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STABILITY OF CHIRAL HIERARCHIES

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ABSTRACT

The need for a fine tuned hierarchy between a strong coupling high energy compositeness scale and a much lower chiral symmetry breaking scale exists in many models of dynamical electroweak symmetry breaking. We explore the stability of such hierarchies against quantum fluctuations. A nonperturbative Wilson renormalization group equation approach is introduced.

A common feature of many of the currently studied models of dynamical electroweak symmetry breaking is the presence of some strong interactions acting at a high energy scale, $\Lambda >$ 10 TeV, which produces an essentially composite scalar bosonic degree of freedom. This dynamics also plays an important role in the electroweak symmetry breaking whose characteristic scale is much lower; $\Lambda_F \simeq 250GeV$. Thus these models require that a significant hierarchy can be established between these scales. The hierarchy is achieved by a fine tuning of parameters close to the critical value for the chiral symmetry breaking. A prototype of this behavior is exhibited by the Nambu Jona-Lasinio (NJL) model[1], where a fine tuning of the four-fermion coupling allows the emergence of a chiral symmetry breaking scale far below the compositeness scale. In more recent work, such fine tunings are necessary ingredients in strong extended technicolor models[2], models involving heavy quark condensation[3] and their generalizations. In order for this hierarchy to be maintained and not have the electroweak scale driven to be of order Λ, it is necessary that the chiral symmetry phase transition be of second order. That is, the order parameter characterizing the chiral transition must remain zero as the theory is scaled from Λ into the infrared until one reaches the electroweak scale. If, on the other hand, the transition turns first order at a scale $e^{-t_0}\Lambda >> \Lambda_F$, then the order parameter will jump discontinuously to be of this value and it will be impossible to maintain the hierarchy all the way down to the electroweak scale. Instead the hierarchy will destabilize after t_0 e-foldings. It is important to recognize that this question is distinct from that of the naturalness of the fine tuning of additive quadratic divergences. Clearly, it becomes necessary to explore[4] when the hierarchy can be self consistently maintained and not destroyed by quantum fluctuations (Coleman-Weinberg phenomenon[5]).

Since models with a single scalar quartic self coupling or the minimal Nambu-Jona Lasinio model exhibit a Gaussian second order chiral transition, we are led to investigate generalizations containing multiple scalar quartic self couplings. In particular, we focus on a model possessing a global chiral $U(2)_L \times U(2)_R$ symmetry which has two independent scalar quartic self couplings. The model degrees of freedom include left and right handed chiral fermions ψ_{iL} and ψ_{iR}, $i = 1,2$, transforming as the fundamental representation of the left and right handed $U(2)$ groups respectively which further carry the fundamental, N_C, representation of a gauged symmetry. It is assumed that this chiral symmetry is spontaneously broken as a consequence of some unspecified dynamics acting at scale Λ. This symmetry breaking is further assumed to produce a gauge singlet scalar composite Σ_{ij} which has the $U(2)_L \times U(2)_R$ quantum numbers of the fermion bilinear $\bar{\psi}_{jR}\psi_{iL}$ and whose vacuum expectation value, $v/\sqrt{2}$, acts an an order parameter for the chiral symmetry

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breaking. Since we are assuming the chiral symmetry phase transition is second order, we are led to study a Ginzburg-Landau effective Lagrangian at scale Λ which includes the Yukawa coupling $\frac{\pi}{\sqrt{2}}g(0)(\bar{\psi}_L\Sigma\psi_R + \bar{\psi}_R\Sigma^\dagger\psi_L)$ and the invariant potential function

$$V(x,y,0) = \frac{1}{2}m^2(0)x + \frac{\pi^2}{12}\lambda_1(0)x^2 + \frac{\pi^2}{6}\lambda_2(0)y\,. \tag{1}$$

Here $x = tr(\Sigma^\dagger\Sigma), y = tr(\Sigma^\dagger\Sigma)^2$ are independent $U(2)_L \times U(2)_R$ invariants. The Coleman-Weinberg instability is signalled by the appearance of a non-trivial global minimum of the effective potential with vanishing renormalized mass appearing at the scale $v = e^{-t_0}\Lambda$. In such a case, the phase transition is driven first order by quantum fluctuations and one can technically achieve a hierarchy of only t_0 e-foldings.

Using the 1-loop perturbation theory improved effective potential while keeping the Yukawa coupling fixed, the renormalization group trajectories are found[4] to either run to the infrared quasi fixed point near the origin, lead to a first order transition or simply run away in which case the model is ill defined. In particular, for the region of coupling space corresponding to the initial couplings $\lambda_1(0) = 0$ and $\lambda_2(0)$ and $g^2(0)$ both large and positive, the transition goes first order for sufficiently large $\lambda_2(0)/g^2(0)$ ratio ($\gtrsim 7$). Moreover, this occurs, in general, near to the compositeness scale. For example, for $\lambda_2(0) = 10$ and $g^2(0) = 1$ (and $N_C = 3$), the hierarchy destabilizes after ~ 1.3 e-foldings (which corresponds to $v \sim 0.27\Lambda$). However, since the couplings are very large, the 1-loop perturbative approximation can certainly be called into question. For instance, using the 2-loop perturbative renormalization group functions for these large initial couplings, the renormalization group trajectories simply run away. Clearly, some nonperturbative approximation scheme is required to properly deal with the system in the vicinity of the strong coupling compositeness scale. The purely bosonic $U(2)_L \times U(2)_R$ model (no chiral fermions) has been simulated using lattice Monte Carlo techniques[6] and was seen to undergo a Coleman-Weinberg instability. Another approach[7] includes the chiral fermions and uses a large N_C approximation in the vicinity of the compositeness scale so that the Yukawa coupling dominates. Retaining only it and the fine tuned scalar mass term needed to cancel the additive quadratic divergence, the model at scale Λ reduces to the minimal NJL model which is exactly soluble in the large N_C limit and is known to exhibit a (trivial) second order chiral transition. Thus the large N_C approximation allows any sized hierarchy to be technically achieved. Running the couplings using this approximation (in which $\lambda_1(t) = 0$) until $\lambda_2(t) = 3g^2(t) = \frac{48}{N_C lnt}$ has decreased sufficiently to be smoothly joined onto a 2-loop perturbative (including Yukawa coupling) running, it is found that sizeable hierarchies can be established. To the extent that the large N_C approximation is nonperturbative, this procedure is a self consistent one. On the other hand, it can be reliably employed for only a very limited range of the initial parameter space.

An alternate nonperturbative method is provided by the continuous Wilson renormalization group equation (WRGE)[8] which has been extended to include chiral fermions[9]. This approach incorporates the contributions from a complete set of local operators (including irrelevant ones). Using a local action approximation[10] which ignores anomalous dimensions and derivative interactions and further neglecting operators higher than bilinear in the fermion fields, the WRGE for fixed Yukawa coupling reduces to an equation for the potential function, $V(x,y,t)$, at scale $e^{-t}\Lambda$ given by

$$\begin{aligned}
\frac{\partial V}{\partial t} &= 4V - 2xV_x - 4yV_y \\
&+\frac{1}{8\pi^2}\ell n[(1+V_x+2xV_y)^2 - 2(x^2-y)V_y^2] \\
&+\frac{1}{16\pi^2}\ell n[(1+V_x)^2 + 2x(1+V_x)V_y \\
&+2(x^2-y)V_y^2] + \frac{1}{16\pi^2}\ell n[(1+V_x)(1+V_x \\
&+6xV_y + 2xV_{xx} + 8yV_{xy} + 4x(3y-x^2)V_{yy}) \\
&+6(x^2-y)V_y(3V_y + 2V_{xx} + 4xV_{xy} \\
&+4yV_{yy}) + 8(x^2-y)(x^2-2y)(V_{xy}^2 - V_{xx}V_{yy})] \\
&-\frac{N_C}{4\pi^2}\ell n[1 + \frac{\pi^2}{2}xg^2(0) + \frac{\pi^4}{8}(x^2-y)g^4(0)]\,,
\end{aligned} \tag{2}$$

with $V_x = \frac{\partial V}{\partial x}$ etc, subject to the initial condition of Eq.(1). For $t \geq 0$, each action constructed using the $V(x,y,t)$ satisfying this equation lies on the same Wilson renormalization group trajectory and produces the same physics on all scales less than $e^{-t}\Lambda$. Unfortunately, the solution to Eq.(2) is currently beyond our numerical abilities. Thus we make the further truncation of retaining terms only up to linear in y with coeficients which are arbitrary functions of x. Eq.(2) then reduces to two coupled equations which are of a similiar (although considerably more complicated) form to what we previously solved in obtaining nonperturbative mass bounds[9,11]. While the truncations used are drastic and uncontrolled, they still include contributions from an infinite number of operators.

The resulting equations are then numerically solved for t values up to some t^*, where t^* lies in a region where $V(x,y,t)$ is found to be linear in $t - t^*$ with a slope of the same form as the linearized in $t - t^*$ 1-loop effective potential. The effective potential is then constructed as $V_{eff}(x,y) = V(x,y,t^*) + V_{1-loop}(x,y,t^*)$, where $V_{1-loop}(x,y,t^*)$ is the 1-loop effective potential which accounts for the effects of the degrees of freedom carrying momentum less than $e^{-t^*}\Lambda$. A Coleman-Weinberg instability is signalled by a non-trivial global minimum of $V_{eff}(v) = V_{eff}(x,y)_{y=\frac{1}{2}x^2=\frac{1}{2}v^4}$ with vanishing renormalized mass. If such a minimum appears at $v = e^{-t_0}\Lambda$, then the system can sustain a hierarchy only over t_0 e-foldings. For the special case of $\lambda_1(0) = 0, \lambda_2(0) = 10, g^2(0) = 1$, we integrated the WRGE and found that for $t^* \sim 1$, $V(x,y,t)$ was linear in $t - t^*$ and smoothly joined onto the 1-loop effective potential. The transition was seen to go first order at $t_0 \sim 1.6$. Thus a hierarchy of only ~ 1.6 e-foldings can be established ($v \sim 0.20\Lambda$). This result is in qualitative agreement with that found using the 1-loop perturbative approximation. In a similar manner, one can nonperturbatively investigate[12] the initial parameter space and map out the model phase structure.

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References

1. Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122** (1961) 345; see also W.A. Bardeen and S.T. Love, Phys. Rev. **D45** (1992) 4672.

2. For a review, see T. Appelquist in the *Proceedings of the Fourth Mexican School of Particles and Fields*, Mexico City 1990, ed. by L.J. Lucio M and A. Zepeda, (World Scientific, 1991) 1 and references therein.

3. For a review, see W.A. Bardeen in *Gauge Theories, Past and Future*, ed. by R. Akhoury et al., (World Scientific, 1992) 51 and references therein.

4. R.S. Chivukula, M. Golden and E. Simmons, Phys. Rev. Lett. **70** (1993) 1587.

5. S. Coleman and E. Weinberg, Phys. Rev. **D7** (1973) 1888.

6. Y. Shen, Phys. Lett. **315B** (1993) 146.

7. W.A. Bardeen, C.T. Hill and D. Jungnickel, Phys Rev. **D49** (1994) 1437.

8. K. Wilson and J. Kogut, Phys. Rept. **12C** (1974) 75; K. Wilson, Rev. Mod Phys. **55** (1983) 583; F. Wegner and A. Houghton, Phys. Rev. **A8** (1973) 401.

9. T.E. Clark, B. Haeri and S.T. Love, Nucl. Phys **B402** (1993) 628.

10. P. Hasenfratz and J. Nager, Z. Phys. **C37** (1988) 477.

11. S.T. Love in *Proceedings of the International Europhysics Conference on High Energy Physics*, Marseilles 1993 ed. by J. Carr and M. Perrottet (Editions Frontieres, 1994) 208; T.E. Clark, B. Haeri, S.T. Love, M.A. Walker and W.T.A. ter Veldhuis, Phys. Rev. **D50** (1994) 606.

12. T.E. Clark and S.T. Love, PURD-TH-94-10.