg} = \frac{1}{4\pi^2}\,\frac{1}{|x(\tau') - x(\tau) - b\,n(\tau)|^2}, \tag{22}$$

$$\begin{aligned} B^{reg}_{\mu\nu} = & \frac{1}{8\pi^2}\,\frac{\delta_{\mu\nu}}{|x(\tau') - x(\tau) - b\,n(\tau)|^2} \\ & -\frac{1}{4\pi^2}\,\frac{(x_\mu(\tau') - x_\mu(\tau) - b\,n_\mu(\tau))\,(x_\nu(\tau') - x_\nu(\tau) - b\,n_\nu(\tau))}{|x(\tau') - x(\tau) - b\,n(\tau)|^4}. \end{aligned} \tag{23}$$

Note that τ' serves to denote the independent running along one of the ribbon edges with respect to the other; it does not amount to a different parametrization.

We encounter now the evaluation of the integral

$$I^{(b)} = g^2 \int_0^T d\tau \int_\tau^T d\tau' \dot{x}_\mu(\tau)\dot{x}_\nu(\tau')\Big(\delta_{\mu\nu} A^{reg}[x(\tau), x(\tau')] + (1-\lambda) B^{reg}_{\mu\nu}[x(\tau), x(\tau')]\Big), \tag{24}$$

the leading contribution of which results from contigous points placed on opposites edges of the ribbon., i.e., for $x(\tau) \simeq x(\tau')$. Thus we employ a Taylor expansion to write $x_\mu(\tau') \simeq x_\mu(\tau) + (\tau' - \tau)\,\dot{x}_\mu(\tau)$ on account of which for $T \gg b$ ($|\dot{x}| = 1$),

$$\left\langle \exp\left\{ ig \int_0^T d\tau\; \dot{x}_\mu(\tau) A_\mu[x(\tau)] \right\} \right\rangle_A^{reg} = \exp\left(-\alpha\,\frac{T}{2b} + a \ln\frac{T}{b} + \text{finite terms} \right), \tag{25}$$

where $\alpha = 1/137$ is the fine-structure constant and $a \equiv \frac{\alpha}{2\pi}(3-\lambda)$. Introducing a bare-mass dependence on the width of the ribbon, the physical mass m is read off the relation $m \overset{b\to 0}{=} \left(m_0(b) + \frac{\alpha}{2b}\right)$, whereas the logarithmic term in (25) furnishes a wave function renormalization constant Z_2 giving rise to an anomalous dimension γ_F for the fermion field. Introducing a renormalization scale μ, we write $\ln\frac{T}{b} = \ln T\mu - \ln b\mu$, from which we deduce $Z_2 = \mathrm{e}^{-a\ln\mu b}$, so that $\gamma_F = \frac{\mu}{Z_2}\frac{\partial Z_2}{\partial\mu} = -a$. Finally, the full renormalized two-point Green function, which exhibits a cut near the mass shell, is

$$\begin{aligned} G_{ren}(x,y) \sim \mu^a \int_{0+}^{\infty} dT T^a \int_{\substack{x(0)=x \\ x(T)=y}} [dx(\tau)][dp(\tau)] \exp\left\{ i\int_0^T d\tau\; p(\tau)\cdot\dot{x}(\tau) \right\} \\ \times\; \mathcal{P}\exp\left\{ -i\int_0^T d\tau\; [m + i\not{p}(\tau)] \right\} \end{aligned} \tag{26}$$

and after analytic continuation to Minkowski space

$$G_{ren}(x,y) \sim \mu^a \int \frac{d^4p}{(2\pi)^4}\,\frac{\mathrm{e}^{-ip\cdot(x-y)}}{(\not{p} - m + i\epsilon)^{1+a}}. \tag{27}$$

Replacing the γ-matrices by four-velocity vectors [1], this result coincides with that found previously within the Bloch-Nordsieck model (see, e.g., [2]). The effect of the soft-photon cloud, controlled by the anomalous dimension γ_F, smears out the mass shell's behavior of the charged fermion endowing it with an infraparticle structure [5].

4. Universal vertex function and hadronic weak decays

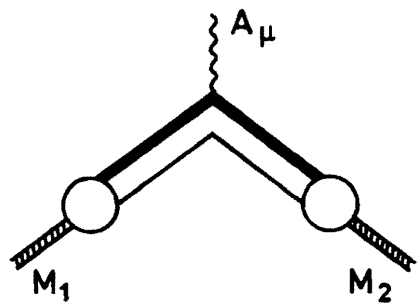

Figure 2: Heavy meson decay in which a heavy quark Q changes flavor by emitting a weak current.

The basic aim of this section is to derive an universal vertex function (form factor) capable to emulate the role of the Isgur-Wise function in the weak decays of heavy mesons in the heavy quark effective field theory (see Fig. 2). We propose to simulate this transition by the following process. An infraparticle on route along a straight-line path from x to y with four-velocity u_1 is suddenly derailed at the spacetime point z and then proceeds to propagate, still as an infraparticle, on a different straight-line path characterized by the four-velocity u_2. As we shall see below, such a process can be described by a single universal function, which depends only on the invariant velocity change $w = u_1 \cdot u_2$. Within this context, infraparticles represent heavy quarks, whereas the complicated hadronic soft dynamics governing the overlap of the light degrees of freedom will be simulated by an infinitesimal loop which causes a cusp in the contour. The claim is that since the light degrees of freedom in the weak transition are "blind" to the flavor (mass) and spin orientation of the heavy quark and "feel" only its color field with wavelenghts much larger than the Compton wavelength of the heavy quark, whereas the typical momenta exchanged between the heavy quark and the light constituents are of order $\Lambda_{QCD} \ll m_Q$, this is a realistic assessment of the physical picture.

Technically, this is achieved by introducing the following three-point function

$$\Gamma(x,y,z) \overset{c\to 0}{=} \int_c^\infty dT \int_0^T ds \int [dp(\tau)]\, \mathrm{tr}\mathcal{P} \exp\left\{\int_0^T d\tau\, (ip(\tau)\cdot\gamma + m)\right\} G(x,y,z), \quad (28)$$

where

$$G(x,y,z) = \int_{\substack{x(0)=x\\ x(s)=z\\ x(T)=y}} [dx(\tau)] \exp\left\{i\int_0^T d\tau\, p(\tau)\cdot \dot{x}(\tau)\right\} \left\langle \exp\left\{ig\int d\tau \dot{x}(\tau)\cdot A[x(\tau)]\right\}\right\rangle_A . \quad (29)$$

The calculation involves rearrangement of the contour into three distinct branches, one of which forms a loop. Hence, the gauge-field fluctuation term acquires the form

$$\left\langle exp\left\{ig\int_0^T d\tau \dot{x}(\tau)\cdot A[x(\tau)]\right\}\right\rangle_A \simeq \left\langle \exp\left\{ig\int_{L^z_{x,y}} dw_\mu\, A_\mu(w)\right\}\right\rangle_A \times \left\langle \exp\left\{ig\int_{s_1}^{s_2} d\tau\, \dot{x}(\tau)\cdot A[x(\tau)]\right\}\right\rangle_A , \quad (30)$$

where we have combined the two infraparticle contour pieces into a single composite path, denoted $L^z_{x,y}$, which is the the broken contour from x to $z-0^+$ and from $z+0^+$ to y (Fig. 3a). Note that the intersection point z, along with infinitesimal prongs to the left and to the right, has been excised from $L^z_{x,y}$. In a literal sense, the sharp bend at point z causes a break in the infraparticle path, so that the three contour branches become disconnected. We now introduce a coordinate rendering of $L^z_{x,y}$, which adapts itself to

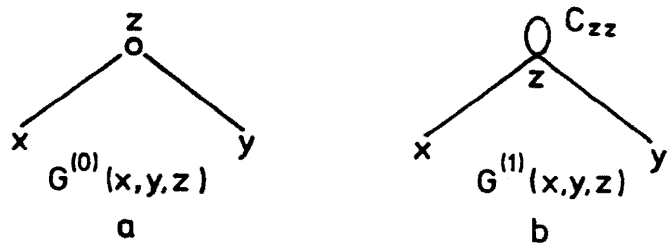

Figure 3: (a) pieces of the contour $x \longrightarrow z \longrightarrow y$ which correspond to an independent propagation of two infraparticles. (b) nontrivial part of the same contour involving a minuscule loop which serves to simulate the interaction at the point z, excised in the broken contour shown in (a).

a parametrization in the interval $[0,1]$ with the interaction point removed. Specifically, we define the variable $x^L(t)$ by

$$x^L(t) = \begin{cases} x\left(t(T-\tilde{\tau})\right), & 0 \le t < \tilde{t} \\ x\left(t(T-\tilde{\tau})+\tilde{\tau}\right) & \tilde{t} < t \le T, \end{cases} \tag{31}$$

where $\tilde{t} = \frac{s_1}{T-\tilde{\tau}} = \frac{s_2-\tilde{\tau}}{T-\tilde{\tau}}$, $\tilde{\tau} = s_2 - s_1$, $x^L(\tilde{t}) = z$ and $\tilde{t}$ is excised from $[0,1]$. The remaining branch of the three-point process has the geometric characteristics of a closed contour, designated C_{zz} (Fig. 3b). The explicit contribution of this branch to $G(x,y,z)$ depends on the topology of C_{zz}. Recalling Eq. (30), the three-point function factorizes:

$$G^{(i)}(x,y,z) = G(x,y|z)\, G^{(i)}(z,z), \tag{32}$$

where $G^{(i)}(z,z)$ represents the contribution from loop configurations and

$$\begin{aligned} G(x,y|z) = & \int_{\substack{x^L(0)=x \\ x^L(1)=y}} [dx^L(t)]\delta[x^L(\tilde{t})-z] \exp\left\{-ig\, \mathcal{P}_{\tilde{t}} \int_0^1 dt\, \dot{x}^L(t)\cdot p^L(t)\right\} \\ & \times \left\langle \exp\left\{ig\, \mathcal{P}_{\tilde{t}} \int_0^1 dt\, \dot{x}^L(t)\cdot A[x^L(t)]\right\}\right\rangle_A \end{aligned} \tag{33}$$

is a purely kinematical factor describing coexisting in and out four-velocity states. In (33), $\mathcal{P}_{\tilde{t}}$ denotes a principal value prescription: $\mathcal{P}_{\tilde{t}} \int_0^1 dt \cdot = \int_0^{\tilde{t}-0^+} dt \cdot + \int_{\tilde{t}+0^+}^1 dt \cdot$. In the absence of the factor $G^i(z,z)$, $G(x,y|z)$ represents two infrapropagators, separated by at least the resolution scale b, i.e., the ribbon width, giving rise to two independently renormalized fermion propagators. The factor $G^i(z,z)$ simulates the "dynamical pinch" at z which causes a discontinuity in the four-velocity and derails the infraparticle. A flavor of the loop computation involved is now presented. The zeroth order term $G^{(0)}(x,y,z) = G(x,y|z)G^{(0)}(z,z) = G(x,y|z)$ constitutes an overall kinematical factor, since $G^{(0)}(z,z) = 1$ corresponds to a loop that shrinks to zero ($s_1 = s_2$). The first nontrivial contribution comes from the configuration shown in Fig. 3b which contains a cusp in the contour with a deficiency angle complementary to the one formed by the in and out four-velocities u_1 and u_2. It is understood that the cusped loop is to be entered and exited *smoothly*, which calls for an appropriate parametrization of the loop integral $\oint_{C_{zz}^{(1)}} dx_\mu\, A_\mu(x)$, provided by $\tau \longrightarrow t = \tau - (s_1+s_2)/2$ and accompanied by the coordinate readjustment $x(\tau) \longrightarrow x^c(t) = x\left(t + (s_1+s_2)/2\right)$ with $t \in [-\frac{\tilde{\tau}}{2}, \frac{\tilde{\tau}}{2}]$. As a result, contributions from terms containing the gauge parameter λ, which are proportional to $\oint dw_\mu \oint dz_\nu B_{\mu\nu}(w,z)$ (c.f., Eqs. (21), (24)), vanish identically by appreciating

that $B_{\mu\nu}(w,z)$ is a total derivative. Then the only term we have to evaluate is

$$I = -\frac{g^2}{8\pi^2}\int_{-\tilde{\tau}/2}^{\tilde{\tau}/2} dt \int_{-\tilde{\tau}/2}^{\tilde{\tau}/2} dt' \frac{\dot{x}^c(t')\cdot\dot{x}^c(t)}{|x^c(t')-x^c(t)|^2}\Bigg|_{\substack{\dot{x}^c(-\tilde{\tau}/2)=u_1\\ \dot{x}^c(\tilde{\tau}/2)=u_2}}. \tag{34}$$

Adopting the picture of a sudden impulse process, which demands a minuscule loop, we are led to express the entrance segment of the loop at z by $x^c(t) = z_- + (t+\tilde{\tau}/2)u_1$ and the exit segment by $x^c(t') = z_+ + (t'-\tilde{\tau}/2)u_2$. Obviously, this corresponds to propagating along a contour in the vicinity of the distortion point z, once entering and once exiting, with the rest of the contribution being negligible on account of the infinitesimal size of the loop. Using the above parametrizations and regularizing via the point-splitting method of the ribbon, we find

$$I(\theta) \simeq -\frac{g^2}{8\pi^2}\,\theta \int_{-\tilde{\tau}/2}^{\tilde{\tau}/2} dt \int_{-\tilde{\tau}/2}^{\tilde{\tau}/2} dt' \frac{1}{t^2+t'^2-2\,t\,t'\,\theta+b^2}, \tag{35}$$

where $\theta = u_1\cdot u_2$. The length scale in the above relation is of the order of the diminishing loop size (due to the sudden impulse approximation) and serves as an IR regulator. The result of the integration is [10]

$$I(\theta) \simeq -\frac{g^2}{4\pi^2}\ln\left(\frac{\tilde{\tau}}{2b}\right)\frac{\pi\theta}{\sqrt{1-\theta^2}}. \tag{36}$$

In order to disentangle IR from UV effects, we make a scale readjustment according to which the coupling constant is tuned to values that are relevant to our effective description. Thus we write $g^2(b^2)\ln\left(\frac{\tilde{\tau}}{2b}\right) = g^2(\mu^2)\ln(\tilde{\tau}\mu) = g^2(\mu^2)\ln k$. We may view k as the factorization scale which separates long- from short-distance effects at the vertex. The former are associated with light degrees of freedom that remain undedected, while the latter are absorbed into the renormalized vertex function. Hence for the loop contribution to the vertex we obtain after renormalization ($x^c(-\frac{\tilde{\tau}}{2}) = x^c(\frac{\tilde{\tau}}{2}) = z$)

$$G^{(1)}_{ren}(z,z) = \exp\left\{-\frac{g^2}{4\pi}(\ln k)\frac{\theta}{\sqrt{1-\theta^2}}\right\}\int[dx^c(t)]\exp\left\{-i\int_{-\tilde{\tau}/2}^{\tilde{\tau}/2} dt p^c(t)\cdot x^c(t)\right\}, \tag{37}$$

and accordingly

$$\Gamma^{(1)}_{ren}(x,y,z) = -\exp\left\{-\frac{g^2}{4\pi}\frac{\theta}{\sqrt{1-\theta^2}}\ln k\right\}\Gamma^{(0)}_{ren}(x,y,z). \tag{38}$$

To isolate the nonperturbative part of the vertex function, we remove the kinematics by defining

$$F(x,y,z) = \frac{\Gamma^{(0)}_{ren}(x,y,z)+\Gamma^{(1)}_{ren}(x,y,z)}{\Gamma^{(0)}_{ren}(x,y,z)}. \tag{39}$$

Thereby we determine

$$F(x,y,z) = F(\theta) = 1-\exp\left\{-\frac{g^2}{4\pi}\frac{\theta}{\sqrt{1-\theta^2}}\ln k\right\}. \tag{40}$$

Analytic continuation to Minkowski space entails $\theta \to w \equiv \tilde{u}_{1\mu}\tilde{u}_2^{\mu} = 1/\theta$, where $\tilde{u}_1$ and $\tilde{u}_2$ are, respectively, the Minkowski counterparts of u_1 and u_2. Renaming $F(\theta)$ in $\xi(w)$ and replacing e^2 by $\frac{4}{3}g_s^2$, we finally obtain [11]

$$\xi(w) = 1 - \exp\left\{-\frac{4}{3}\alpha_s\frac{1}{\sqrt{w^2-1}}\ln k\right\}, \tag{41}$$

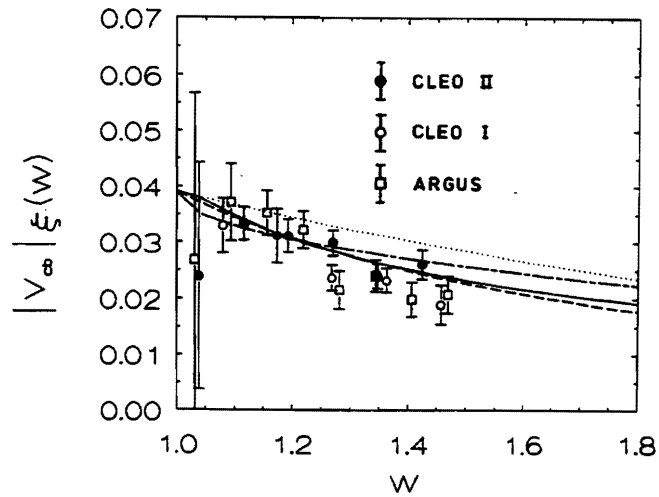

Figure 4: Theoretical predictions for the Isgur-Wise function in comparison with existing data ($|V_{cb}| = 0.039$).

with $\alpha_s = \frac{g_s^2}{4\pi}$. This result represents an universal vertex function, since it depends only on one single variable, *viz.* the invariant four-velocity change $w = \tilde{u}_1 \cdot \tilde{u}_2$. It satisfies the correct normalization for the Isgur-Wise function $\xi(w = 1) = 1$ at zero recoil, and has a functional form which decreases monotonically at large recoil, as expected from hadronic dynamics. Adapting our result to the scales of the heavy quark effective field theory, we set $k = \frac{m_Q}{\bar{\Lambda}}$, $\bar{\Lambda}$ being the energy scale of the light degrees of freedom. Using rather typical values for the various parameters [11], we find $k \approx 8$. The obtained universal function (solid line) is shown in Fig. 4, in comparison with other theoretical predictions (see [11]) and current data. We emphasize the excellent agreement with the recent CLEO II data without any tuning of the parameters entering our calculation.

5. Conclusions

We have reviewed some of the results obtained so far within an approach which recasts fermionic field systems in terms of a particle-based path-integral formulation with a built-in resolution scale and keeping intact the full γ-structure of the theory. Within this approach dominant contributions are identified by geometry, thus enabling resummation of the perturbation series in the coupling constant. It appears that simulation of hadronic dynamics is a most helpful expendient for obtaining nonperturbative results within our effective low-energy Abelian field theory.

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