PURD-TH-93-11

Fermilab Library
0 1460 0036749 0

MASS BOUNDS IN YUKAWA MODELS: WILSON RENORMALIZATION GROUP ANALYSIS

S.T. Love[1]
Department of Physics
Purdue University
West Lafayette, IN 47907-1396

ABSTRACT

Nonperturbative triviality and vacuum stability mass bounds are obtained for the scalar and fermion degrees of freedom in a Yukawa model using Wilson renormalization group techniques. Particular attention is given to the effect of the generalized Yukawa coupling on the scalar mass upper bound which is to slightly increase it.

Numerous studies of self coupled $\lambda\varphi^4$ field theory using 1-loop perturbation theory as well as nonperturbative lattice Monte Carlo simulations conclude that the model can only be consistently defined as an effective description of the dynamics below some momentum cutoff Λ, with the $\Lambda \to \infty$ limit characterized by a vanishing renormalized coupling. This in turn constrains the parameters of the model and leads to an upper (triviality) bound on the scalar mass. When a huge hierarchy exists between the cutoff and the scalar mass (generally accomplished by a fine tuning of parameters), then the infrared Gaussian (trivial) fixed point focuses the renormalization group trajectories to a low energy value which is basically insensitive to the exact form of the short distance couplings and in a way which is amenable to a perturbative analysis. On the other hand, when determining scalar mass absolute upper bounds, the mass value will not be too far removed from the cutoff and some form of nonperturbative calculational procedure is required. One such nonperturbative approach to the study of the critical dynamics in this system is to use the continuous Wilson renormalization group equation (WRGE)[1]. This functional differential equation satisfied by the action is constructed by integrating out the degrees of freedom between the original cutoff and a new, lower cutoff while demanding that the physics (ie- the renormalized Green functions) remain unchanged on all momentum scales less than the lower cutoff. The change in the cutoff is compensated by changing the coupling constants of a complete set of local operators. This includes irrelevant as well as relevant and marginal operators. The various action functionals satisfying this equation correspond to points on a particular (Wilson) renormalization group trajectory each producing the same physics on momentum scales less than the UV cutoff used in its definition. Moreover, as the construction is independent of the strength of the initial coupling(s), it constitutes a nonperturbative approach to the study of the critical behavior. Using a local action approximation to this exact functional equation, an upper bound on the scalar mass consistent with the perturbative[2] and the nonperturbative lattice bounds[3] was extracted[4].

We have extended this approach to construct the Wilson renormalization group equation for models containing fermions in addition to scalars[5]. The need to include fermions in such a nonperturbative analysis is clearly motivated by the large top quark mass and its associated Yukawa coupling. Once again employing the local action approximation which ignores anomalous dimensions and derivative couplings, we study a Euclidean space action at the scale $e^{-t}\Lambda$ which involves a scalar φ and a Dirac fermion ψ and takes the form

$$S[\varphi,\psi,\bar{\psi};t] = \int d^4x[\frac{1}{2}\partial\varphi\cdot\partial\varphi + \bar{\psi}\gamma\cdot\partial\psi + U(\varphi,\sigma;t)], \tag{1}$$

[1] *Invited talk given at the International Europhysics Conference on High Energy Physics, Marseille 1993*

where $\sigma = \bar{\psi}\psi$. Here

$$U(\varphi, \sigma; t) = V(\varphi; t) + \sigma G(\varphi; t) \qquad (2)$$

is a generalized potential which is the sum of the effective potential V which dictates the vacuum structure and a generalized Yukawa coupling G. Defining $F(\varphi; t) = V_\varphi(\varphi; t)$, (subscripts denote differentiation) so that the ground state corresponds to the zeros of F, the WRGE reduces to the coupled partial differential equations

$$\frac{\partial F}{\partial t} = 3F - \varphi F_\varphi + \frac{1}{16\pi^2}\frac{F_{\varphi\varphi}}{1+F_\varphi} - \frac{1}{2\pi^2}\frac{GG_\varphi}{1+G^2},$$

$$\frac{\partial G}{\partial t} = G - \varphi G_\varphi + \frac{1}{16\pi^2}\frac{G_{\varphi\varphi}}{1+F_\varphi} - \frac{1}{8\pi^2}\frac{GG_\varphi^2}{(1+G^2)(1+F_\varphi)}. \qquad (3)$$

To complete the description of the model, the form of the generalized potential at the initial UV cutoff $\Lambda(t=0)$ is specified as

$$U(\varphi, \sigma; t=0) = \frac{1}{2}\mu^2(0)\varphi^2 + \frac{1}{4}\lambda(0)\varphi^4 + g(0)\varphi\sigma. \qquad (4)$$

Note that the $t = 0$ model contains only relevant and marginal operators and moreover, possesses the discrete γ_5-symmetry: $\varphi \to -\varphi, \psi \to \gamma_5\psi, \bar{\psi} \to -\bar{\psi}\gamma_5$, which forbids an explicit fermion mass term. We seek solutions for F, G which spontaneously break this discrete γ_5-symmetry, ie- $< \varphi > \neq 0$. This in turn restricts the $\mu^2(0), \lambda(0), g(0)$ parameter space. Specifically, μ^2_{cr} is defined as the maximum value of $\mu^2(0)$ for a given $\lambda(0), g(0)$ which results in a nontrivial zero of $F(\varphi; t)$ as t increases into the infrared. To determine μ^2_{cr}, we evaluate $F(\varphi; t)$ for $\mu^2(0)$ well into the broken phase and then increase $\mu^2(0)$ until the zero of F decreases as t increases. The transition value of $\mu^2(0)$ defines μ^2_{cr}. So doing, we establish the infrared Gaussian fixed point. From the solution, we also see that the induced irrelevant operators can give sizeable contributions for small t values and play an important role in driving the theory toward his fixed point. Furthermore, the numerically generated solution to the WRGE is seen to smoothly join onto the 1-loop solution (which includes irrelevant operators) for t values beyond some t^*.

To extract the scalar and fermion mass bounds, we choose a point in the allowed $\lambda(0), g(0), \mu^2(0) < \mu^2_{cr}$ parameter space corresponding to the spontaneous symmetry breaking solution and then numerically integrate the WRGE to obtain F and G for

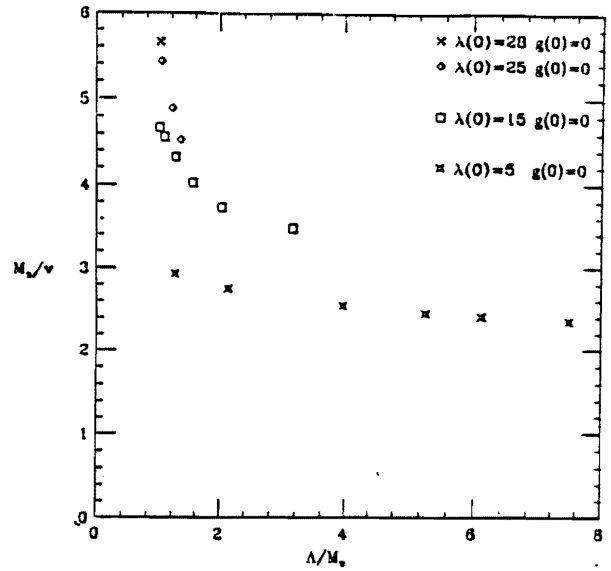

Figure 1: M_s/v as a function of Λ/M_s for different initial $\lambda(0)$ values and $g(0) = 0$.

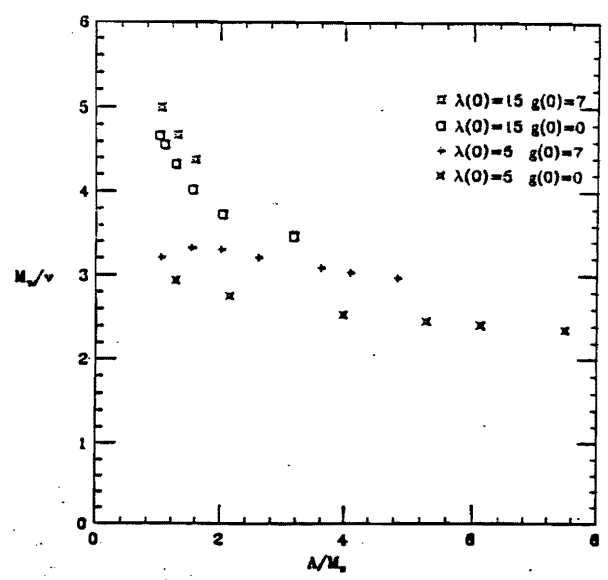

Figure 2: M_s/v as a function of Λ/M_s for different initial $\lambda(0)$ and $g(0)$ values.

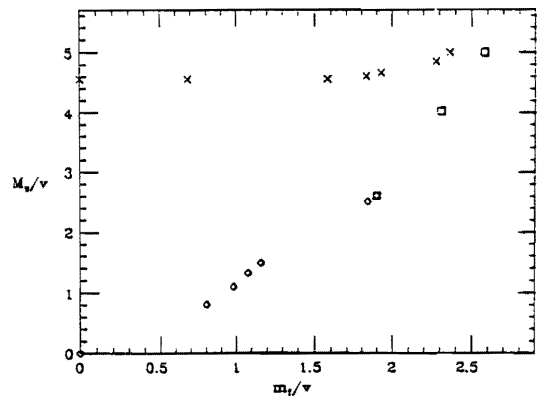

Figure 3: Allowed range of M_s/v and m_f/v values for $\Lambda/v = 5$.

$0 < t < t^*$. We next include the degrees of freedom with $|p| < e^{-t^*}\Lambda$ via the 1-loop perturbative solution which smoothly joins onto the numerically generated solution at t^*. Summing the contributions produces the functions $F^{eff}(\varphi)$ and $G^{eff}(\varphi)$ from which the nontrivial scalar vacuum expectation value (v) and scalar and fermion masses (M_s and m_f respectively) are secured as

$$F^{eff}(\frac{v}{\Lambda}) = 0; \ \frac{M_s^2}{\Lambda^2} = F_\varphi^{eff}(\frac{v}{\Lambda}); \ \frac{m_f}{\Lambda} = G^{eff}(\frac{v}{\Lambda}). \tag{5}$$

The value of t^* is chosen such that the computed masses are stable to within an error the size of the t-grid spacing. Finally, the computation is repeated varying over the allowed parameter space.

In the absence of any initial Yukawa coupling, $g(0) = 0$, we see from Fig.1 that as the initial $\lambda(0)$ coupling is increased, a dynamical envelope is seen to be established. When combined with the physical restriction that $M_s \leq \Lambda$, it follows that for fixed Λ/v, the ratio M_s/v converges to an upper limit. The scalar mass absolute upper bound is estimated as ~ 900 GeV for $v = 175$ GeV which is in line with the lattice estimates. This scalar mass bound increases somewhat in the presence of a nonzero initial Yukawa coupling as is shown in Fig. 2. The amount of increase depends on the strength of the initial scalar self coupling. For $\lambda(0) = 5$, M_s/v increases by 15-20% as $g(0)$ varies from 0 to 7, while for $\lambda(0) = 15$, the increase in M_s/v is 6-10% as $g(0)$ varies from 0 to 7. The allowed domain of M_s/v and m_f/v for $\Lambda = 5v$ is displayed in Fig. 3. The upper portion of the boundary, denoted by the ($\times$) marks is the scalar triviality bound discussed above. The ($\square$) markings correspond to triviality bounds on m_f, obtained by finding the largest $g(0)$ values for a given $\lambda(0)$ producing an envelope in the m_f/v versus Λ/m_f plane. Finally, the ($\diamond$) points arise from the vacuum stability requirement that $lim_{|\varphi|\to\infty}F(\varphi;t) > 0$ for all t. Note that for larger fermion masses, the fermion mass triviality constaint provides a more stringent bound than that due to vacuum stability. This is not the case for huge Λ/v ratios where the vacuum stability always provides the more stringent bound[6].

This work was performed in an enjoyable collaboration with T.E. Clark and B. Haeri. I thank the Aspen Center for Physics for providing an atmosphere allowing for the preparation of this talk. This work was supported in part by the U.S. Department of Energy under contract DE-AC02-76ER01428 (Task B).

References

1. K.G. Wilson and J.B. Kogut, Phys. Rept. **12C** (1974) 75; K.G. Wilson, Rev. Mod Phys. **55** (1983) 583; F.J. Wegner and A. Houghton, Phys. Rev. **A8** (1973) 401.

2. R. Dashen and H. Neuberger, Phys. Rev. Lett. **50** (1983) 1897; M.A.B. Beg et al, Phys. Rev. Lett. **52** (1984) 833; D.J.E. Callaway, Nucl. Phys. **B223** (1984) 189; M. Lindner, Z. Phys. **C31** (1986) 295.

3. M. Lüscher and P. Weisz, Phys. Lett. **212B** (1988) 472; Nucl. Phys. **B318** (1989) 705; J. Kuti, L. Lin and Y. Shen, Phys. Rev. Lett. **61** (1988) 678; G. Bhanot, K. Bitar, U. Heller and H. Neuberger, Nucl. Phys. **B353** (1991) 551.

4. P. Hasenfratz and J. Nager, Z. Phys.**C37** (1988) 477.

5. T.E. Clark, B. Haeri and S.T. Love, Nucl. Phys **B402** (1993) 628.

6. L. Maiani, G. Parisi and R. Petronzio, Nucl. Phys. **B136** (1978) 115; N. Cabibbo et al, Nucl. Phys. **B158** (1979) 295.