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Replica symmetry breaking effects in spin glasses

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Abstract

Two competing pictures have been proposed which describe the low temperature properties of spin-glasses. We discuss some recent results which try to discern what is the correct description of the spin-glass phase. We review results for the spin glass order parameter distribution in case of zero and finite magnetic field and for the static chaos problem. We conclude that it is possible to solve this controversy, the crucial point being the existence of phase transition with magnetic field.

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1 Introduction

One of the main issues in spin glasses which still remains unsolved concerns the correct description of the low temperature phase [1, 2]. There is mean-field theory which gives a very complex description of the spin-glass phase with a huge number of equilibrium states separated by different energy barriers of all possible heights [3, 4]. Even though there are still open problems in mean-field theory, it is relevant to know if its main results also apply in case of more realistic systems like short-range models. In statistical mechanics, the usual strategy to obtain properties in models which are not exactly solvable is to take mean-field theory as a zeroth order approximation and do some perturbative analysis around it. This is a correct procedure if the solution around which we are perturbing is physically close to the true solution. This strategy has revealed very difficult and progress in this direction is slow [5].

The most well known version of mean-field theory is the SK model [6]. In this model all spins interact among them and there are no characteristic length scales. During the past different approaches to spin glasses have been proposed in which the length scales have a more specific role. These approaches are of a phenomenological type and they are known as droplet models [7]. These models are phenomenologic because they do not start from a full microscopic analysis but these take for granted some abinitio hypothesis from which they can derive general results. The main ingredient underlying these approaches is the zero-temperature scaling hypothesis [8, 9]. According to this hypothesis, the low temperature phase in spin-glasses is determined by the zero-temperature fixed point in the renormalization group flow equations. Droplet approaches can go further in making predictions because they take for granted some hypothesis. But in this way also they loose the mean-field approximation since it becomes unsense in their framework. It would be better if one could retain some kind of phenomenological theory without loosing the zeroth order or mean-field approach.

In this talk we want to emphasize the different predictions in both pictures and we present the most recent numerical results which try to discern what is the correct description of the spin-glass phase. We think that a correct answer to this question is relevant also from the experimental point of view [10]. In the experiments it is not possible to reach equilibrium in a time window of the same order of the experimental realization time. We expect that a correct description of the statics will reveal important (at least in case of Ising spin-glass models with full replica symmetry breaking [11, 12]) for a full understanding also of the off-equilibrium dynamics.

This talk is divided in four parts. In the next section we will explain what are the different predictions in both pictures. The third section contains some of the main results at zero magnetic field in case of short-range models. In the fourth section we discuss the chaos problem and we propose a scaling theory of the spin-glass phase in case of magnetic field perturbations. Section 5 is devoted to the crucial problem about the existence of the de Almeida-Thouless critical line in magnetic field. The last part contains the conclusions.

2 Mean-field approach versus droplet models

According to mean-field theory, the low temperature phase in spin glasses consists of an infinity of equilibrium or pure states separated by energy barriers of all heights. These pure states differ in their statistical weights even though they are thermodynamically equivalent. These states should be considered like equilibrium states except by the fact that they are in some sense critical. By this we mean that spatial (time) correlation functions decay very slowly with the distance (time). In mean-field theory this criticality appears in the study of the spectrum of fluctuations [13]. The existence of zero modes reveals that this systems can be quite sensitive to external perturbations.

Droplet models assume that there is a unique ground state and that the spectrum of fluctuations is given by a mechanism of droplet excitations.

This mechanism consist in the reversal of compact domains of spins of size L characterized by a rough surface of fractal dimension d_s with $d-1 \leq d_s \leq d$. The typical energies of these excitations scale like L^θ with θ being the thermal exponent. This exponent is given by the zero-temperature fixed point in the renormalization-group flow equations. In this picture a mean-field theory is unclear because the volume of the droplet coincides with its surface.

One more difference among both approaches concerns the behavior of the system against a external perturbation. For example, if we apply a small magnetic field h, the mean-field theory predicts that the spin-glass phase is not destroyed. A more detailed study of this effect reveals that a great number of states are destroyed but an infinity of them still remain [14]. When the intensity of the applied field increases we reach the AT (de Almeida-Thouless) critical line [15] and the system goes into the paramagnetic phase. In droplet models the effect of the magnetic field is the suppression of all the excitations of size L larger than a characteristic length ξ. According to D. S. Fisher and D. A. Huse [7]:

$$\xi \sim (q_{EA}\, h^2)^{\frac{1}{2\theta - d}} \tag{1}$$

with q_{EA} the Edwards-Anderson order parameter. After applying the magnetic field h the system acquires a finite correlation length ξ and it enters in the paramagnetic phase. Then the AT line is given by $h = 0$.

The existence of a unique equilibrium state and the effect of the magnetic field on the spin-glass phase are the two most important differences between both approaches which we will investigate in the following sections.

3 The order parameter P(q) at zero magnetic field

The Ising spin glass model is given by a hamiltonian of the type:

$$H = -\sum_{(i,j)} J_{ij}\sigma_i\sigma_j \; - \; h\sum_i \sigma_i \tag{2}$$

where the couplings J_{ij} are quenched variables distributed according to a probability function $P(J)$ of zero mean and finite variance. The interaction is restricted to nearest neighbours and h is the magnetic field. The Ising spins σ_i take the two possible values ± 1 and live in a d-dimensional hypercubic lattice. In the limit $d \to \infty$ one expects to converge to mean-field theory, i.e. the SK model. In the SK model all spins interact one to each other and the couplings J_{ij} are normalized by a factor $1/\sqrt{N}$ where N is the number of spins.

Let us suppose now we take two copies $\{\sigma, \tau\}$ with the same realization of the disorder and we define the overlap

$$q_{\sigma\tau} = \sum_i \sigma_i \tau_i \tag{3}$$

From this overlap we can construct its probability distribution defined by

$$P(q) = \overline{\langle \delta(q - \frac{1}{N} \sum_i \sigma_i \tau_i) \rangle} \tag{4}$$

where $\langle ... \rangle$ and $\overline{(...)}$ mean the usual statistical Gibbs average over configurations and the average over the quenched disorder respectively.

This function $P(q)$ (which gives the probability of taking two configurations α and β to have overlap $q_{\alpha\beta} = q$) can be analytically computed in mean-field theory. It is symmetric around $q = 0$ and consists of a delta function centered at a maximum value q_{max} plus a continuous part. This function extends down to $q = 0$ and the value of $P(0)$ is nearly independent of the size of the system. Numerical simulations for the SK model [16] are in agreement with this prediction and we want to know if this mean-field result also is true in finite dimensions. Droplet models predict that the $P(q)$ should be single peaked at $q = q_{max}$ and the tail of the $P(q)$, which extends down to $q = 0$, should be suppressed as the size increases. More precisely we expect the $P(0)$ should scale like $L^{-\theta}$ [21] where L is the lattice size. Numerical studies in the chaos problem predict a value of θ close to 0.5 in four dimensions. Monte Carlo numerical results are shown in figure 1 in case of

the $4d \pm J$ Ising spin glass at $T = 1.4$ ($T_c = 2.05$). The value of $P(0)$ seems to be constant with the size in agreeement with mean-field predictions. For this regime of sizes, mean-field theory seems to describe qualitatively well the behavior of the system.

Another interesting result in mean-field theory regards the freezing of some critical properties in the spin-glass phase. In the critical point the order parameter vanishes and the probability function $P(q)$ scales with the size according to the expression[17]:

$$P(q) \sim N^{\frac{1}{3}} exp(-a\, N\, q^3) \tag{5}$$

where a is a coefficient and N is the size of the system. In the spin-glass phase one can analitically compute the finite-size corrections to the tail of the $P(q)$ around $q = q_{max}$ [18]. The same cubic exponent is found and the result is

$$P(q > q_{max}) \sim N^{\frac{1}{3}} exp(-a\, N\, (q - q_{max})^3) \tag{6}$$

Figure 2 shows this result in case of the SK model at $T = \frac{T_c}{2}$. This freezing of the exponent of the tail seems also to be a general characteristic in short-range models. The corresponding exponent which gives the cost of the free energy for configurations with q different from zero, is given by $\frac{d}{dq}$ where $dq = \frac{d-2+\eta}{2}$ and d is the dimension. In four dimensions $\eta \simeq -0.25$ [19, 20] and the correct scaling is given by [22, 23]

$$P(q > q_{max}) \sim P(q = q_{max})\, exp(-a\, N\, (q - q_{max})^{4.5}) \tag{7}$$

Figure 3 shows the same result for $T = 1.4$ in the $4d \pm J$ Ising spin glass. Within errors the exponent coincides with the exponent one finds in the critical point. Now the position q_{max} of the peak of the $P(q)$ has increased from 0 at T_c to 0.50 at $T = 1.4$. It can be objected that all these simulation results are close to the critical point ($T \sim 0.7T_c$.) This is unvoidable because one cannot perform standard simulations down to very low temperatures even though new numerical simulation techniques like simulated tempering [29, 30] should be able to avoid any critical effect. In any case,

these results are very suggestive and we will see that the freezing of some exponents can be taken as a good starting hypothesis to understand other interesting results like chaoticity in spin glasses. We remind the reader that this result suggests that the low temperature phase in spin-glasses should be described by properties derived from the finite-temperature fixed point exponents in the renormalization group flow equations. This is precisely the opposite assertion of droplet models.

4 Static chaos in spin glasses

By static chaos we understand the fact that a small perturbation of the Hamiltonian is enough to reshuffle the Boltzmann weights of the different equilibrium configurations. This result is a natural feature of mean-field approach and droplet models. In the first case the fact that there is a very large number of equilibrium states. This implies that a small addition of energy to the system can be enough in order to reshuffle all the free energy differences among the states changing their statistical weights. In droplet models,the ground state is marginally stable and it can be quite sensitive to perturbations. There are several type of perturbations and the most studied up to now are a change of the magnetic field or a change of the temperature [24]. We want now to focuse in the case in which a small magnetic field is applied to the system which is initially at zero magnetic field.

Let us suppose an Ising spin glass system with Hamiltonian $H_1[\sigma]$. Let us apply a perturbation P to the system and the new Hamiltonian for a different copy of spins $\{\tau_i\}$ is

$$H_2[\tau] = H_1[\tau] + P[\tau] \tag{8}$$

We consider now a full Hamiltonian whichis the initial system $H_1[\sigma]$ plus the perturbed one $H_2[\tau]$, i.e. $H[\sigma,\tau] = H_1[\tau] + H_2[\tau]$. We define the adimensional quantity (chaos parameter)

$$a = \frac{\overline{\langle \sigma_i \tau_i \rangle_H^2}}{\langle \sigma_i^a \sigma_i^b \rangle^2{}_{H_1}} \tag{9}$$

Now $\langle \sigma_i^a \sigma_i^b \rangle$ means the order parameter evaluated taking two copies a, b of the initial system H_1. The numerator is obtained averaging over the full Hamiltonian $H[\sigma,\tau]$.

The spin-glass phase is chaotic if

$$\lim_{P\to 0} \lim_{N\to\infty} a(P) < 1 \tag{10}$$

Chaoticity in spin glasses reflects the fact any small but finite perturbation P causes the parameter a to fall abruptly to a value smaller than $a = 1$. This can only happen in the thermodynamic limit because for a finite size N the chaos parameter a will be always a smooth function of N. This means that one has to take the thermodynamic limit before applying the perturbation P.

In droplet models there is chaos against magnetic field because of the sudden jump of the correlation length which becomes finite as soon as h is finite. This mechanism is similar to the effect of a magnetic field when the system stays at the critical point. In fact using eq.(1):

$$a \equiv f(Nh^{\frac{2\delta}{d-2\theta}}) \tag{11}$$

The mechanism of chaoticity in mean-field theory is different. After the application of the magnetic field the system remains in the spin-glass phase and not all states have been suppressed. We argue that some equilibrium states are destroyed by the effect of the magnetic field and the free energy cost of the suppressed states is given by the finite-temperature fixed points exponents [14]. The suppressed states acquire a finite correlation length ξ (the chaos correlation length). This hypothesis gives exact results in mean-field theory and interesting predictions in case of short-range models.

Accordin to this hypothesis, the singular part of the free energy scales according to

$$f_{sing} \sim \sum_{a<b} Q_{ab}^{\frac{d}{dq}} \tag{12}$$

where $dq = \frac{\beta}{\nu} = \frac{d-2+\eta}{2}$ (β, ν, η are critical exponents) and d is the dimension. The difference between the singular part of the free energy at field h and zero magnetic field gives the free energy cost of the suppressed states:

$$f_{sing} = \sum_{a<b} Q_{ab}^{\frac{d}{dq}}(h) - \sum_{a<b} Q_{ab}^{\frac{d}{dq}}(0) = q_{min}^{\frac{d}{dq}+1} \tag{13}$$

The main ingredient that we have used in this derivation is the fact that the order parameter $q(x)$ in short range models is characterized by a continuous part plus a plateau where q_{min} and x_{min} are the value of $q(x)$ and the breakpoint of the plateau which depends on the magnetic field.

The adimensional chaos parameter a scales according to

$$a \equiv r \sim f(N\, h^{\frac{2(d+dq)}{\lambda dq}}) \tag{14}$$

where the exponent λ defined by $q_{min} \sim h^{\frac{2}{\lambda}}$ and dq depends on the critical exponents.

In mean-field theory one gets $a \equiv f(Nh^{\frac{8}{3}})$ which is in agreement with replica calculations [24, 14]. Within mean-field theory the chaos correlation length scales like $\xi \sim h^{-\frac{2}{3}}$. Figure 4 shows numerical results in mean-field theory. It can be seen [14] that, at the mean-field level, 3 is the lower critical dimension (d_l) and 4 corresponds to an upper critical dimension ($d_u = 4$). We point out that this upper critical dimension corresponds to a different universality class than the one corresponding to the usual Ising spin-glass transition which is 6. Figure 5 shows finite-size scaling for the chaos parameter a in the $4d$ $\pm J$ Ising spin glass at $T = 1.5$. Data fit very well the mean-field prediction. Within this approach one can also derive the nature of the AT line in short range models (we only need to know the critical exponents). In $d = 4$ we obtain $h \sim \tau^{1.3\pm0.1}$ which is close to the mean-field

theory result. The result $d_l = 3$ is in agreement with recent numerical simulations [28]. In three dimensions we also expect a pseudocritical AT line. This seems paradoxical if $d_l = 3$. The solution to this paradox is that only for sizes (times) smaller than the true correlation length (time) one has some kind of pseudocritical AT line. The same mean-field approach reveals that the critical line is given by $h \sim \tau^{1.8}$ which is close to experimental results in bulk CuMn [25].

All this results can also be explained in the framework of the droplet picture. It is interesting to note that droplet models also predict $d_l = 3$ and the zero-temperature exponent θ is very well approximated by the formula $\frac{d-3}{2}$. Static chaos reveals interesting properties of the spin-glass phase but it is very difficult to use it in order to distinguish what is the correct picture of the spin-glass phase. The crucial difference between both pictures concerns the existence of the AT line. In the droplet picture this should not be a real transition but only a dynamical freezing line. The study of the existence of the AT line is the purpose of the next section.

5 The order parameter P(q) at finite magnetic field

In the previous section we said that some states are suppressed after applying the magnetic field. At zero magnetic field the order parameter distribution $P(q)$ is symmetric under the exchange $q \to -q$. The magnetic field breaks this symmetry and the $P(q)$ is expected to be non-zero only if $q > 0$. In mean-field theory we expect that only those states $\{\alpha\}$ which give overlaps $\{q_{\alpha\beta} \leq q_{min}, \forall\beta\}$ are simply erased by the magnetic field. The suppression of the states conserves the ultrametricity property. In a magnetic field the $P(q)$ is non zero inside the interval (q_{min}, q_{max}) and has two singularities at the extremes of this interval

$$P(q) = P_0(q) + a\delta(q - q_{min}) + b\delta(q - q_{max}) \tag{15}$$

Figure 6 shows simulation results [26] in case of mean-field theory at $T = 0.5$ and $h = 0.3$ (the AT line is at $h \sim 0.57$). This result is a stronger test of Parisi solution to mean-field theory. The position of the singularities q_{min} and q_{max} can be otained within replica theory. Calculations at first order of replica symmetry breaking give $q_{min} = 0.45$, $q_{max} = 0.63$ [27]. This is in agreement with the numerical results.

In case of short-range systems the situations is much more subtle. Figure 7 shows the results in case of the $4d$ $\pm J$ Ising spin-glass at $T = 1.2$ and $h = 0.4$ [26]. These results, in function of the size of the lattice, show a similar trend to what appears in the simulations for the SK model. Anyway it would be desiderable to locate precisely the singularity corresponding to $q = q_{min}$. The main problem in these simulations is that the correlation length ξ predicted in the framework of the droplet model is of the order of the size of the system. It is crucial to make simulations in the region $L >> \xi$ where ξ can be estimated from numerical studies on chaos (see figure 5). This can be achieved using new numerical techniques like simulated tempering [29, 30]. We expect to obtain more conclusive results in this direction in the near future.

6 Conclusions

Two competing approaches have been proposed to understand the low temperature phase in spin glasses. In the first approach one tries to understand the spin-glass phase using mean-field theory as a starting point. The other approach is the phenomenological droplet model in which the thermodynamics is given by droplet excitations.

The predictions are different in both pictures and it would be very interesting to know what is the relevant description of the low temperature phase. Up to date, numerical simulations in the regime of small sizes favour the mean-field approach. But it is also true that in that regime of sizes the finite-size effects could reproduce very well the typical features of mean-field

theory (several states, non self-averaging and similar concepts [31].)

Chaoticity of spin glasses against magnetic field perturbations give rich information about the nature of the spin-glass phase. But it is very difficult to make a clear distintion between both pictures. It is possible that chaoticity in case of temperature perturbations will reveal much more conclusive but this is still an open problem.

The crucial difference between both picture concerns the existence of transition with magnetic field. Recent numerical simulations for small lattices show that it is very difficult to get a conclusive answer on this problem. Anyhow we think that it is reliable and possible to go to the region of sizes for which the correlation length (according to droplet picture) is smaller than the lattice size. Then it is possible to investigate this region in the field-temperature plane using numerical tecniques like simulated tempering. Even though it is very difficult to decide between the mean-field picture and droplet pictures we hope that conclusive results in this direction will be obtained in the near future.

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Figure Caption

Fig. 1 $P(q)$ at $T = 1.4$ in the $d = 4$ case for several sizes. Error bars are of order 10 per cent for all sizes.

Fig. 2 Tails of $P(q)/P_{max}$ with $P_{max} = N^{\frac{1}{3}}$ in the SK model versus $N(q-0.62)^3$ for several sizes $N = 32 - 768$ at $T = 0.5$.

Fig. 3 Tails of $P(q)/P_{max}$ at $T = 1.4$ in the 4d case for several sizes.

Fig. 4 Chaos with magnetic field in the SK model at $T = 0.6$. Field values range from $h = 0.2$ up to $h = 1.0$ for the smaller sizes and up to $h = 0.4$ for the largest ones.The number of samples range from 200 for $N = 32$ down to 40 for $N = 1632$. Typical error bars are of order 5 per cent in all cases.

Fig. 5 Chaos with magnetic field in the $4d$ $\pm J$ Ising spin glass at $T = 1.5$. Magnetic field values range from $h = 0.1$ up to $h = 0.8$. The number of samples is approximately 100 for all lattice sizes. Typical error bars in this case are of the size of the symbols.

Fig. 6 $P(q)$ for the SK model at $T = 0.5$, $h = 0.3$. The number of samples are 500 for $N = 736$, 30 for $N = 1408$ and 20 for $N = 3200$. Error bars run from 20% for the largest size down to 5% for the smallest one.

Fig. 7 $P(q)$ with magnetic field in the $4d$ $\pm$ Ising spin glass at $T = 1.2$, $h = 0.4$. The number of samples are $320, 128, 100, 50$ for $L = 3, 5, 6, 8$ respectively. Error bars are smaller than 15% in all cases.

figure 1

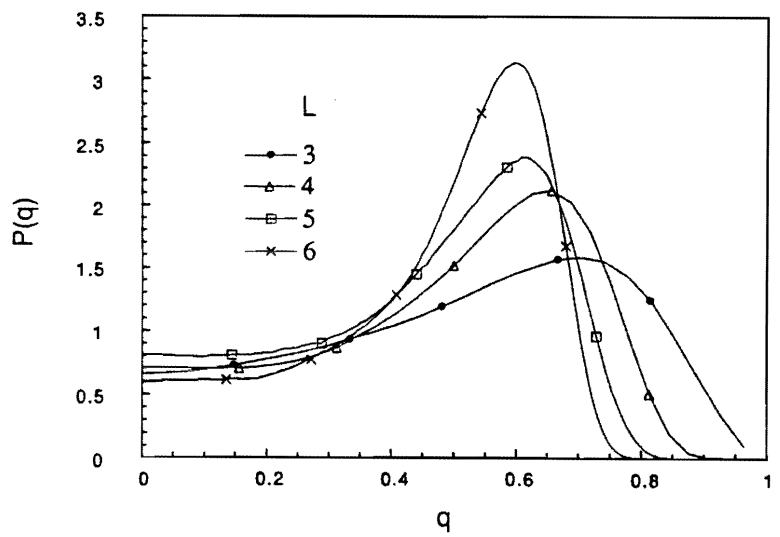
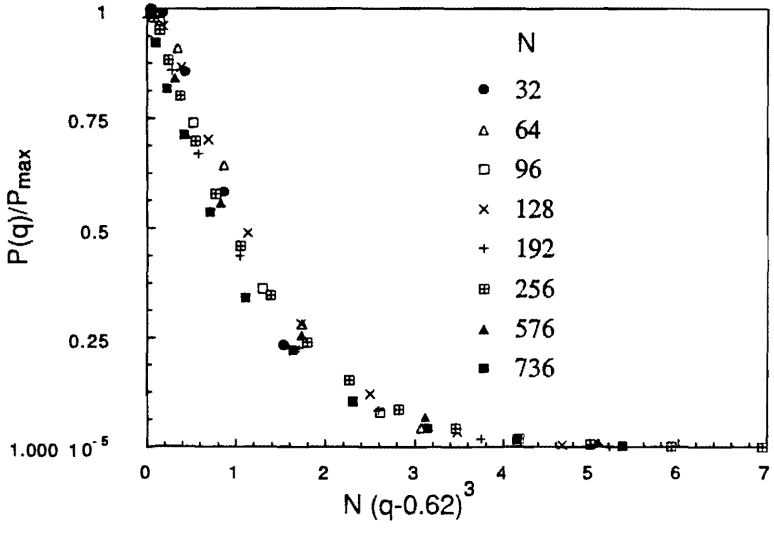


figure 2



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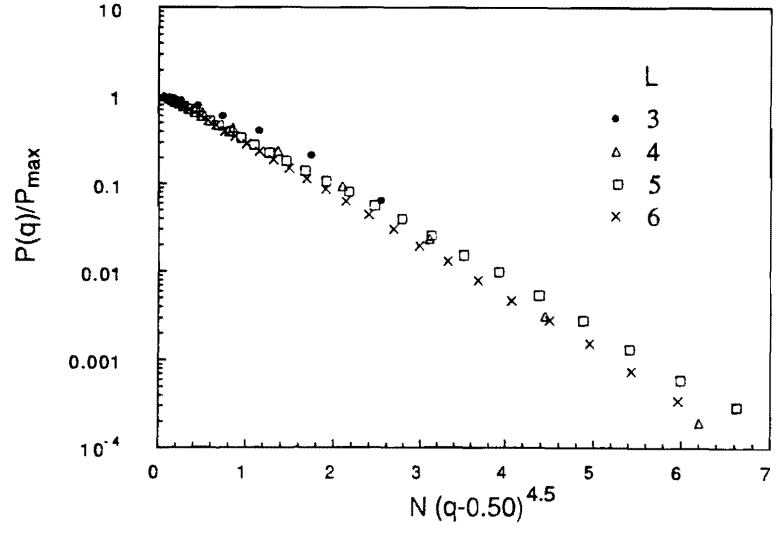

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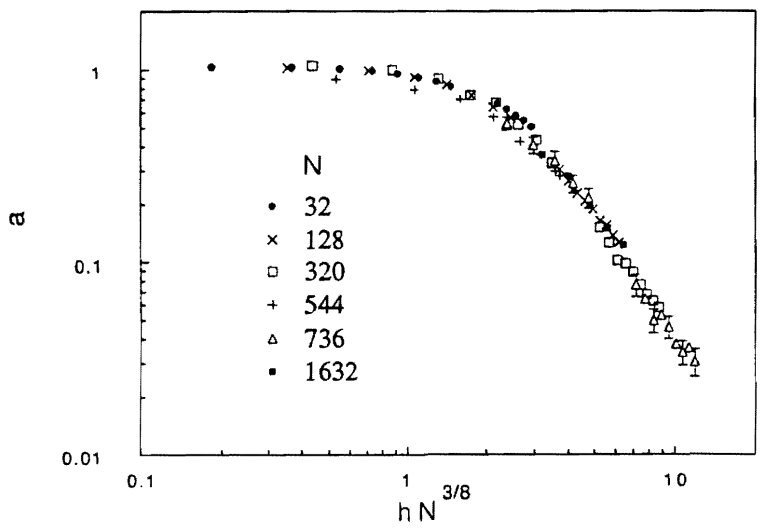

figure 5

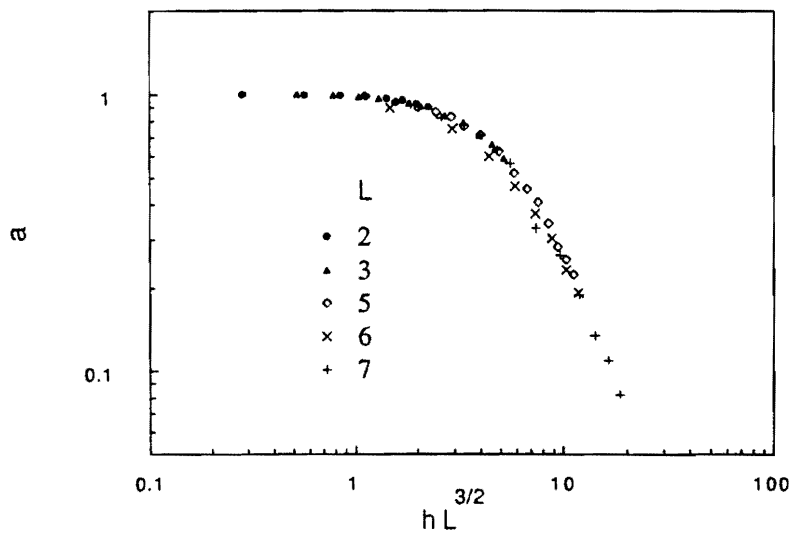

figure 6

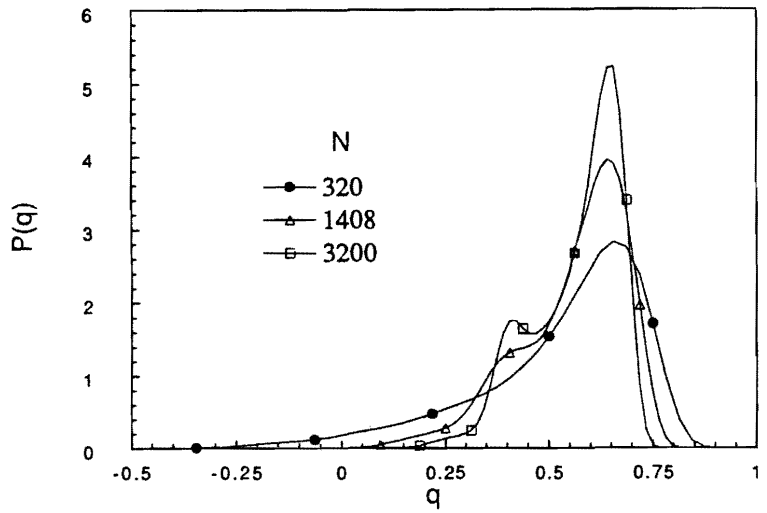
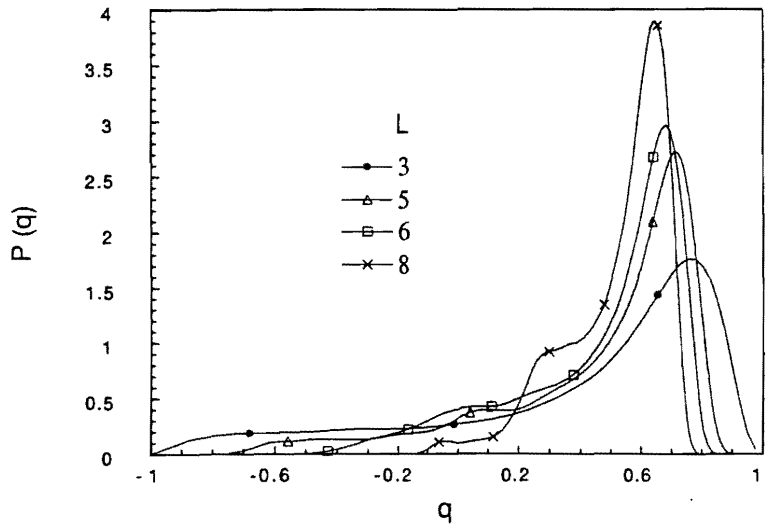
figure 7