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Signals in a Resistive Plate Chamber *

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Zusammenfassung. Es wird ein einfaches Model für eine Resistive Platten Kammer (RPC) vorgestellt. Dies ist ein unendlicher ebener Plattenkondensator mit zwei Schichten; die eine ist Vakuum und stellt den Gasspalt dar. Die andere ist ein Dielektrikum mit der relativen Dielektrizitätskonstante 2 - 4. Die geringe Leitfähigkeit des Dielektrikums wird vernachläßigt, weil in Schöpf und Schnizer (1992) gezeigt worden ist, daß diese keinen Einfluß auf die Signale hat und daß die quasistatische Näherung verwendet werden kann. Daher wird das Potential einer Punktladung im Vakuum und daraus die Ladungsverteilung auf einer durchgehenden Anodenplatte berechnet. Die gesamte Ladung in einem Anodenstreifen ergibt sich daraus durch Integration über einen unendlich langen und endlich breiten Streifen. Die nun eingeführte Bewegung der Punktladung macht die Anodenstreifenladung zeitabhängig; deren Zeitableitung gibt den gesuchten Strom durch den Streifen mit gegebenem Abstand von der Teilchenbahn. Dieser Strom wird durch ein unendliches Integral ausgedrückt. Dieses kann für eine relative Dielektrizitätskonstante = 1 analytisch ausgewertet werden; in den anderen Fällen numerisch mittels Mathematica. Es zeigt sich, daß das Dielektrikum die Signale nur um einige Prozent erhöht.

Abstract. A simple model for a resistive plate chamber is developed. This is an infinite plane condensor with two layers; one is vacuum and represents the gas gap. The other one neighbouring the anode is a dielectric with a relative dielectric constant between 2 and 4; its weak conductivity is neglected as it has been shown by Schöpf and Schnizer (1992) that is has no influence on the signals so that the quasi-static approximation can be applied. So the potential is computed for a point charge at rest in the vacuum. From this the charge distribution on the continuous anode is derived. The total charge on a strip is found by integrating this distribution over the area of a strip with infinite length and given finite width and distance from the point charge. Assuming that the position of the latter is time-dependent renders the charge on the strip time-dependent; whose time-derivative gives the wanted current induced in the anode strip represented as an infinite integral. The latter is evaluated analytically for a relative dielectric constant equal to unity, numerically by Mathematica in all other cases. It turns out that the dielectric layer only enhances the signals by a few percent.

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1 Introduction

Resistive plate chambers (RPC's) are a new type of particle detectors now under developement at CERN, Geneva. Such a device consists of two weakly conducting dielectric panels bounding a gap filled by a gas (see Bild 1). The panels consist either both of melamine-phenolic laminate (Cro et al., 1995) or one of melamine-phenolic laminate and the other one of glass (Cerron Zeballos et al., 1995). They are in turn covered by the electrodes. The cathode is a continuous sheet of metal. The anode consists of strips; this division permits one to get information on the position of the event. Such a design permits the application of mass production techniques developed in the semiconductor industry. A high energy particle passing through the detector produces ion clusters by ionizing atoms of the gas in the gap. A static voltage (about 1 kV) applied to the electrodes sets the ions and the electrons into motion. The latter grow into an avalanche and hit the melamine-phenolic laminate panel in front of the anode strips. These moving charges induce a current on the opposite anode strip or strips. The contribution to the signal due to ion motion is negligible. We present a simple model to compute the signal induced on an anode strip by the slow motion of the electrons in the gas gap. These are approximated by a point charge moving with constant speed along a straight trajectory perpendicular to the panels. From an earlier investigation (Schöpf and Schnizer, 1995) we know that in view of the slow motion of the charge and the low conductivity of the panels a quasi-static method is applicable and the influence of the conductivity on the signals is negligible. An infinite plane condensor filled with two isolating layers is considered. The layer neighbouring the cathode is vacuum representing the gas gap; that adjacent to the anode strips is a dielectric, whose relative dielectric constant is about 2 - 4, see Bild 3. The potential for a point charge at rest in the vacuum is computed. From this the charge distribution on the continuous electrode joining the dielectric panel is derived. The total charge in an infinitely long strip of finite width is found by integrating the charge distribution over the area of the strip. Assuming the position of the inducing point charge to vary with time renders the total strip charge

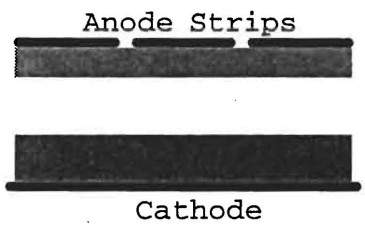

Bild 1. Schematic drawing of a Resistive Plate Chamber.

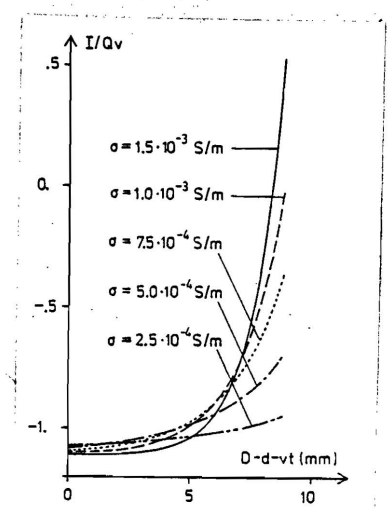

Bild 2. The current in a plane condensor containing a conducting layer.

time-dependent; its time derivative gives the wanted current. The corresponding expression is an infinite integral, which is evaluated by Mathematica (Wolfram, 1996). This current is computed and shown in its dependence on the distance between the point charge and the strip. This provides information how many strips are excited by a single event. The expression for the current can be evaluated analytically if the relative dielectric constant of the dielectric panel is unity. The numerical evaluations show that for values of the panel thickness and of the dielectric contant as used in the designs the influence of the panel is just an enhancement of the signal by a few percent.

2 General Considerations and Estimates

The application of the quasi-static approximation can be justified by some estimates. The dielectric properties of the gas do not differ appreciably from those of the vacuum. The properties of the dielectric plates as given in the Tabelle 1 are taken from Cerron Zeballos et al. (1995). Of course, the conductivity follows from the specific resistance and is given in view of later applications.

We approximate the electron avalanche moving through the gas gap by a constant point charge moving with a constant velocity $v \approx 5 \times 10^4\ m/s$. This moving charge produces a time dependent electromagnetic field, which may be regarded as a superposition of plane waves of all frequencies. These pass through the dielectric layers to the electrodes. Sommerfeld and Brillouin investigated how the field builds up in a dispersive dielectric half-space in response to a plane electromagnetic wave impinging upon the plane interface between the vacuum and the medium. The response of an electron bound to an atom and experiencing the electric field of the wave is modelled by a damped harmonic oscillator. These researches have been collected and extended in Brillouin (1960). The results are also summarized in Stratton (1941, pp.330-340). They show that a first and a second precursor of the wave with very small amplitudes arrive before the principal part of the wave arrives. We shall disregard these subtleties about the various velocities.

For our rough estimates we shall just consider the phase velocity v_{ph} of a plane wave in a conducting medium as given in Stratton (1941, p. 276): $v_{ph}/c = \left[\varepsilon/2\ \left(\sqrt{1+1/(\tau_R^2\omega^2)}+1\right)\right]^{-1/2}$. $\tau_R = \varepsilon_0\varepsilon/\sigma$ is the relaxation time required for a charge to decay in a conducting dielectric to $1/e$ of its original value. For the values given in the Tabelle 1., this relaxation time lies between $0.3s$ and $0.003s$. For the same values and a frequency $\omega/2\pi = 100MHz$ the phase velocity is about $.5c$ to $.7c$. The time $\tau_w = d/v_{ph}$ the wave needs to cross the dielectric layer of thickness $d = 1mm$ is at least 8 orders of magnitude smaller than the relaxation time. So the conduction electrons have negligible influence on the building-up and the propagation of the electromagnetic field.

This is also confirmed by the investigation performed by Schöpf and Schnizer (1992). There the time-dependent problem corresponding to that treated here has been solved by Fourier transforms and the field due to moving point charge has been calculated. In Fig. 3 of this reference the current on an electrode as function of the position of the point charge is shown. From this it can be seen (see Bild 2.) that for a conductivity $\sigma \leq 10^{-4}S/m$ of the dielectric the current is

Tabelle 1. Electric Properties of the Dielectric Panels

material	relative D.K.	spec. resist. ($\Omega\ cm$)	conductivity σ (S/m)
melamine-phenolic laminate (with 1 % water vapor)	2	10^{11}	10^{-9}
Schott A4 glass	4	10^{12}	10^{-10}

independent of the position. As explained in section 3 on p. 341 of the reference just quoted the numerical value of the quasi-static approximation agrees with the result of the more rigorous treatment.

3 Theory to the Model

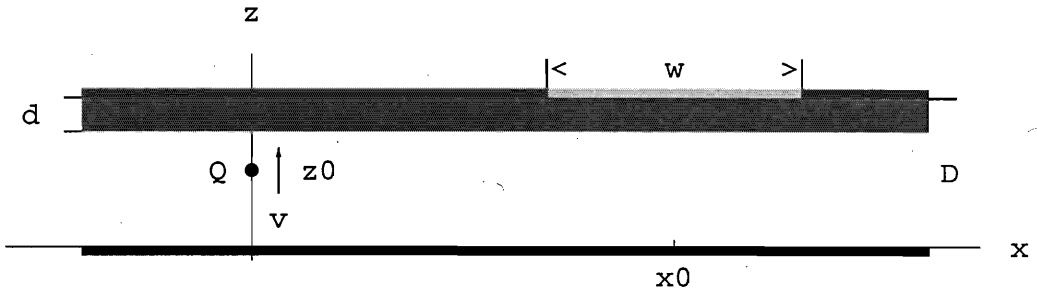

Bild 3. The plane condensor of width D filled with 2 layers. Lower layer vacuum. Upper layer has width d and relative dielectric constant ε. The center of the anode strip of width w has a distance x_0 from the trajectory of the point charge Q. The (instantaneous) position of Q is $(0, 0, z_0)$.

The theory is quite simple. The configuration assumed in our model is shown in Bild 3. In the first subsection we give the formulae, in the second one we discuss the evaluation of the main formula.

3.1 General Outline of the Theory

The static field in the two layers of the plane condensor of width D may be derived from a scalar potential Φ, which in turn may be found from the Green's function of the two layer problem with homogeneous boundary conditions along the electrodes. The potential produced by a point charge Q located in the lower layer (representing the gap, relative dielectric constant = 1) and observed within the upper layer (corresponding to the dielectric of thickness d with relative dielectric constant ε; the conductivity σ is disregarded) is:

$$\Phi(x,y,z;0,0,z_0) = (Q/\varepsilon_0)\, G(x,y,z;0,0,z_0)\,; \quad 0 < z_0 \text{ (or } z') < D-d, \quad D-d \le z \le D. \tag{1}$$

$$G(x,y,z;x',y',z') = \left(\frac{1}{2\pi}\right)^2 \int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} dk_y \, \frac{2}{\kappa} \frac{\sinh[\kappa(D-z)]\,\sinh(\kappa z')}{(\varepsilon+1)\sinh(\kappa D) + (\varepsilon-1)\sinh[\kappa(D-d/2)]} \times \tag{2}$$
$$\times\, e^{ik_x(x-x')} e^{ik_y(y-y')}; \qquad \text{with} \qquad \kappa = \sqrt{k_x^2 + k_y^2}.$$

The Green's function can be calculated by standard methods from the boundary conditions and the continuity conditions prescribed to the potential along the interface between the two layers. From the above equation the charge distribution on the upper plate is found by:

$$\eta = \varepsilon_0 \varepsilon \left.\frac{\partial \Phi}{\partial z}\right|_{z=D}. \tag{3}$$

The charge on an anode strip is:

$$q_s(x_0, w; z_0) = Q\varepsilon \int_{-\infty}^{\infty} dy \int_{x_0-w/2}^{x_0+w/2} dx \left.\frac{\partial G(x,y,z;0,0,z_0)}{\partial z}\right|_{z=D}. \tag{4}$$

The strip of width d extends into infinity at both directions of the y-axis. Its median line has a distance x_0 perpendicular to the straight trajectory of the point charge, which is perpendicular to the electrodes.

Now the slow constant motion of the charge Q is taken into account by inserting

$$z_0 = z_{00} + vt$$

at the right hand side of eq.(4). This renders the total charge q_s on the strip time dependent. The current is calculated according to:

$$I = \frac{dq(x_0, w; z_{00} + vt)}{dt}. \tag{5}$$

As a final result we get the following integral for the current flowing through an anode strip:

$$\frac{ID}{Qv} = \frac{1}{2\pi}\int_0^\infty d\zeta\left[\sin\left(\zeta\frac{x_0 - w/2}{D}\right) - \sin\left(\zeta\frac{x_0 + w/2}{D}\right)\right]\frac{\cosh\left(\zeta z_0/D\right)}{\sinh(\zeta)}\, f(\varepsilon, \frac{d}{D}, \zeta) \tag{6}$$

with

$$f(\varepsilon, \frac{d}{D}, \zeta) = \frac{2\varepsilon}{(\varepsilon+1) + (\varepsilon-1)\,\sinh[\zeta(1-2d/D)]/\sinh(\zeta)} = \frac{2\varepsilon}{\varepsilon+1}\sum_{n=0}^{\infty}\left[\frac{\varepsilon-1}{\varepsilon+1}\right]^n\left[\frac{\sinh[\zeta(1-2d/D)]}{\sinh(\zeta)}\right]^n. \tag{7}$$

The factor $f(\varepsilon, d/D, \zeta)$ accounts for the dielectric layer. If the relative dilectric constant $\varepsilon = 1$ then $f(\varepsilon, d/D, \zeta) = 1$. Then the integral in (6) can be evaluated analytically to give:

$$\frac{ID}{Qv} = \frac{1}{4}\frac{\sinh(\frac{\pi(x_0 - w/2)}{D})}{\cos(\frac{\pi z_0}{D}) + \cosh(\frac{\pi(x_0 - w/2)}{D})} - \frac{1}{4}\frac{\sinh(\frac{\pi(x_0 + w/2)}{D})}{\cos(\frac{\pi z_0}{D}) + \cosh(\frac{\pi(x_0 + w/2)}{D})} := \mathcal{J}_{00}(x_0, w, D); \quad (\text{for } \varepsilon = 1\ !). \tag{8}$$

If the factor f is not constant then the integrals in (6) must be evaluated by more elaborate methods described in the next paragraph.

3.2 Evaluation of the Current Integral

The factor $f(\varepsilon, d/D, \zeta)$ is constant if either $\varepsilon = 1$ or $d = D/2$. In all other cases the integral (6) must be evaluated numerically or by series expansions which are again evaluated numerically. All analytical (symbolic) and numeric computations are donbe in *Mathematica* (Wolfram, 1996). The integral (6) is an infinite integral with an oscillating integrand. So in some cases the routine NIntegrate[] of Mathematica fails, even if the option Method → Oscillatory is used. It was found more expedient to insert the series given in formula (7) into eq.(6) and to integrate term by term. The evaluations show that in most cases the results obtained from eq.(6) for $\varepsilon > 1$ do not differ very much from those obtained for $\varepsilon = 1$, which can be computed from the simple expression (8), see Bild 4 5 . This means that the influence of the dielectric panel on the signal is small. Therefore the expression for the current (6) is rewritten as:

$$\begin{aligned}\frac{ID}{Qv} &= \mathcal{J}_{00}(x_0, w, D) \\ &+ \frac{\varepsilon-1}{\varepsilon+1}\mathcal{J}_{00}(x_0, w, D) + \sum_{n=1}^{\infty}\left[\frac{\varepsilon-1}{\varepsilon+1}\right]^n\left[\mathcal{J}_n\left(\frac{x_0 - w/2}{D}, \frac{z_0}{D}, 1 - \frac{2d}{D}\right) - \mathcal{J}_n\left(\frac{x_0 + w/2}{D}, \frac{z_0}{D}, 1 - \frac{2d}{D}\right)\right].\end{aligned} \tag{9}$$

The first term above gives the contribution for a point charge in an empty condensor. The sum of the expressions in the second line gives the relatively small correction due to the dielectric layer. The integrals

$$\mathcal{J}_n(a, b, c) := \frac{1}{2\pi}\int_0^\infty d\zeta\ \sin(\zeta\, a)\cosh(\zeta\, b)\sinh^n(\zeta\, c)\,/\sinh^{n+1}(\zeta), \qquad 0 \le b < 1,\ 0 \le c < 1, \tag{10}$$

can be done analytically in Mathematica. This is described and results are listed in the appendix. In the cases for which evaluations were done this was performed in the following way: The analytic expressions were used up to $n = 6$ as NIntegrate[] had still troubles to evaluate them. These gave sufficient accuracy as could be checked by summing 6 further terms in which the integrals were done by NIntegrate[]; for $n > 6$ this command performed without problems. Both the evaluations of the analytic expressions and the numeric integrations run into severe trouble for $a = 0$; the obvious value 0 must be inserted by a branching command.

4 Results

As results we give two diagrams displaying the current flowing through the anode strips. A more detailed discussion and optimization of the parameters will be published elsewhere. Fig.3 shows the various parameters used. The trajectory of the charge Q is always perpendicular to the electrodes and to the interface of the dielectrics.

In Bild 4 the dependence of the anode strip current on the on the distance x_0 of the strip from the trajectory of the point charge is shown. The curves containing dots belong to $\varepsilon = 1$ and are computed from the simple expression (8). The continuous line and the dashed lines correspond to $\varepsilon = 2$ and are computed from eq.(9). It is obvious that the dielectric layer gives only a small enhancement of the anode strip current. The dependence of this current on the distance x_0 between the strip and the charge trajectory is strongest if the charge moves just at the interface separating the gap from the dielectric panel. The variation becomes the smaller the farther the charge from the interface, thus also from the anode. The end of the anode strip is at $x_0 = 5mm$.The current has maximum absolute value if the charge moves just a little inside the strip. Remember that our anode is a continous sheet so there are no edge effects in our model.

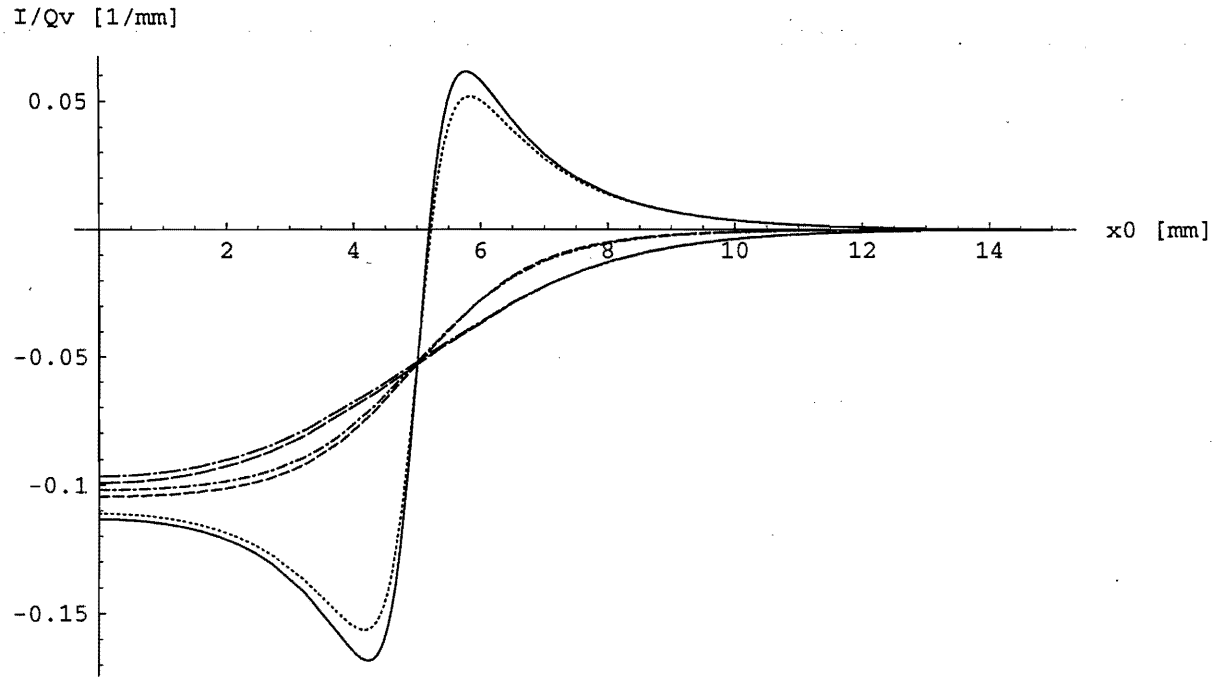

Bild 4. Current through anode strip versus distance (in mm) from trajectory of the charge. Gas gap width $D - d = 4mm$, dielectric layer width $d = .8mm$, relative dielectric constant $\varepsilon = 2$ or 1 (lines with dots), anode strip width $w = 10mm$. $z0 = 4mm$: — and $\cdots$; $z0 = 2mm$: - - - and - · - · -; $z0 = 0mm$: —- —- —- and —- · —- · —- .

In Bild 5 the currents through two neigbouring anode strips are compared. This is done for four positions of the trajectory w.r.t. the strips. The upper part of each picture shows the two strips and the trajectory. In the lower parts the two currents are shown. In some cases one of the currents is so small as compared to the other so that its curves coincide with the abscissa. In the last picture the two currents are equal due to the symmetric position of the trajectory. The continuous curves correspond to the full formula (9) including the effects of the dielectric layer; the dashed cuves have been drawn with the help the simple formula (8) obtained for $\varepsilon = 1$. Again the differences between the two results are small.

The maxima and minima of the curves near to the boundary of the strip may indicate that the image on the anode strip grows while the points charges becomes nearer. It appears that one strip draws charge from his neighbour. This explains also the change of the sign of the current. In our model this is possible as our anode is a continuous sheet. In reality the strips are separated by isolating gaps. But they may be coupled via the signal amplifiers or networks preceeding these. So the validity of our model needs a careful appraisal of this situation. At least a current of opposite sign on the neighbouring strip has been observed (Croty et al., 1995, p.514).

5 Discussion and Conclusions

A simple model for the currents flowing through the anode strips in a resistive plate chamber has been developed and evaluated. It is found:

The influence of the conductivity of the resistive panels on these currents is neglible. The dielectric constant of the panels enhances the signal by less than 10 percent. So a simple formula for the current strength corresponding to a value unity of the relative dielectric constant gives a satisfactory approximation. The dependence of the current on the distance between the strip and the trajectory has also been given, but the underlying assumptions should be discussed in the context with the measuring network.

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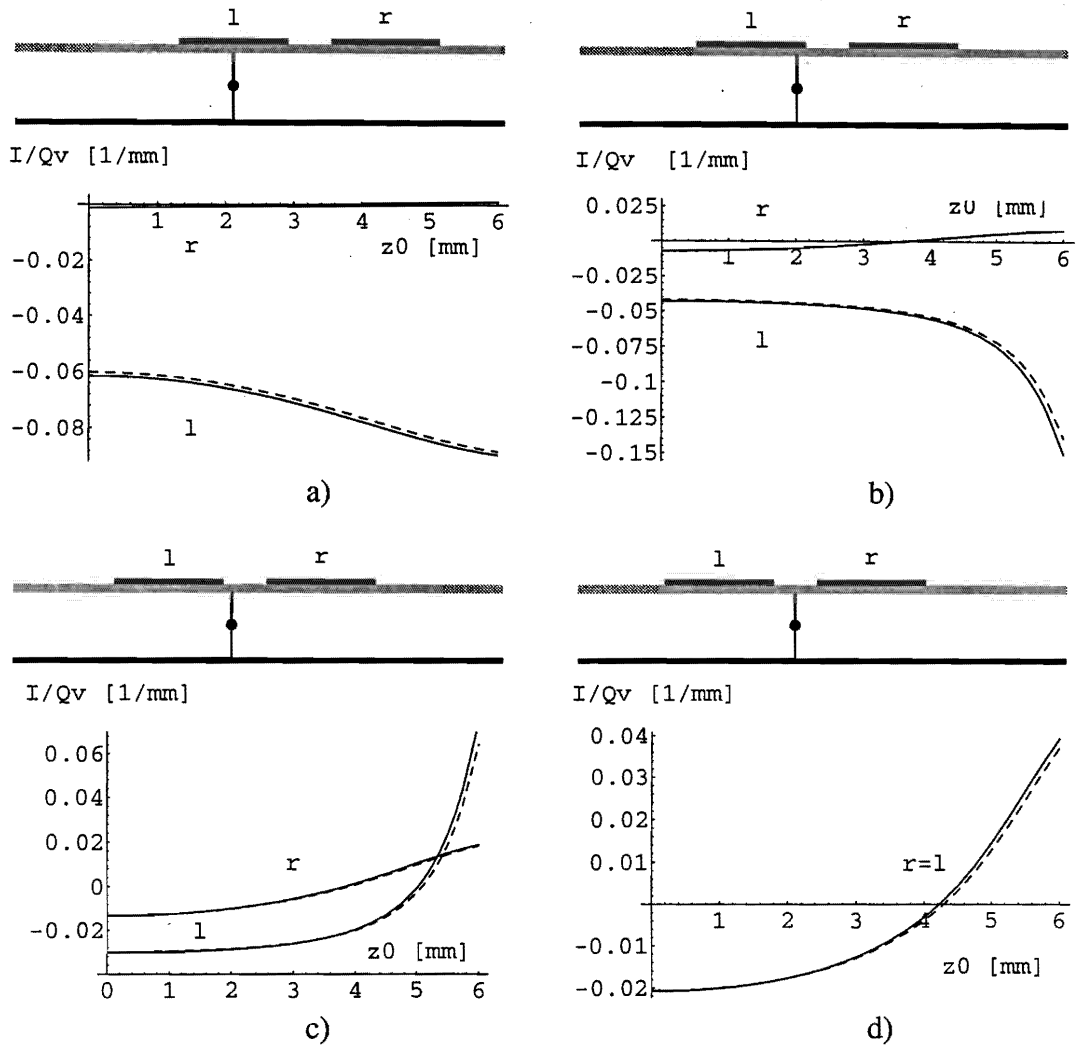

Bild 5. Current through two neighbouring anode strips versus distance (in mm) of the charge from the cathode for various positions of the trajectory w.r.t. the strips. Continuous curves correspond to full formula (9) with a relative dielectric constant $\varepsilon = 2$; dashed curves are computed with the simple formula (8) corresponding to $\varepsilon = 1$. Gas gap width $D - d = 4mm$, dielectric layer width $d = .8mm$, anode strip width $w = 10mm$. Width of gap separating the strips $g = 3mm$. In pict.a) the trajectory is in the center of the left strip. The current through the strip at the right is so small that its curve coincides with the abscissa. The position of the trajectory in pict. b) corresponds to the minimum in Bild 4; that in pict. c) to the maximum. The trajectory in pict. d) is located in the middle of the gap separating the strips.

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