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The Coordinate Systems in Physics

and

Einstein's Equivalence Principle

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Abstract

Einstein makes clear that a physical coordinate system must satisfy the equivalence principle. He further demonstrated the meaning of this principle by explicit calculations. Unfortunately, his viewpoint is not presented fully in current text books, which implicitly substitute the equivalence principle with a mathematical requirement of an appropriate metric signature. Recently, it was discovered that the current notion of gauge, which ignores the equivalence principle, is problematic and is obstructive to the developments of relativity, which is necessitated by the Taylor–Hulse experiment. It is shown: i) Einstein's theory is incompatible with such viewpoints; ii) There are unphysical solutions (with a proper signature) which do not satisfy the equivalence principle; iii) There are solutions (with a proper signature) any of which is not diffeomorphic to a physical solution. Thus, it is necessary to recover the essence of general relativity as Einstein presented.

The Coordinate Systems in Physics and Einstein's Equivalence Principle.

1. Introduction.

A physical foundation of Einstein's theory is his equivalence principle [1]. This principle's satisfaction is necessary for the validity of a coordinate system in physics. This validity, in turn, is the basis for the notion of gauge in general relativity. Thus, to understand general relativity, it is essential and necessary to make clear the equivalence principle in connection with the validity of a coordinate system in physics.

However, it has been discovered [2] that the current notion of gauge, which ignores the equivalence principle, is problematic. Moreover, it has been proven [3] that this notion of gauge is also obstructive to the developments of relativity since such a notion is a major stumpling block to understanding the far reaching implications of the Taylor-Hulse experiment [4] on binary pulsar PSR 1913+16.

Since such a gauge is due to insufficient understanding of physics, one may wonder whether the current viewpoints are in agreement with Einstein's. This question is strengthened by Eddington's claim [5] that few people understand relativity in spite of many existing text books. We found our suspicion confirmed by Einstein's work, "The Meaning of Relativity" [1] edited by Einstein in 1954. This work reveals that his viewpoints are self-consistent, in agreement with experiments, but incompatible with those viewpoints in current text books. This conclusion is also supported by his other articles [6-9]. Thus, to have a meaningful physical theory, it is necessary to recover the essence of general relativity as Einstein presented it.

Historically, as shown by Hilbert [10], Einstein's equation,

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = - KT_{\mu\nu} , \tag{1}$$

where $R_{\mu\nu}$ is the Ricci curvature tensor, $g_{\mu\nu}$ is the spacetime metric, $T_{\mu\nu}$ is an energy-stress tensor, and K is the coupling constant, is insufficient to obtain a solution. Because of the identity,

$$\nabla^{\mu} G_{\mu\nu} \equiv 0, \tag{2}$$

four more conditions are needed. These conditions, which result in a choice of coordinates, is called the

gauge. But, the choice of gauge is not entirely arbitrary because a physical space must satisfy the equivalence principle. An often used condition is the harmonic gauge [11],

$$\frac{\partial}{\partial x^a}(|g|^{1/2}g^{ab}) = 0. \tag{3}$$

However, this gauge can be incompatible with the equivalence principle [12].

It should be noted that it is because of the equivalence principle that the geodesic equation,

$$\frac{d^2x^\mu}{ds^2} + \Gamma^\mu{}_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds} = 0, \tag{4}$$

where $\Gamma^\mu{}_{\alpha\beta}=(\partial_\alpha g_{\nu\beta}+\partial_\beta g_{\nu\alpha}-\partial_\alpha g_{\nu\beta})g^{\mu\nu}/2$, can be considered as the equation of motion for a particle.

The current viewpoint on gauge is based on the diffeomorphism, which is one-one onto, C^∞ (infinitely differentiable) map between two manifolds and its inverse map is C^∞. It is claimed that two diffeomorphic manifolds have physically identical properties [13-16]. One may note, however, that a diffeomorphism ignores the physical nature of the space time, and therefore provides no assurance to the physical reality.

2. Einstein's Viewpoints on the Coordinate Systems in Physics

However, the above viewpoint differs from Einstein's theory in the understanding and the interpretation of the physical principles; and results in invalid criteria for a physical solution. Einstein's viewpoints are:

1) A physical coordinate system must be physically realizable.

   Einstein [6] made clear in *What is the Theory of Relativity?* (1919) that "In physics, the body to which events are spatially referred is called the coordinate system." Furthermore, Einstein wrote "If it is necessary for the purpose of describing nature, to make use of a coordinate system arbitrarily introduced by us, then the choice of its state of motion ought to be subject to no restriction; the laws ought to be entirely independent of this choice (general principle of relativity) [6]. Thus, Einstein's coordinate system has a state of motion and is usually referred to a physical body.

2) A physical coordinate system is a Gauss coordinate system in which the equivalence principle satisfies.

The current viewpoint implies a physical coordinate is just a Gauss system. One might attempt to justify this viewpoint by pointing out that Einstein [1] also wrote that "In an analogous way (to Gauss curvillear coordinates) we shall introduce in the general theory of relativity arbitrary co-ordinates, x_1, x_2, x_3, x_4, which shall number uniquely the space-time points, so that neighbouring events are associated with neighbouring values of the coordinates; otherwise, the choice of co-ordinate is arbitrary." But, he [1] qualifies this with a physical statement that "In the immediate neighbour of an observer, falling freely in a gravitational field, there exists no gravitational field."

3) The equivalence principle requires not only, at each point, the existence of a local Minkowski space

$$ds^2 = c^2dT^2 - dX^2 - dY^2 - dZ^2, \tag{5}$$

but it must be obtained by free falling. Einstein [1] wrote, "According to the principle of equivalence, the metrical relation of the Euclidean geometry are valid relative to a Cartesian system of reference of infinitely small dimensions, and in a suitable state of motion (free falling, and without rotation)." Thus, in addition to having the proper metric signature, a solution of eq. (1) must satisfy the equivalence principle as an independent physical requirement.

3. Einstein's Illustration of a Physical Space

Einstein [1] illustrates the meaning of the equivalence principle by considering a space with metric

$$ds^2 = c^2(1 - \frac{K}{4\pi} \int dV_0 \frac{\sigma}{r})dt^2 - (1 + \frac{K}{4\pi} \int dV_0 \frac{\sigma}{r})(dx^2 + dy^2 + dz^2). \tag{6}$$

According to eq. (6), the unit measuring rod has the coordinate length

$$(1 - \frac{K}{8\pi} \int dV_0 \frac{\sigma}{r})$$

in respect to the system of coordinate selected. The law of the propagation of light in general co-ordinates is,

according to general theory of relativity, characterized, by the light-cone condition,

$$ds^2 = 0.$$

Then, the velocity of light is expressed in our selected coordinates by

$$\frac{[dx^2 + dy^2 + dz^2]^{1/2}}{dt} = c(1 - \frac{K}{4\pi} \int dV_0 \frac{\sigma}{r}). \tag{7}$$

One can therefore draw the conclusion from this, that a ray of light passing near a large mass is deflected [1]. Thus, Einstein demonstated that a coordinate system has physical meaning. Therefore, Hawking's claim [17] that there is no difference between a space coordinate and a time coordinate, is simply not valid.

4. The Physical Validity of a Metric

However, if the metric does not satisfy the equivalence principle, $ds^2 = 0$ would lead to an incorrect light velocity because the space is not physical. Let us demonstate this by considering the metric,

$$ds^2 = \alpha^2 dt^2 - dx^2 - dy^2 - dz^2, \tag{8}$$

where α (>2c) is a constant. Metric (8) is a solution of the Einstein equation $G_{\mu\nu} = 0$. Then, $ds^2 = 0$ implies that the velocity of light is α. One might argue that metrice (8) can be transformed to

$$ds^2 = c^2 dt'^2 - dx'^2 - dy'^2 - dz'^2, \tag{9}$$

by the following diffeomorphism,

$$x' = x, \quad y' = y, \quad z' = z, \quad \text{and} \quad t' = t\alpha/c. \tag{10}$$

Eq. (10) implies, however, that the units of t and t' are distinct and the light speed remains α but not c.

Now, let us consider another metric,

$$ds^2 = [dz' + (c - v)dt'][-dz' + (c + v)dt'] - dx'^2 - dy'^2, \tag{11}$$

which also satisfies the Einstein equation $G_{\alpha\beta} = 0$. Then, for light rays in the z'-direction, $ds^2 = 0$ would imply the light speeds were

$$\frac{dz'}{dt'} = c + v, \quad \text{or} \quad \frac{dz'}{dt'} = -c + v. \tag{12}$$

Clearly, eq. (12) also does not give a correct light speed since (12) violates relativistic causality, i.e. no cause event can propagate faster than the velocity of light in a vacuum.

Now, let us check whether the equivalence principle is satisfied. Since (8) and (11) are constant metrices, all their Christoffel symbols $\Gamma^{\mu}{}_{\alpha\beta}$ are zero. Therefore, the manifolds are flat and, according to the geodesic equation (4), there is no gravitational acceleration. Then, in a non-rotating free falling, the velocity of an observer is a constant. According to special relativity, this observer carries with himself a new coordinate system which must be obtained by a Lorentz transformation. But, a Lorentz transformation cannot transform metric (8) nor (11) to a local Minkowski space. Thus, the equivalence principle is not satisfied. The cause of an incorrect light speed is identified as the failure of satisfying the equivalence principle.

Moreover, metric (11) is obtained from the flat metric ($ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$) by

$$x' = x, \quad y' = y, \quad z' = z + vt, \quad \text{and} \quad t' = t, \tag{13}$$

where t is the time coordinate and v is a constant. The Galilean transformation (13) is clearly a diffeomorphism. But, one cannot justify transformation (13) as physically realizable.

According to $z' = z + vt$, the prime coordinate system would be associated with an observer on an object moving with a uniform velocity in the z-direction. However, if this prime observer measures light

speeds from this object, he finds, as in the Michelson–Morley experiment and etc, that a light velocity in the z'-direction is again $\pm c$, but not according to eq. (12). Thus, transformation (13) and therefore metric (11) is ***not*** physically realizable, and the prime system is merely a manifold in mathematics.

In conclusion, due to the requirement of the equivalence principle, a diffeomorphism may not be valid in physics (see also §§ 5–7) although such a diffeomorphism could be useful for the purpose of calculation in mathematics. In particular, the Galilean transformation (13) is also not valid in general relativity.

However, one may notice that all the above manifolds are diffeomorphic to physical spaces although these manifolds themselves are not physical. This is perhaps the underline thinking of the current viewpoint.

5. An Invalid "Definition" of the Light Speed

Currently, to circumvent the problem of a physical space and to avoid addressing the issue of light speeds in a coordinate system, some theorists "define" a light speed at any point of a manifold in terms of a local Minkowski space. (Then, according to this, both metrices would be valid.) However, if the manifold is not a physical space, a light speed in a local Minkowski space is not only meaningless, but also misleading in physics. As illustrated by manifold (8), its "light speed" is larger than 2c. More important, there are manifolds any of which is not diffeomorphic to a physical space. As Kramer et al. [18] pointed out, almost any metric could be considered as a solution since the source has not been determined.

An example of such an intrinsically unphysical manifold has a metric [2] as follows:

$$ds^2 = du\, dv + h_{ij}(u)x^i x^j\, du^2 - dx^i\, dx^j \,, \tag{14}$$

where $u = t - z$, $v = t + z$ (the light speed in a vacuum is denoted as 1), $h_{ii}(u) \geq 0$, and $h_{ij} = h_{ji}$. Its physical cause can be an electromagnetic plane wave [19]. This metric clearly violates the principle of causality [2] since the gravitational force (related to $\Gamma^z{}_{tt} = (1/2)\partial(h_{ij}x^i x^j)/\partial t$) has arbitrary parameters (the coordinate origin). Metric (13) also does not satisfy relativistic causality.

This shows that it is futile to attempt in resolving a physical problem, without addressing the physics, but by mathematical manipulations alone.

6. Relativistic Causality as a Criterion

Since the principle of equivalence physically implies relativistic causality, a physical space necessarily satisfies relativistic causality. It can, therefore, be used as a convenient criterion. Then, the laborious process of showing the satisfaction of the equivalence principle, can be circumvented if only the necessary condition for the equivalence principle are needed to be considered. Obviously, the requirement that a speed of light is smaller than or equal to c (the light speed in a vacuum) may not be valid for some diffeomorphic manifolds. ***Nevertheless, the principle of covariance is compatible with relativistic causality since its violation means that the choice of coordinates is not valid in physics.***

It is not difficult to see that relativistic causality is satisfied by the Schwarzschild solution [13],

$$ds^2 = (1 - \frac{C}{r})dt^2 - (1 - \frac{C}{r})^{-1}dr^2 - r^2 d\Omega^2, \tag{15}$$

where C (= KM/4π) is a positive constant, $d\Omega^2 = (d\theta^2 + \sin^2\theta\, d\phi^2)$, and (r,θ,ϕ) are spherical coordinates. Thus, for $r > C$, the light speeds in the r-direction and θ-direction are respectively,

$$\frac{dr}{dt} = \pm\,(1 - \frac{C}{r}), \quad \text{and} \quad \frac{rd\theta}{dt} = \pm\,(1 - \frac{C}{r})^{1/2}\,. \tag{16}$$

Eq. (16) shows that, due to gravity, light speeds are slower.

One might argue on the ground that, based on the simultaneous distance and the local time, light speeds remain to be ± 1. However, a speed in terms of a local Minkowski space is not a speed in the manifold. Moreover, from metric (11), for a light ray in the z'-direction, the simultaneous distance is determined by $ds^2 = -dz'^2$, and the local time is determined by $ds^2 = (c^2 - v^2)dt^2$. If such a z'-directional light speed were ± c, then one obtains $ds^2 = 2vdt'dz' \neq 0$. For a light ray in the x'-direction, althought light cone condition is not violated, one would still obtain the unphysical relation that $dx'^2/dt^2 = c^2 - v^2$.

Now, let us examine the validity of relativistic causality of a slightly different metric,

$$ds^2 = \alpha^2(1 - \frac{C}{r})dt^2 - (1 - \frac{C}{r})^{-1}dr^2 - r^2 d\Omega^2, \tag{17}$$

where α (>2) is a constant. Then, for light speeds in the r-direction, they are

$$\frac{dr}{dt} = \pm\, \alpha(1 - \frac{C}{r}). \tag{18}$$

Thus, for a sufficiently large r, the light speeds would be faster than 1, and relativistic causality is violated. Therefore, metric (17) does not satisfy the equivalence principle, and is not physically realizable.

From metric (11), the cross terms might be blamed for not satisfying relativistic causality. This is simply not valid and can be shown by the Kerr metric [13] as follows:

$$ds^2 = (1 - \frac{2rM}{\Sigma})dt^2 + \frac{4rMa\,\sin^2\Theta}{\Sigma}dtd\phi - (r^2 + a^2 + \frac{2rMa^2\sin^2\Theta}{\Sigma})\sin^2\Theta d\phi^2 - \frac{\Sigma}{\Delta}dr^2 - \Sigma d\theta^2,$$

where

$$\Sigma = r^2 + a^2\cos^2\Theta \text{ , and } \Delta = r^2 + a^2 - 2rM \text{ .} \tag{19}$$

From metric (19), one can check easily that relativistic causality would be satisfied if r > a and 2M.

One may ask whether metric (15) remains valid when r < C. Note that for r < C, the coefficients g_{rr} and g_{tt} both change sign so t becomes a space-like and r a time-like coordinate. Then, the metric is neither static nor spherically symmetric. Such problems are usually "explained" in terms of the Kruskal metric. Bonnor [20] finds, however, that the Kruskal extension and other remedies are physically unrealistic. Since Einstein equation (1) has been experimentally proven to be only approximately valid [3], r < C would probably be beyond its valid range of application. The Kerr metric (19) also has similar problems.

7. Conclusions and Discussions

Some relativists were unable to distinguish physical from mathematical solutions because the physical requirements have not been considered adequately. It is often said that a coordinate system is arbitrary. However, this arbitrariness is only among physical coordinate systems [7]. It is often said that coordinates have no meaning. This only means that the metric g_{ik} is essential to a physical space [8,9]. In Einstein's calculation of the light speeds, a coordinate system clearly provides more than an identification. Also, Haw-

king makes clear in 'The Arrow of Time' [17] that a time coordinate is distinct from a space coordinate. As pointed out by Weinberg [11], the mathematical viewpoint based on geometry may not be appropriate.

Moreover, the principle of equivalence puts a ***restriction*** on physical coordinates. In particular, just as in special relativity, once the space coordinates are chosen, the time coordinate is fixed. This is why, without mentioning the time coordinate, Einstein [6] identifies the coordinate system with a referring body only.

A Galilean transformation is not valid in physics because this diffeomorphism is in disagreement with experiments and theoretically incompatible with the equivalence principle. Thus, the equivalence principle necessarily restricts the general covariance. Moreover, from the viewpoint of physics, it is necessary to reduces the covariance to among physical coordinate systems only. This restricted mathematical covariance should be called the ***physical-covariance*** to distinguish the mathematical general covariance.

On the issue of whether a manifold is physical and a coordinate system is realizable, some theorists simply ignore the issue and instead "define" a light speed for any point with a local Minkowski space. However, such a "definition" is not valid because it is meaningless to consider the light speed unless the manifold is a physical space. Moreover, there are solutions of which one cannot transform any to a physical space [2]. This shows that such a circumvention is not valid and one cannot resolve a physical problem by mathematical manipulations alone. It should be noted that the incorrect belief that a gauge is arbitrarily applicable, is also obstructive to theoretical developments in relativity [3].

In short, the Einstein equation and the equivalence principle are independent physical requirements. Thus, for a physical coordinate transformation, not only is it a diffeomorphism, but also the resulting manifold must satisfy the equivalence principle. Otherwise, theoretical inconsistency and disagreements with experiments would result [2,3,20]. Moreover, as Einstein [1] illustrated clearly, a satisfaction of the equivalence principle assures that the light speeds can be calculated with the light-cone condition $ds^2 = 0$.

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