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UNIFIED GRAVITATIONAL WAVE EQUATIONS OF ELECTROMAGNETIC PLANE WAVES AND DUALITY IN GENERAL RELATIVITY

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Abstract

The Einstein equation of an electromagnetic plane wave is analyzed in connection with causality and the principle of equivalence. It is concluded that there is no physical solution unless an additional energy-stress tensor with an anti-gravity coupling is included in the source. Furthermore, a physical solution requires that, on the time average, the electromagnetic energy-stress tensor and the additional tensor must be equal in the flat metric approximation. Based on physical considerations, such a tensor is identified to be the distinct tensor for photons. The resulting gravitational waves have matching polarizations. For monochromatic waves, the frequency ratio between gravitational and electromagnetic wave components is two. For a circularly polarized wave, unexpectedly the gravitational wave component can be zero. The necessary inclusion of the photon tensor explains also the long standing problem why, **while the light bends, the static electromagnetic energy does not "fall"**. Moreover, with the required photon energy-stress tensor, the source is the real part of the electromagnetic energy-stress tensor obtained with the complex electromagnetic plane wave in quantum theory. This manifests that there is a connection between quantum theory and general relativity.

UNIFIED GRAVITATIONAL WAVE FORMS OF ELECTROMAGNETIC PLANE WAVES AND DUALITY IN GENERAL RELATIVITY

I. Introduction.

Relativity suggests the existence of gravitational waves [1]. Although there are indirect observational evidences [2-4], gravitational waves have not been directly observed [5]. Theoretically, the existence of gravitational waves had been assumed a certainty to the first approximation [6]. However, Einstein himself discovered [7] in 1936 that linearized gravity is not reliable. (In fact, for gravitational radiation, linearized gravity is not self-consistent [8,9].) Nevertheless, during 1950s theorists reached a consensus [10] that gravitational waves did indeed exist, although a valid proof remains to be shown [9]. Also, exact physical solutions of gravitational waves are not yet available [11]. The main difficulties appears to be: 1) Einstein equation is non-linear; 2) The related physical requirements remain to be investigated.

To demonstrate the existence of gravitational waves theoretically, one should consider, as physicists often do, some simple physical situations. Analysis indicates that an electromagnetic wave would generate an accompanying gravitational wave [11-13]. Such unified wave forms would be calculable because the energy-stress tensor of an electromagnetic wave and the related physics are well-studied.

However, the validity of an electromagnetic wave energy-stress tensor as the only tensor in the source (of the Einstein equation $G_{ab} = -KT_{ab}$) should be examined because of duality. In general relativity, duality was used in the calculation of the star light deflection where light is considered as consisting of massless particles, photons [6,9]. Naturally, one may ask whether duality should be considered in Einstein's field equation, which usually includes the equation of motion [14]. To be more specific, is there a distinct energy-stress tensor for photons as part of the source tensor?

While particles and waves are conceptually distinct, experimentally electromagnetic waves and photons are inseparable. If they are really distinct objects, one may conjecture that, for some cases, without a photon tensor, Einstein's field equation cannot have a *physical* solution. (Note that the variational principle assures mathematical compatibility [15] only, but not the existence of a physical solution [8].) Then the validity of

general relativity, as a whole, would be in question because the electromagnetic energy–stress tensor must be included in the source. On the other hand, the necessary involvement of a distinct photon tensor in the Einstein equation is an unmistakable manifestation that there is an intrinsic connection between relativity and quantum theory. In term of physics, such a connection is very plausible because both theories have their foundations on different aspects of a common physical phenomenon -- The velocity and quantum of light.

Here, we consider a simple case when the source is the energy–stress tensor of an electromagnetic plane wave. Our conjecture is supported by the fact [16] that the solutions obtained by Peres and Bonner [17] are not physical because both the equivalence principle and causality are violated.

The principle of causality implies that an electromagnetic plane wave would generate an accompanying gravitational plane wave (see Appendix A). However, since the time average of the time–time component G_{tt} is positive, there is no physical solution unless another tensor is subtracted from the source (see §4). Moreover, a physical solution requires [13] that, in the flat metric approximation, this unknown tensor, on the time average, is the same as the electromagnetic energy–stress tensor. Thus, this unknown tensor would satisfy the condition for a photon energy–stress tensor required by experiments.

However, this would also mean that the photons have an *anti-gravity coupling*. Although this is beyond the theory originally proposed by Einstein, Pauli [18] pointed out in 1921 that in general relativity such a coupling is possible. Thus, the antigravity coupling should be a necessary feature to be verified.

In this paper, based on that photons travel along a geodesic and other physical considerations, a photon tensor is obtained for monochromatic electromagnetic plane waves (see §5). Then, physical solutions of gravitational plane waves are obtained, and the polarization of the gravitational wave matches that of the related electromagnetic wave (see §6). Thus, the anti–gravity coupling for photons is confirmed theoretically.

Nevertheless, the skeptics might argue that, in addition to singularities [19,20], the necessary existence of antigravity coupling is another evidence for the breaking down of general relativity since Newtonian gravity does not allow an antigravity coupling. To settle the question of whether the antigravity coupling exists, it seems, experimental evidences are needed. A direct measurement for the accompanying gravitational waves of electromagnetic waves would be technically very difficult. However, if the antigravity coupling is necessary for electromagneic radiation, according to the Einstein tensor, such a coupling is also necessary for

a pure gravitational radiation (see §9). Surprisingly, this indirect verification has actually been confirmed [21,22] by the Taylor and Hulse binary pulsar PSR 1913+16 experiment [2] 20 years ago!

Thus, duality is also a necessary feature of general relativity. To see its compatibility with quantum theory, one should note that the wave functions of photons are complex. Calculation (see §8) shows explicitly that there is a connection between the photon energy-stress tensor and the complex waves used in quantum theory. This can be interpreted as duality is implicitly contained in complex waves, and the physical meaning of using complex waves in quantum theory becomes clearer. The existence of the antigravity coupling further manifests the intrinsic differences between relativity and Newtonian theory.

However, to keep the paper on the focus of duality, the profound meaning of the antigravity coupling to space-time and the life of very massive stars will be discussed in a separated paper [23]. This paper is a continuation of an earlier paper [13] which shows the necessity of a photon tensor. For convenience and clarity, its main arguments are briefly presented in §§2-4.

2. Causality and Duality

Let us consider a ray of electromagnetic waves propagating in the z-direction. Within the ray, one can assume a strong cylindrical condition (i.e. the wave amplitude is independent of x, and y.) Thus, as in the literature [6,11], the electromagnetic potentials are plane waves,

$$A_k = A_k(t - z) \ . \tag{1}$$

Due to the principle of causality (see Appendix A), the metric g_{lk} is functions of u (= t-z), i.e.,

$$g_{lk} = g_{lk}(u) \ . \tag{2}$$

Let P^k be the momentum of a photon. Then, one obtains [13] the conditions,

$$P^z = P^t, \quad P^x = P^y = 0, \quad \text{and} \quad P^m g_{mk} = P_k = 0 \ , \tag{3a}$$

for k = x, y, and v ($\equiv$ t + z). Eq. (3a) is equivalent to

$$g_{xt} + g_{xz} = 0, \; g_{yt} + g_{yz} = 0, \text{ and } g_{tt} + 2g_{tz} + g_{zz} = 0 , \tag{3b}$$

or

$$g^{xt} - g^{xz} = 0 , \; g^{yt} - g^{yz} = 0, \text{ and } g^{tt} - 2g^{tz} + g^{zz} = 0. \tag{3c}$$

The transversality of an electromagnetic wave would imply

$$P^m A_m = 0 \text{ , or equivently } \qquad A_z + A_t = 0 . \tag{4}$$

Eqs. (2) to (4) imply that not only the geodesic equation, the Lorentz gauge, but also Maxwell's equation are satisfied. Moreover, the Lorentz gauge becomes equivalent to a covariant expression.

The above analysis suggests also that an electromagnetic plane wave can be an exact solution. The scalar $\int P_m dx^m$ would equal to $P_m x^m$ in a coordinate system where P_m are constants. Then, $P_m x^m$ represents a scalar even though the space is not flat.

3. The Reduced Field Equation

Then, eq. (2) and eq. (3) reduce the Einstein equation $G_{lk} = -K\, T(E)_{lk}$ to a single equation,

$$R_{tt} \equiv -\frac{\partial \Gamma^m{}_{tt}}{\partial x^m} + \frac{\partial \Gamma^m{}_{mt}}{\partial t} - \Gamma^m{}_{mn}\Gamma^n{}_{tt} + \Gamma^m{}_{nt}\Gamma^n{}_{mt} = -K\, T(E)_{tt} = K\, g^{mn} F_{mt} F_{nt}, \tag{5a}$$

where F_{lk} is the electromagnetic field tensor, $T(E)_{lk}$ is the electromagnetic energy-stress tensor, G_{lk} is the Einstein tensor, and K is the coupling constant. Note that eq. (5a) is compatible wth the relation,

$$R_{tt} = - R_{tz} = R_{zz} , \tag{5b}$$

and the other components are zero. Then, eq. (5) is simplified to a differential equation of u as follows:

$$G'' - g_{xx}'g_{yy}' + (g_{xy}')^2 - G'(g'/2g) = 2K\,(F_{xt}^2 g_{yy} + F_{yt}^2 g_{xx} - 2F_{xt}F_{yt}\, g_{xy}), \qquad (6a)$$

where

$$G \equiv g_{xx}\, g_{yy} - g_{xy}^2, \quad \text{and} \quad g = |g_{lk}| \qquad (6b)$$

is the determinant of the metric. The metric elements are connected by the following relation:

$$-\,g = G\, g_t^2\ , \quad \text{where} \quad g_t \equiv g_{tt} + g_{tz}. \qquad (7)$$

If g is a constant, the metric shall be called ***semi-unitary.***

Equations (3), (4), (6), and (7) allow A_t, g_{xt}, g_{yt}, and g_{zt} to be set to zero (or equivalently $g_{uk} = 0$ for k = x, y, u). These orthogonal conditions are valid because there is no physical reason to suggest otherwise. In any case, these assigned values have little effect in subsequent calculations. Note that equation (3) requires only $g_{tt} + g_{zz} = 0$. This allows g_{tt} to have a wave component.

Now, there are four metric elements (g_{xx}, g_{xy}, g_{yy}, and g_{tt}) to be determined. However, there is only one differential equation (since eq. (7) is not really an equation if g is not specified by other means). Nevertheless, to show that there is no physical solution, eq. (6) is sufficient.

4. Physical Solution and Necessary Condition on the Source Tensor.

It has been shown [13] that, in general, there is no physical solution for eq. (6) by using the required periodic nature of the metric due to causality (see Appendix A) and that the curvature tensor, on the time average, is necessarily non-negative for a plane wave. Moreover, if one assumes that, for an electromagnetic plane wave, the metric is a function of only t and z, then the Einstein equation implies $g(u)_{\mu\nu}$ [13]. Thus, the principle of causality supports the Einstein tensor $G_{\mu\nu}$. For some special cases, it will be shown further that the Einstein tensor $G(u)_{\mu\nu}$ implies that $g(u)_{\mu\nu}$ is periodic.

Let us consider a circularly polarized monochromatic electromagnetic plane wave,

$$A_x = \frac{1}{\sqrt{2}}\, A_0 \cos \omega u\ , \quad \text{and} \quad A_y = \pm\frac{1}{\sqrt{2}}\, A_0 \sin \omega u\ . \qquad (8)$$

The rotational invariants with respect to the z-axis are constants. These invariants are: G, $(g_{xx} + g_{yy})$, R_{tt}, $T(E)_{tt}$, g_{tz}, g, g_{tt}, and etc. Let us assume the invariant,

$$g_{xx} + g_{yy} = -2 - 2C, \quad \text{then} \quad g_{xx} = -1 - C + B \ , \text{ and } \ g_{yy} = -1 - C - B. \tag{9}$$

Thus,

$$B^2 + g_{xy}^2 = (1+C)^2 - G \ , \text{ and } \ (B')^2 + (g_{xy}')^2 = 2GR_{tt} \geq 0 \tag{10}$$

are constants. It follows that eq. (10) imply

$$B = B_\alpha \cos(\omega_1 u + \alpha), \quad \text{and} \quad g_{xy} = \pm B_\alpha \sin(\omega_1 u + \alpha) \ , \tag{11a}$$

where

$$\omega_1^2 = 2R_{tt}G/B_\alpha^2, \quad \text{and} \quad B_\alpha^2 = (1+C)^2 - G \geq 0. \tag{11b}$$

Thus, it is proven that the metric is a periodic functions. Also, as implied by causality, the metric is <u>not</u> an invariant under a rotation (since a transverse electromagnetic wave is not such an invariant).

Since $T(E)_{tt}$ is a constant, it is necessary to have

$$\omega_1 = 2\omega, \quad \text{and} \quad T(E)_{tt} = \frac{1}{2G}\, \omega^2 A_0^2 (1 + C - B_\alpha \cos\alpha) > 0. \tag{12}$$

Eq. (11) implies that the metric is a circularly polarized wave with the same direction of polarization as the electromagnetic wave (8). However, if the photon tensor were zero, it is not possible to satisfy Einstein's equation because $T(E)_{tt}$ and R_{tt} have the same sign. (Note that $G > 0$; and the equation of motion of a charged particle does not allow changing the sign of the coupling constant K)

The above calculation illustrates that there is no possibility, within the current theory, to construct an acceptable metric representing the accompanying gravitational wave. One might regard this as an example that general relativity, being so physical a theory, is generally unable to represent entities of infinite extent such as a plane sheet of mass. But, the electromagnetic energy is not equivalent to mass as shown by the facts that

the trace of an electromagnetic energy-stress tensor is zero and that static electromagnetic energy does not "fall" under the influence of gravity. Furthermore, the plane wave is a well-tested idealization in electrodynamics. If general relativity is fundamentally correct as experiments demonstrated, there must be a way to modify the equation such that a physical solution can be obtained for an electromagnetic plane wave.

Moreover, since the Einstein tensor is supported by causality, this suggest that it would be sufficient to modify the source tensor. (Einstein pointed out in 1955 that [24] his theory is "far from completed ...") The additional term should be a constant of different sign, and is larger in absolute value. The above calculation also suggests that the time average of the source stress tensor must be negative and of the second order of deviations. Also, ***in the flat metric approximation, an electromagnetic wave tensor and the unknown tensor carry, on the average, the same energy-momentum*** [13].

5. Anti-Gravity Coupling and the Photon Energy-Stress Tensor.

To verify the conjecture that a non-physical field equation is due to an inappropriate source tensor, one must show that a modification can be done within the theoretical framework of general relativity. To this end, one must find a photon tensor for electromagnetic plane waves. Let us assume that the source is

$$T_{\mu\nu} = T(E)_{\mu\nu} + \alpha T(P)_{\mu\nu} , \tag{13a}$$

where $T(E)_{\mu\nu}$ and $T(P)_{\mu\nu}$ are the energy-stress tensors for the electromagnetic wave and the related photons, and α is a constant to allow a possibly different coupling. Since both $T(E)_{\mu\nu}$ and $T_{\mu\nu}$ are divergence free and traceless, $T(P)_{\mu\nu}$ must also be divergence free and traceless. Moreover, the photon tensor should accommodate the fact that photons move along a geodesic.

It is observed that there is very little interaction, if any, among photons of the same ray. Theoretically, since photons travel in the velicity of light, there should not be any interaction among photons of a plane wave. Therefore, for a monochromatic wave, one may assume a dust-like model,

$$T^{ab}(P) = \rho P^a P^b, \tag{13b}$$

where ρ is a scalar which, according to causality, is a function of u. The geodesic equation, $P^a\nabla_a P^b = 0$, is implied by $\nabla_a(\rho P^a) = 0$, and $\nabla_a T(P)^{ab} = 0$. $\rho(u)$ should be a non-zero function of the electromagnetic potentials and/or fields. This implies $\rho = \lambda\ A_m g^{mn} A_n$, where λ is a scalar constant to be determined. Note that $\rho(u)$ is Lorentz gauge invariant because of eqs. (2) and (3). In classical theory, light intensity is proportional to the square of the wave amplitude. Thus, ρ can be considered as the density function of photons if $\lambda = -1$. Also, without any lost of generality, α can be selected since λ can be adjusted accordingly.

In anticipation of an anti-gravity coupling, one may choose $\alpha = -1$ in eq. (13a), and obtain

$$T_{\mu\nu} = T(E)_{\mu\nu} - T(P)_{\mu\nu} = T(E)_{\mu\nu} - \lambda\ A_m g^{mn} A_n\ P_\mu P_\nu. \tag{13c}$$

Thus, a photon tensor changes nothing in the calculation, but only gives another term for eq. (6).

To determine λ, let us consider a circularly polarized monochromatic wave (8). Then, we have,

$$T_{tt} = \frac{1}{2G}\omega^2 A_0{}^2 [(1 + C)(1 + \lambda) - (1 - \lambda)B_\alpha \cos\alpha] \leq 0 \tag{14}$$

since causality requires R_{tt} to be of second order and positive. Eq. (14) requires that $\lambda \leq -1$ because the constants C and B_α are much smaller than 1. Causality requires that, in a flat metric approximation, the time average of T_{tt} is zero. This implies that, as expected,

$$\lambda = -1,$$

and

$$T_{tt} = -\frac{1}{G}\omega^2 A_0{}^2 B_\alpha \cos\alpha \leq 0, \qquad \text{since} \quad B_\alpha = \frac{K}{2}\ A_0{}^2 \cos\alpha. \tag{15}$$

where B_α is the amplitude of the gravitational wave and α is its phase difference to the electromagnetic wave. Note that, ***pure electromagnetic waves can exist*** since $\cos\alpha = 0$ is possible.

To confirm the general validity of $\lambda=-1$, consider a wave linearly polarized in the x-direction,

$$A_x = A_0 \cos\omega(t - z) \ . \tag{16a}$$

Then, one has

$$T_{tt} = \frac{g_{yy}}{2G} \omega^2 A_0^2 \{ (-\lambda - 1) + (1 - \lambda)\cos[2\omega(t - z)] \}. \tag{16b}$$

Thus, the flat metric approximation again requires that $\lambda = -1$. Then,

$$T_{tt} = \frac{g_{yy}}{G} \omega^2 A_0^2 \cos[2\omega(t - z)] . \tag{16e}$$

Eq. (16e) implies that the related Einstein tensor and $(g_{xx} + g_{yy})'$ are of <u>first</u> order of deviations. Thus, its polarization has to be different. However, physically the time average of T_{tt} should also be <u>negative</u>.

6. Unified Polarizations and Physical Solutions.

If a circularly polarized electromagnetic plane wave results in a circularly polarized gravitational wave, one may expect that a linearly polarized electromagnetic plane wave results in a linearly polarized gravitational wave. From the viewpoint of physics, it would be meaningful to require that, for an x-directional polarization, gravitational components related to the y-direction, remains the same. In other words,

$$g_{xy} = 0 \ , \quad \text{and} \quad g_{yy} = -1 \ . \tag{17a}$$

Mathematically, condition (17a) is compatible with semi-unitary (i.e. g is a constant, see Appendix B). Equation (17a) means that the gravitational wave is also linearly polarized. In the literature [3,4,11], there are other proposals. However, they all lead to unphysical solutions (see Appendix B).

It follows that equation (6) becomes

$$G'' = -2 K G T_{tt}, \quad \text{and} \quad G = -g_{xx} \ . \tag{17b}$$

Then, the general solution for equation (17) is:

$$- g_{xx} = 1 + C_1 - \frac{K}{2} A_0^2 \cos[2\omega(t - z)], \quad \text{and} \quad g_{tt} = - g_{zz} = \sqrt{\frac{g}{g_{xx}}}, \tag{18b}$$

where C_1 is a constant. Note that the frequency ratio is the same as that of a circular polarization. For a polarization in the diagonal direction of the x–y plane, the solution is:

$$g_{xx} = g_{yy} = -1 - C_1/2 + \frac{K}{4} A_0^2 \cos[2\omega(t - z)], \tag{19a}$$

$$g_{xy} = - C_1/2 + \frac{K}{4} A_0^2 \cos[2\omega(t - z)], \tag{19b}$$

$$g_{tt} = - g_{zz} = -g/\{1 + C_1 - \frac{K}{2} A_0^2 \cos[2\omega(t - z)]\}^{1/2}, \tag{19c}$$

Note that for a perpendicular polarization, the metric element g_{xy} changes sign. Solutions (18) and (19) imply that linear superposition of electromagnetic waves is only approximately valid. The time averages of their T_{tt} are also negative as required. If $g = -1$, relativistic causality requires $C_1 \geq K A_0^2/2$.

If the photon tensor were absent (i.e., $\lambda = 0$), then the solution of equation (17) could have been

$$- g_{xx} = 1 + C_1 - \frac{K}{4} A_0^2 \{2\omega^2 (t - z)^2 + \cos[2\omega(t - z)] + C_2 (t - z), \tag{20}$$

where C_1 and C_2 are constants. Solution (20) is not physical because the term $(t - z)^2$ grows very large as time goes by. Thus, $T(E)_{tt}$ has a time limit zero, and therefore disagrees with special relativity.

For a circularly polarized electromagnetic wave, the phase difference controls the amplitude of the gravitational wave (see eq. (17)), and the amplitude of the electromagnetic wave gives an upper bound. This is different from the case of linearly polarized waves for which the amplitude of gravity is fixed. Also, in both cases, there is a small constant C_1 to be determined. Nevertheless, this constant C_1 would not affect the gravitational force, and thus, the forces related to gravitational waves (18) and (19) can be compared with measurements. In particular, the frequency ratio would make such measurements easier. Thus, this calculation can be experimentally verified.

7. A Unified Formalism for Plane Wave Polarizations and Semi-Unitary of the Metric.

In electrodynamics, because the Maxwell equation is linear, real waves are related to complex waves. However, to use complex waves could be beyond mathematical convenience but, as quantum electrodynamics suggests, this would be a necessity. To prepare the discussion of complex waves (see Appendix C), here we consider the polarization of complex waves and the notion of a semi-unitary metric. To this end, we first show that there is a unified formalism for polarizations of real plane waves.

Eq. (17a) can be interpreted as the transverse metric components are subjected to another constraint related to duality and invariance of polarization. Note that both the momentum P^k and the conjugate momentum P_k are in the z-direction. One may conjecture that an electromagnetic plane wave A_k and the corresponding contravariant electromagnetic potential A^k have the same plane of polarization. In other words, the ratios among their corresponding components are the same. Then one has the following additional equation:

$$(\alpha^2 - \beta^2)\, g_{xy} = \alpha\beta\, (g_{xx} - g_{yy}), \tag{21}$$

where

$$A_x = \alpha A, \quad A_y = \beta A, \quad \text{and} \quad |\alpha|^2 + |\beta|^2 = 1.$$

Equation (21) is equivalent to that, for an electromagnetic wave linearly polarized in the x-axis, $g_{xy} = 0$. For a circularly polarized wave, one could extend equation (21) to complex waves. Then, an electromagnetic and the accompanying gravitational wave have similar connection between real and complex waves.

A semi-unitary condition simplifies equation (6) considerably since eq. (7) implies that the last two terms on the left-hand side of equation (6a) cancel each other. Then equation (6a) is reduced into

$$G'' - g_{xx}'g_{yy}' + (g_{xy}')^2 = 2K\,\{(F_{xt}^2\, g_{yy} + F_{yt}^2\, g_{xx} - 2F_{xt}F_{yt}\, g_{xy}) + GT(P)_{tt}\}. \tag{22}$$

Note that equation (22) includes only transverse metric elments, and G" can be of first order of deviations. The unitary condition lead to the same differential equation. But, g_{tt} and g_{zz} would be simpler. It should be noted also that the derivation of eq. (6) is also valid for complex functions.

8. General Relativity and Quantum Theory

Now, it has been established that general relativity requires a distinct energy–stress tensor for photons. On the other hand, in quantum theory, the electromagnetic wave and the related photons are indistinguishable. Thus, one might ask whether general relativity and quantum theory are compatible. It will be shown in this section that not only are they compatible but also inextricably related.

In quantum theory, an electromagnetic wave is not a real but a complex function. For plane waves which are linearly polarized and circularly polarized, their electromagnetic potentials are respectively,

$$A_x = A, \qquad \text{where} \quad A = A_0 \exp\{-i\omega(t - z)\}; \tag{23a}$$

and

$$A_x = \frac{1}{\sqrt{2}} A, \qquad \text{and} \qquad A_y = \pm \frac{i}{\sqrt{2}} A. \tag{23b}$$

Their complex energy–stress tensors are, for the linearly polarized wave (23a),

$$T(\tilde{E})_{tt} = - g^{mn}F_{mt}F_{nt} = - \frac{g_{yy}}{G}(-\omega^2)A^2 = \frac{g_{yy}}{G}\omega^2 A_0{}^2(\cos 2\omega u - i \sin 2\omega u); \tag{24a}$$

and

$$T(\tilde{E})_{tt} = \frac{\omega^2 A^2}{2G}[g_{yy} - g_{xx} - (\pm)2ig_{xy}] = - \frac{\omega^2 A^2}{G} B_\alpha [\cos(2\omega u+\alpha) + i\sin(2\omega u+\alpha)]$$

$$= - \frac{\omega^2 A_0{}^2}{G} B_\alpha \exp(i\alpha), \tag{24b}$$

for the circularly polarized waves (23b). Note that $G = g_{xx}g_{yy} - g_{xy}{}^2$. From these two explicit calculations, it is easy to see that the modified source is the real part of the energy–stress tensor of the complex wave, i.e., $\mathrm{Re}[T(\tilde{E})_{lk}] = \{T(E)_{lk} - T(P)_{lk}\}$. This manifests that while duality is explicitly represented in general relativity, duality is implicitly included in the complex wave functions.

In conclusion, the necessity of using complex waves in quantum theory is not just useful for practical calculations, but it has a deep root in general relativity (see also Appendix C). Thus, there is clearly a connection between general relativity and quantum theory, and they are no longer unrelated.

9. Discussions and Conclusions.

In general relativity, a major problem is to obtain a physical solution among the mathematical possibilities. However, this is possible only if the source tensor is physically appropriate. If the source is not massive matter, it is difficult to determine whether a source term is adequate since some types of energy may not necessarily be equivalent to massive matter [13,25]. (The once prevailing interpretation [5,26,27] that the famous $E = mc^2$ implies energy-mass unconditional equivalence, is incorrect. In fact, Einstein [28] has clarified in 1946 that this equation must be understood in connection with energy-conservation.) Moreover, even when the solution is not physical, the source need not be invalid. For instance, if the gauge is unphysical, the solution would not be physically valid [16,22]. Thus, it is desirable to have criterions which enable one to determine the appropriateness of a source tensor without considering all the mathematically possible solutions. To this end, related physical considerations and physical principles would play a crucial role.

The validity of an electromagnetic energy-stress tensor as the only source term is questioned because of duality. The bending of light was calculated with two distinct methods: first based on the Huyghens principle [29] and later based on the geodesic of photons [6,9]. The agreement of these calculated results suggests that an energy-stress tensor for photons should be included. To examine the appropriateness of a perceived source tensor, one should choose a simple situation and then identify the related physical requirements.

For the case of an electromagnetic plane wave, relativistic causality, the principle of equivalence, and the principle of correspondence are essentially passive requirements which can be used to check the validity of a solution. But, the principle of causality is an active requirement since it relates the symmetries between the physical cause and the solution. This principle implies that the metric is a plane wave with transverse components. This is consistent with the principle of correspondence which requires the flat metric to be a valid approximation for weak waves. Then, duality and the Einstein tensor imply that, for a circularly polarized plane wave, the metric is also circularly polarized. This gives us added confidence to the Einstein tensor, and therefore the essential validity of the present form of Einstein equation.

However, since G_{tt} of a circularly polarized gravitational plane wave is positive, ***validity of the Einstein equation is impossible unless there is a source tensor with an anti-gravity coupling.*** One might regard this as an example of general relativity being so physical a theory that it is generally unable

to represent entities of infinite extent such as a plane sheet of mass. But, the electromagnetic energy is not equivalent to mass [13,25]. An electromagnetic wave, moving with the maximum possible velocity, should be intrinsically different from the massive matter.

Furthermore, the plane wave is a well-tested idealization in electrodynamics. Therefore, to be compatible with electrodynamics and special relativity, electromagnetic plane waves must be considered as meaningful physical idealizations in general relativity. If general relativity is fundamentally correct as experiments demonstrated, there must be a way to modify the equation such that a physical solution can be obtained for an electromagnetic plane wave. To this end, adding to the source an energy-stress tensor for the photons with an antigravity coupling would be the most satisfactory solution in terms of physics.

Now, the question becomes what is a photon tensor and can it satisfy the above physical requirements. There are three physical conditions that a photon tensor should satisfy. They are: i) It produces the null geodesic equation for the photons; ii) In the flat metric approximation, on the time average it should equal to the electromagnetic tensor as required by quantum theories; iii) The polarization of the resulting gravitational wave matches the polarization of the electromagnetic wave. Condition i) makes a photon tensor distinct from an electromagnetic energy-stress tensor. But, condition ii) requires them to be intimately related. These two conditions seem to be contradictory. Nevertheless, calculation shows that, for a physical solution, condition ii) must be satisfied. This strong confirmation leads to a required photon tensor which can also satisfy condition iii). In view of the difficulties, one cannot help believing that nature has a way to made things work.

The modified source implies that an energy-stress tensor of photons consists of two parts. One part is associated with the electromagnetic wave, and the other part provides for space-time curvatures of the gravitational wave components. Thus, in general relativity, electromagnetic waves and the photons are related but distinct objects. This would be a step to reconcile the difficulty between physical concepts and experiments.

Moreover, the bending of light has been interpreted (incorrectly) as due to the unconditional energy-mass equivalence [5,26,27]. On the other hand, the static electromagnetic energy does not "fall" under the influence of gravity. The necessary inclusion of the photon tensor resolves such an *apparent inconsistency*, and the non-equivalence between mass and electromagnetic energy is further confirmed. The explanation of this difficult paradox is another triumph of general relativity.

For a linearly polarized electromagnetic waves, gravity is generated by a term of second order in time-average and therefore this is only a very small portion of the total energy. Thus, the non-equivalence between mass and the electromagnetic energy is further demonstrated. Moreover, for a circularly polarized wave, the gravitational wave component can be zero. Thus, ***similar to Newtonian theory, some forms of energy do not generate gravity even in general relativity.***

It is conceivable that this non-equivalence of mass and energy in terms of gravity would have a profound influence on the life of a massive star. Perhaps, the antigravity coupling would provide an insight on why a very massive star would explose and becomes a supernova.

This calculation illustrates that, not only the energy, but the form of energy is important for gravity. This new source form indicates that the radiation would be related to antigravity coupling. This suggests that gravitational radiatioin would be associated with an anti-gravity coupling. Consequently, for an energy-stress tensor $T(m)_{\mu\nu}$ of massive matter, the Einstein equation should be modified to the following form:

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu}R = - K \left[T(m)_{\mu\nu} - t(g)_{\mu\nu}\right], \tag{28}$$

where $t(g)_{\mu\nu}$ is the energy-stress tensor for gravity. Then, $\nabla^{\mu}G_{\mu\nu} \equiv 0$ implies both conservation laws,

$$\nabla^{\mu}T(m)_{\mu\nu} = 0, \qquad \text{and} \qquad \nabla^{\mu}t(g)_{\mu\nu} = 0, \tag{29}$$

because of the minus sign between $T(m)_{\mu\nu}$ and $t(g)_{\mu\nu}$. (It should be noted that, for both the wave and photon energy-stress tensors in eq. (13), $\nabla^{\mu}T(E)_{\mu\nu} = 0$ and $\nabla^{\mu}T(P)_{\mu\nu} = 0$.) Thus, eq. (28) remains compatible with the principle of equivalence. If gravity is generated by massive matter, then $K\, t_{\mu\nu}$ is expected to be of second order. It has been shown [22] that Einstein's radiation formula [18] and the binary pulsar experiments [2-4] necessarily lead to the modified eq. (28). Note that "current" Einstein equation is supported by experiments only for the cases of weak and static gravity [22,30].

The existence of the antigravity coupling which is necesitated by the gravitational waves is the most profound difference between relativity and Newtonian gravity which does not allow an antigravity coupling.

Also, there is no conclusive evidence to support the extension of Newtonian universal coupling beyond massive matter [25,31]. An important implication of the antigravity coupling in equation (28) is that the energy conditions in the singularity theorems [19,20,32] are not valid in physics. Consequently, one can no longer claim that general relativity supports the notion of black holes, which is originally due to Newtonian gravity [27]. To keep the paper from becoming too long, the implications and detailed discussions of the antigravity coupling will be provided in a separate paper [23].

The necessary inclusion of a photon tensor demonstrates that there is a connection between relativity and quantum theory. Such a connection is further manifested by the fact that the photon tensor can be derived form the electromagnetic energy-stress tensor of the related complex wave in quantum electrodynamics. It is interesting to note that both general relativity and the concept of photon were proposed by Einstein. As shown, these two seemingly independent theories are actually inextricably related.

Moreover, the gravitational effects are crucial in distinguishing the photons from the related electromagnetic waves although the gravitational waves are comparatively very small. Thus, this analysis supports the foresight of Einstein that general relativity would encompass every fundamental aspects of physics. Einstein [15] wrote, "The comparative smallness of what we know today as gravitational effects is not a conclusive reason for ignoring the principle of general relativity in theoretical investigations of fundamental character".

Einstein maintained the hope for a total solution on the lines of a classical field theory until the end of his life. Pauli [18] considered this would be the great open problem of the relation of relativity theory to quantum theory. But, he pointed out that a clear connection between the general theory of relativity and quantum mechanics is not yet in sight. Now a connection between relativity and quantum theory is clearly there. Would Einstein's dream occupy physicists for a long while to come?

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Appendix A: The Principle of Causality, Validity of an Equation, and Symmetry.

The concept of causality describes the ideas of cause and effect (which needs not be deterministic). There are two aspects in causality: its relevance and its time ordering. In time ordering, a cause event must happen before its effects. This is further restricted by relativistic causality that no cause event can propagate faster than the light speed in vacua. This requirement restricts also the choice of physical coordinates [13,16]. The time-tested assumption that phenomena can be explained in terms of identifiable causes will be called the principle of causality. This principle is the foundation of scientific studies. Here, this principle will be elucidated first in connection with symmetries, and then in the validity of an equation in physics.

In practice, we assume certain properties (such as symmetries etc.) for a "normal" state whose existence is without any specific cause. Then, any deviation from the normal state must have physically identifiable cause(s). Since the principle of causality implies that symmetry breaking must have cause(s), a symmetry must be preserved if no cause breaks it. For example, in electrodynamics, the electromagnetic field is zero in a normal state. The implication of causality to symmetry has been used in deriving the inverse square law from Gauss's law. Although a related potential may not be spherically symmetric, at least one is, the Coulomb potential. This shows that, at least, a gauge can be compatible with such symmetries.

In general relativity, matter is the cause of gravity. The normal state of a metric is the flat metric in special relativity. (This is a gauge choice.) The constant flat metric possesses all the symmetry allowed by special relativity. Thus, if a non-constant metric does not possess a certain symmetry, then there must be physical cause(s) which has broken such a symmetry. In other words, the metric should have at least the same symmetry as its physical cause(s). For example, in the Schwarzschild solution, causality requires that the metric is spherically symmetric and asymptotically flat. Note that, in a sperical coordinate system, the angular dependence of this metric is related to the solid angle $d\Omega$ because such a dependence exists inherently for the normal state. Also, without the principle of causality, the flat metric is not the only constant solution for Einstein's equation in empty space.

However, the physical cause(s) should not be confused with the mathematical source term in the field equation. Such a confusion would be possible because, for some situations, such a distinction does not seem to be meaningful. For instance, in electrodynamics, the physical cause of an electromagnetic field and the source

term in Maxwell's equation, are the same charged currents. In general relativity, the cause of gravity remains the physical matter, but not the source term in Einstein's field equation. The energy-stress tensors (for example the perfect fluid model) may explicitly depend on the metric. Since nothing should be a cause of itself, such a source tensor does not represent the cause of a metric. For the accompanying gravitational wave of an electromagnetic wave, the physical cause is the electromagnetic wave. In the Schwarzschild case, the cause is the mass distribution. Thus, it does not make sense, without directly using causality, to infer the symmetries of the metric from the source term although their symmetries are not unrelated.

Moreover, inferences based on the source term can be misleading. Sometimes, the source term may have higher symmetries than those of the cause and the metric. For instance, a transverse electromagnetic plane wave is not rotationally invariant with respect to the direction of propagation. But the related electromagnetic energy-stress tensor can be rotationally invariant and even be a constant [13]. In the literature (see [17], eq. (35.40) in [6] and also §13 & §21 of [11]), the metric is incorrectly assumed to be rotationally invariant. This assumption violates causality and results in theoretical difficulties (see [16]).

Classical electrodynamics and experiments imply that the flat metric is an accurate approximation of the metric which is caused by the presence of a weak electromagnetic plane wave. This physical requirement is supported by the principle of causality which implies that such a metric is a bounded periodic function. However, this requirement is not satisfied by solutions in the literature, because they are not bounded, independent of how weak the electromagnetic plane waves are (see [16]). Also, they violate causality.

This compatibility of symmetry due to causality is a physical requirement. On the other hand, since any field equation and its physical solutions must be compatible with the principle of causality, symmetry consideration can be used as a criterion, which is independent of the field strength.

For some mathematical equations, the symmetries of a solution can be very different from that of the source term (which may or may not be the physical cause). For example, consider the following equation,

$$\eta^{ab}\partial_a\partial_b F = f(u), \tag{A1}$$

where η_{ab} is the flat metric $(+---)$ and $u \equiv (t - z)$. If F is a function of only t and z, then the

inhomogeneous solution of eq. (A1) is

$$F(t,z) = \frac{v}{4} \int^{u} f(t)\, dt, \tag{A2}$$

where $v \equiv (t + z)$. Solution (A2) depends not only on u, but also v.

Then, one may examine a field equation after the related physical cause is identified. The left-hand side of eq. (A1) can be considered as a Maxwell's equation or an equation in linearized gravity. For the case of Maxwell's equation, the principle of causality implies that the source term may not be in the form of plane waves. This restriction is satisfied physically because, in nature, a charged particle is invariably massive. For linearized gravity, Function F relates to the deviations from a flat metric. (An implicit assumption of weak gravity is that an empty space has a flat metric. This assumption is identical to the requirement of a normal state.) If the physical cause is an electromagnetic plane wave propagating in the z-direction, then the related source energy-stress tensor can be a function of u [6,11,17], and its lowest order approximation is a function of u, and thus the source term is linearized gravity would have the form f(u). Then, according to solution (A2), F(t,z) and therefore the metric has a factor v.

On the other hand, the principle of causality implies that the metric is a function of u only [6,11,13]. This contradiction suggests that, for gravitational waves, eq. (A1) is not an appropriate form. Thus, causality implies that there are weak gravity exact solutions, which cannot be approximated with linearized gravity. In other words, causality supports Einstein's observation that linearized gravity is not reliable [7]. One might argue that a solution could be a function of only u through a gauge transformation. This is not possible physically nor mathematically since a flat space-time has to remain flat.

In short, the principle of causality may appear to be questionable from the viewpoint of mathematics, but nature requires that this principle is satisfied. In classical electrodynamics, it is well-known that causality restricts the possible form for the radiation reaction force. Here, causality explains physically that, even for weak gravity, the field equation of gravity must go beyond a linear equation of Maxwell type because the physical causes of gravity include electromagnetic waves.

Appendix B: Compatibility of Polarizations.

Einstein's field equation shall be examined with different polarizations. Then, the relations between the polarizations of electromagnetic and gravitational wave components are established.

It has been established that for a circularly polarized electromagnetic wave, the solution is also a circularly polarized gravitational wave. Its curvature tensor and electromagnetic energy-stress tensor are:

$$R_{tt} = 2\omega^2 B_\alpha{}^2/G \ , \quad \text{where} \quad G = (1 + C_1)^2 - B_\alpha{}^2, \tag{B1}$$

and

$$T(E)_{tt} = \omega^2 A_0{}^2 (1 + C_1 - B_\alpha \cos \alpha)/2G \tag{B2}$$

where 2ω and B_α are the frequency and the amplitude of the gravity wave. B_α and C_1 are small numbers. The determinant g and g_t are constants.

The frequency ratio suggests that $-K A_0{}^2$ is of first order of deviations. Thus, $\omega^2 A_0{}^2/2G$ in (B2) must be canceled by the photon tensor. (This means that, in the flat metric approximation, an electromagnetic wave and its photons carry, on the average, the same energy-momentum.) To support this, consider a linearly polarized electromagnetic wave

$$A_x = A = A_0 \cos[\omega(t - z)] \ . \tag{B3}$$

From equation (6), the equation of the lowest order terms is

$$f''/2 = - K (A')^2, \quad \text{where} \quad -f \equiv (g_{xx} + g_{yy}) + 2 \tag{B4}$$

Indeed, there is a constant $-K\omega^2 A_0{}^2/2$ to be canceled. To have a physical solution, the modified equation of the lowest order should be

$$f'' = - 2K T(u), \tag{B5}$$

and the time average of T(u) is zero. Now, equation (6a) becomes

$$G'' - g_{xx}'g_{yy}' + (g_{xy}')^2 - G'(g'/2g) = 2K\, g_{yy}T(u). \tag{B6}$$

There are four unknowns in eq. (B6). The previous case suggests that eq. (B6) can be reduced.

From eqs. (7) and (B4), only one of g and g_t can be a constant. (The Schwartzchild solution suggests that g would likely be a constant.) Nevertheless, it is still possible to simplify (B6). To this end, define

$$F_1'' \equiv -2K\, T(u)\ , \quad \text{and} \quad f \equiv f_1 + F_1 + F_2\ , \tag{B7}$$

where F_2 is of second order and f_1 is the time average of f. Then, one obtains

$$G'' - g_{xx}'g_{yy}' + (g_{xy}')^2 = -(fg_{yy}' + F_2'g_{yy})' - F_1''g_{yy} + F_2'g_{yy}'$$

$$- [2\gamma_{yy}\gamma_{yy}'' + (\gamma_{yy}')^2 + 2g_{xy}g_{xy}'' + (g_{xy}')^2]. \tag{B8}$$

where γ_{ab} $(= g_{ab} - \eta_{ab})$ is the deviation from the flat metric η_{ab}. Then (B6) is reduced to

$$G'(g'/2g) + (fg_{yy}' + F_2'g_{yy})' - F_2'g_{yy}' =$$

$$- [2\gamma_{yy}\gamma_{yy}'' + (\gamma_{yy}')^2 + 2g_{xy}g_{xy}'' + (g_{xy}')^2]. \tag{B9}$$

where

$$G' = -(fg_{yy})' - 2[\gamma_{yy}\gamma_{yy}' + g_{xy}g_{xy}'].$$

For a physical solution, it requires that the time average of the term $(fg_{yy}' + F_2'g_{yy})'$ is zero.

If g is a constant, then eq. (B9) implies

$$g_{yy}' = g_{xy}' = 0 \ , \tag{B10}$$

since the time average of the right hand-side of eq. (B9) cannot be positive. These equations in turn imply

$$F_2'' = 0 \ , \tag{B11}$$

and consequently $F_2 = 0$. Thus, both g_{yy} and g_{xy} are constants, and equation (B6) is reduced to

$$g_{xx}'' = 2K\,T(u). \tag{B12}$$

If the constants are independent of the wave amplitude, then one has

$$g_{xy} = 0 \ , \text{ and } \ g_{yy} = -1 \ . \tag{B13}$$

On the other hand, in general, (B13) implies

$$G'(g'/2g) - F_2'' = 0 \tag{B14}$$

It follows that g_t cannot be a constant in a linear polarization. To illustrate this, let us assume

$$g_{tt} = 1 \ , \ g_{xy} = 0 \ , \text{ and } \ g = -1 \ . \tag{B15a}$$

as suggested by linearized gravity. Then equation (6) becomes

$$(g_{xx}')^2 = 2K\,g_{xx}\left[(A')^2 + g_{xx}T(P)_{tt}\right] \ . \tag{B15b}$$

Without a photon tensor ($\lambda = 0$), the solution for eq. (B15) is:

$$g_{xx} = -\left(1 + C \pm i\sqrt{\frac{K}{2}}\,A\right)^2 \simeq -1 - 2C \mp i\sqrt{2K}\,A\ , \tag{B16a}$$

$$g_{yy} = g_{xx}^{-1} \simeq -1 + 2C \pm i\sqrt{2K}\,A\ , \tag{B16b}$$

where C is a constant which is zero when $A = 0$. The imaginary sign comes from the fact that special relativity is an accurate approximation. Solution (B16) is not physical because it is essentially imaginary for a real electromagnetic wave. Moreover, the frequency ratio between gravitational and electromagnetic wave components should be two. Also, it is easy to see that no value of λ can make g_{xx} a real wave function.

If one assumes, as Misner et. al. [6], that

$$g_{tt} = 1\ ,\ g_{xy} = 0\ ,\ \text{and}\ \ g_{xx} = g_{yy}\ , \tag{B17a}$$

then the resulting equation is

$$2LL'' = -K\left[(A')^2 + L^2T(P)_{tt}\right], \quad \text{where} \quad (-g_{xx}) = L^2. \tag{B17b}$$

For $\lambda = 0$, equation (B17b) implies that L is not bounded. For $\lambda = -1$, equation (B17b) becomes

$$2LL'' = K\,\omega^2 A_0^2 \cos(2\omega u)\ . \tag{B18}$$

However, equation (B18) implies that L is not a periodic function of u. In fact, there is no λ which can make L a periodic function.

It should be noted that, in the above calculations, the sign of R_{tt} is crucial to the physical conclusions. Because R_{tt}, on the time average, is positive, it is necessary to have an additional energy-stress tensor with an anti-gravity coupling

Appendix C: Real–Complex "Wave–Duality" and Particle–Wave Duality.

In § 8, it is not clear how the imaginary part of the energy–stress tensor $\mathrm{Im}[T(\tilde{E})_{lk}]$ is related to the metric g_{lk}. Here, we consider this problem further from the viewpoint of duality. In classical electrodynamics, it is well–known that a real wave is the real part of a complex wave. These waves satisfy related Maxwell's equations in which the source term of the real equation is the real part of the complex source. It seems that such a "wave duality" is only a mathematical convenience and that complex waves are mathematical auxiliaries. ***However, quantum electrodynamics (QED) suggests that this wave-duality should have a physical origin, particle-wave duality*** because complex wave functions must be used.

If "wave–duality" indeed has a physical origin, then wave–duality should also be valid in general relativity eventhough the equation is nonlinear. Then, QED (in which there is no photon tensor) would imply that the real part of the energy–stress tensor $T(\tilde{E})_{lk}$ of a complex wave is the modified tensor (13), i.e.

$$\mathrm{Re}[T(\tilde{E})_{lk}] = T_{lk}(E) - T_{lk}(P). \tag{C1}$$

Eq. (C1) implies that, for a static electromagnetic field, $T(\tilde{E})_{lk}$ is real, and there is no photon. The validity of eq. (C1) is verified for monchromatic plane waves. Since the imaginary part, $\mathrm{Im}[T(\tilde{E})_{lk}]$ may not be zero, from the viewpoint of completeness, complex $T(E)_{lk}$ should satisfy a complex Einstein equation, i.e.,

$$G_{lk} = -K\, T(E)_{lk}; \tag{C2}$$

and the real part of $T(\tilde{E})_{lk}$ of a complex gravitational wave satisfies a modified real Einstein equation. This would be compatible with that, ***whereas particle-wave duality is explicitly manifested in a real Einstein equation; duality is implicitly included in a complex Einstein equation.***

Eq. (C2) is supported by the facts that eqs. (2) and (3) are valid for complex functions and that the geodesic equation and the generalized Maxwell's equation can be extended to a complex metric. Since eq. (20) has only transverse metric elements, one may expect that wave–duality is valid for those elements. However, since the metric is semi–unitary, one may expect wave–duality to be only approximately valid for

g_{tt} and g_{zz}. In short, wave-duality is valid at least for weak gravity.

To see that wave-duality is valid for directionally polarized electromagnetic waves, let us consider the complex wave (23a) related to the wave (16a), the complex gravitational metric elements are:

$$- g_{xx} = 1 + C - \frac{K}{2} A^2, \qquad \text{and} \qquad g_{tt} = - g_{zz} = \sqrt{\frac{g}{g_{xx}}}\ , \tag{C3}$$

where C is a complex constant. These metric elements satisfy equations (2), (3), (7), and (22). For wave (C3), eq. (22) is simplified to the following differential equation

$$G'' = - 2\ K\,(A')^2, \qquad \text{where} \quad G = - g_{xx}\ . \tag{C4}$$

To further support wave-duality, one can calculate the case of circularly polarized electromagnetic waves. A circularly polarized electromagnetic complex wave would be wave (23b). Then, the gravitational complex wave should also be circularly polarized. Since equation (21) implies $T(E)_{tt}$ in (C2) is zero, eq. (21) and eq. (C2) imply that the gravitational complex wave is circularly polarized as follows:

$$g_{xx} = -1 - C + B\ , \quad g_{yy} = -1 - C - B\ , \quad \text{and} \quad g_{xy} = \pm\ iB\ , \tag{C5a}$$

where

$$B = B_\alpha \exp\{-i\,[2\omega(t - z) + \alpha]\}, \qquad C = C_1 \pm iB_\alpha\ ,$$

and

$$G = (1 + C)^2 = (1 + C_1)^2 - B_\alpha{}^2 \pm i2(1 + C_1)B_\alpha. \tag{C5b}$$

Formula (C5) indeed further confirms wave-duality.

The above calculations confirm that the complex Einstein eq. (C2) and Einstein eq. (13) are compatible. Thus, it is confirmed that wave-duality has its origin from particle-wave duality. While duality is represented explicitly by the real energy-stress tensors of photons and electromagnetic waves, duality is implicitly included in complex waves. Now, the photon tensor can be calculated easily from eq. (C1).