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EINSTEIN'S RADIATION FORMULA

AND

MODIFICATIONS IN THE EINSTEIN EQUATION

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Abstract

Einstein's radiation formula is supported by the Taylor-Hulse Experiment, although its derivation is not self-consistent. It is further confirmed, as discovered by Einstein, that his radiation formula is not compatible with his field equation. As suggested by Einstein's own remark, modifications to the source tensor are necessary. Based on the Taylor-Hulse experiment, a theory is developed within the theoretical framework of general relativity. And the radiation formula remains the same for the binary stars. Concurrently, it is determined that, due to radiation, the source tensor is not zero in a vacuum. Anti-gravity coupling, discovered by Pauli as a possibility, is a necessary feature. Also, the current theory of linearized gravity is not valid for radiation.

EINSTEIN'S RADIATION FORMULA

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I. Introduction.

General relativity suggests the existence of gravitational waves [1,2]. Although gravity waves have never been directly observed, the Taylor-Hulse experiment supports energy loss by gravitational radiation [3,4]. While Einstein's radiation formula is supported by the observed data [3], one should not consider this as a verification of Einstein's gravitational radiation theory (see §II and §III) because his theory does not produce the radiation formula in a self-consistent manner [4,5]. Instead, one should first identify the problems in its derivation and deduce its theoretical implications. Accordingly, to support the radiation formula, one may attempt to develop a self-consistent theory within the theoretical framework of general relativity.

Einstein's radiation formula is based on his notion of energy-momentum components of the gravitational field, a pseudotensor $t_{ab}(g)$ [6]. Since his theory is not covariant, doubts have been raised by Lorentz [7], Levi-Civita [8], and Einstein [9] himself. Moreover, it has been proven by Denisov et. al. [10] that the radiation formula is not an invariant; and the rate of energy emission, depending on the choice of the system of coordinates, may be positive, negative or zero. It seems, only a covariant theory can be valid in physics (see Appendix A). Moreover, linearized gravity necessarily implies the linearized conservation law.

This law implies no radiation [4,5]. Thus, it seems, there was little hope to develop a self-consistent theory to support Einstein's formula.

Then, in the question of radiation, theorists seem to be divided into three schools. One school is happy with the radiation formula and accepts the limitations of Einstein's theory [11]. Another school would develop a theory within the theoretical framework of general relativity [4,12]. But, some has gone so far as to justify an alternative gravity theory [13]. The crucial question, whether Einstein's radiation formula can be supported by a self-consistent theory, has not been answered.

However, as noted by Einstein [14], linearized gravity is not reliable. Recently, it is found that the principle of equivalence is also a crucial requirement for a physical coordinate system, and therefore a gauge condition may not be valid [15]. Then, it becomes clear that a previously seemingly impossible task would be feasible (see §II, §III and §IV). Although a covariant theory does not produce the same radiation formula, as far as agreements with data, it is sufficient to show that their rates of energy loss, on the time average, are the same.

In this paper, it will be shown that the radiation formula has important implications (see §III). Because of radiation, the source in a vacuum is necessarily not zero. Anti-gravity coupling, which is a possibility pointed out by Pauli [6], is a necessary feature. The linearized gauge is incompatible with the radiaton formula. Moreover, Einstein's radiation formula can, indeed, be supported with a self-consistent theory within the theoretical framework of general relativity.

II. Derivation of the Radiation Formula and Self-consistence.

To support Einstein's formula, it would be necessary to identify the causes of inconsistence in the derivation. Then, necessary approaches and methods can be developed for a supporting theory.

First, the non-linear Einstein's field equation reads

$$G_{ab} \equiv R_{ab} - \frac{1}{2} R g_{ab} = - K T_{ab} , \tag{1a}$$

where its source $K T_{ab}$ generally depends on the space-time metric g_{ab}. The harmonic coordinate condition [2,16] is

$$\frac{\partial}{\partial x^a}(|g|^{\frac{1}{2}} g^{ab}) = 0, \tag{1b}$$

where g is the determinant of the metric. For weak gravity, it is convenient to consider equations, which is expressed in terms of deviations γ_{ab} ($\equiv g_{ab} - \eta_{ab}$) from the flat metric. Then, eqs. (1a) and (1b) are respectively linearized to [2,4]

$$G_{ab}{}^{(1)} = - K T_{ab}, \quad \text{where} \quad G_{ab}{}^{(1)} \equiv \frac{1}{2}\partial^c\partial_c\bar{\gamma}_{ab} + H_{ab}{}^{(1)}, \tag{2a}$$

where

$$H_{ab}{}^{(1)} \equiv - \partial^c\partial_{(b}\bar{\gamma}_{a)c} + \frac{1}{2}\eta_{ab}\partial^c\partial^d\bar{\gamma}_{cd} ;$$

and

$$\partial^a\bar{\gamma}_{ab} = 0 , \tag{2b}$$

where

$$\bar{\gamma}_{ab} \equiv \gamma_{ab} - \frac{1}{2} \eta_{ab}\gamma , \text{ and } \gamma \equiv \eta^{ab}\gamma_{ab} .$$

The linearized "gauge" (2b) sufficiently reduces (2a) to

$$\frac{1}{2}\partial^c\partial_c\bar{\gamma}_{ab} = - K\ T_{ab}. \tag{2c}$$

It thus follows from eq. (2b) that the linearized conservation law,

$$\partial^a\ T_{ab} = 0\ , \tag{2d}$$

is necessarily <u>exactly</u> satisfied. Note that eq. (2d) is also implied directly by eq. (2a) since $\partial^a G_{ab}{}^{(1)} \equiv 0$.

The effective stress-energy tensor of the gravitational field, valid to second order, is assumed to be [4] (see also Appendix A)

$$t_{ab} = G_{ab}{}^{(2)}/K \tag{3a}$$

where

$$G_{ab}{}^{(2)} = G_{ab} - G_{ab}{}^{(1)}. \tag{3b}$$

Then, the rate of energy loss due to radiation is [4]

$$-\frac{dE}{dt} = \int t_{k0}dS^k = \frac{1}{K}\int G_{k0}{}^{(2)}dS^k\ , \tag{3c}$$

where

$$E = \int t_{00}d^3x + \int T_{00}d^3x$$

where the first integral is the total energy associated with γ_{ab}. Note that E is unchanged if the Landau-Lifshitz "pseudotensor" is used in eq. (3c). Also, eq. (3c) requires only $\int[\partial^a G_{a0}{}^{(2)} + \partial^a T_{a0}]d^3x = 0$.

To evaluate formula (3c), one must solve eq. (2c), and obtain

$$\bar{\gamma}_{ab}\ (x^i,\ t) = -\frac{K}{2\pi}\int\frac{1}{R}T_{ab}[y^i,(t - R)]d^3y \tag{4a}$$

where

$$R^2 = \sum_{i=1}^{3}(x^i - y^i)^2.$$

In the far field from the source, eq. (4a) can be approximated by

$$\bar{\gamma}_{ab}(x^i,t) = -\frac{K}{2\pi}\frac{1}{r}\int T_{ab}[y^i,(t - R)]d^3y\ , \tag{4b}$$

where $r^2 = \sum_{i=1}^{3}(x^i)^2$. Then, one can use eq. (2d) to establish [4]

$$\int T^{jk}\ d^3x = \frac{1}{2}\frac{\partial^2}{\partial t^2}\int T^{00}\ x^k x^j\ d^3x\ . \tag{5}$$

Substituting (5) to formula (4b), one obtains,

$$\bar{\gamma}^{jk} = -\frac{K}{4\pi}\frac{1}{r}\frac{\partial^2}{\partial t^2}\int T^{00}\ x^k x^j\ d^3x\ . \tag{6}$$

Eq. (6) manifests that the metric is periodic for periodic motions.

Then the rate of energy loss formula (3c) becomes

$$-\frac{dE}{dt} = \frac{G}{45}\ (\ddot{q}'_{kj}\ \ddot{q}'^{kj}) \geq 0\ , \tag{7}$$

where q_{jk} is the quadrupole moment tensor of the material system. Eq. (7) is the famed "quadrupole radiation" formula. In this derivation eq. (2c), eq. (2d), and eq. (3c) are the independent equations.

However, Einstein's theory is not self-consistent. As pointed out by Wald [4] and Yu [5] that the linearized conservation law eq. (2d) implies that "two stars would not orbit each other but would move on geodesics of the flat metric." This means $\ddot{q}'_{jk}$ is zero and therefore no gravitational radiation. The usual formula for the rate of change of orbital period has been derived assuming eq. (7) without reference to eq. (2d) and the analysis by Peters and Mathews [17] is based on Newtonian orbits. That derivation is illegetimate as eq. (7) has been derived from eq. (2d). The claim that observed data from PSR1913 + 16 has verified Einstein's gravitational radiation theory is therefore groundless [5].

Nevertheless, this conclusion was not generally accepted [11], and some conjectured that this could be a matter of an appropriate approximation for Einstein equation (1a). The futility of such efforts is manifested in a concluding remark of Damour [18] that "nearly all aspects of approximation methods need to be thoroughly re-investigated." As noted by Einstein [14] in 1936 that eq.(1a) and eq.(2c) are not compatible.

To have radiation, as pointed out by Wald [4], one must obtain a gravitational acceleration. From the conservation law,

$$0 = \nabla_a T^{ab} = \partial_a T^{ab} + \Gamma^b_{ac} T^{ac} + \Gamma^a_{ac} T^{cb} \tag{8}$$

one can see that, for a first order approximation of the metric, the conservation law is accurate upto the second order. Thus, it is possible to describe gravitational radiation based on a first order approximation of the metric. On the other hand, one must also show that the linearized

conservation law (2d) is not necessary. In the derivation, eq. (2d) is used only to obtain eq. (5). Due to weak gravity, eq. (5) can be derived with $\nabla^a T_{ab} = 0$. In this alternative derivation, the accuracy of $G_{ab}{}^{(2)}$, up to second order of deviations, remains the same. Thus, eq. (2d) is indeed not needed to obtain the radiation formula.

However, eq. (2a) directly implies eq. (2d). Moreover, although Einstein's formula is based on the subsequent eq. (2c), "gauge condition" (2b) still implies eq. (2d). Thus, his radiation formula is not only independent of, but inconsistent with linearized gravity. Therefore, a necessary implication of his formula is that eq. (2c) should be justifiable without eq. (2b). Then, analysis by Peters and Mathews becomes valid.

Because observation supports eq. (7) and therefore eq.(2c), it is necessary to modify eq. (1a). Then one must show also such a modification is compatible with eq. (3c). Moreover, this can be done within the theoretical framework of general relativity.

III. Modifications in the Einstein Equation.

The existence of gravitational waves is due to physical considerations which are independent of Einstein's equation [1]. Since such waves should carry energy-momentum [19], an expected modification would be that the source term should not be zero in a vacuum. This means that a source tensor due to gravity energy-stress tensor must exist. Moreover, such a tensor should have an anti-gravity coupling since gravity would not be self-generating. The radiation formula precisely confirm these.

In other words, while his radiation formula supports the Einstein tensor, it is necessary to modify the source tensor.

Although Einstein [14] was aware of the inconsistency between his equation and his formula, due to a lack of an experimental evidence, he could not resolve this difficulty. However, as if to encourage a modification, Einstein once remarked that the left-hand side of his equation was granite, but the right-hand side (the source) was sand [20].

In view of the fact that there is no existing gravity energy-stress tensor, it seems simple and natural to assume that the source tensor T_{ab} is zero in vacuum. Now, let us show that such a current Einstein's field equation is not consistent with Einstein's radiation formula.

Einstein equation can be written in an alternative form,

$$G_{ab}{}^{(1)} = - K\, \tilde{T}_{ab} = - K(T_{ab} + t_{ab}), \tag{9a}$$

where

$$t_{ab} = G_{ab}{}^{(2)} /K .$$

Then,

$$\partial^a \tilde{T}_{ab} = 0 \tag{9b}$$

is exact since $\partial^a G_{ab}{}^{(1)} \equiv 0$. If the linearized gauge (2b), $\partial^a \bar{\gamma}_{ab} = 0$, is applied, then eq. (9a) is reduced to

$$\frac{1}{2}\partial^c \partial_c \bar{\gamma}_{ab} = - K\, \tilde{T}_{ab} . \tag{9c}$$

A formal "solution" of eq. (9c) would be as follows:

$$\bar{\gamma}_{ab}\ (x^i,\ t) = -\frac{K}{2\pi}\int\frac{1}{R}\tilde{T}_{ab}[y^i,(t - R)]d^3y\ , \tag{9d}$$

where

$$R^2 = \sum_{i=1}^{3}(x^i - y^i)^2.$$

In the right-hand side of (9d), the metric is implicitly included. Now, it seems, eq. (9b) implies that there is no difficulty to accommodate the linearized gauge (2b).

However, the problem of compatibility with Einstein's radiation formula remains, unless the contribution of T_{ab} dominates the effect of t_{ab} (= $G_{ab}{}^{(2)}/K$). If KT_{ab} is non-zero only in a finite region, then it is not clear whether the contribution of Kt_{ab} (= $G_{ab}{}^{(2)}$ which may be non-zero almost everywhere) is negligible. Now, one can see that the contribution of a radiating T_{ab} may <u>not</u> be dominating.

It will be shown that, according to eq. (4a), the contribution of $G_{ab}{}^{(2)}$ diverges. At large r, we have approximately [2,4]

$$G_{ab}{}^{(2)} + H_{ab}{}^{(1)} \sim O(1/r^2). \tag{9e}$$

The contribution of $(G_{ab}{}^{(2)} + H_{ab}{}^{(1)})$ to $\bar{\gamma}_{tt}\ (x^i,t)$ would be

$$D_{tt}(x^i,t) = -\frac{1}{2\pi}\int\frac{1}{R}[G_{tt}{}^{(2)}+H_{tt}{}^{(1)}]d^3y = -\frac{1}{2\pi}[\int_{r\le a} + \int_{r>a}]. \tag{9f}$$

Note that

$$\int_{r>a} \sim \int_{r>a}\frac{d\Omega}{r}(\frac{1}{r^2})r^2dr = 4\pi\int_{r>a}\frac{dr}{r}\ , \tag{9g}$$

for large a and x^i near the origin. Thus, (9f) may not be mathematically negligible if the source has been emitting waves long enough [2], and *divergence would occur so long as the source is non-zero only in a finite region*. Moreover, since such a divergence has nothing to do with the emmision process [2], the approximation is not valid in physics.

Thus, the problem of compatibility can be resolved only in terms of physics. The divergent contribution must be canceled by an additional source tensor which must be of second order and non-zero almost everywhere in vacuum. From the viewpoint of physics, this should be the energy-stress tensor $t(g)_{ab}$ for gravity. These suggest the modified Einstein equation would be $G_{ab} = -K[T_{ab} - t(g)_{ab}]$. Tensor $t(g)_{ab}$ should have the anti-gravity coupling because gravity should not be self-generating. Thus, the radiation formula implies that the assumption of a zero source in vacuum is incorrect; and current Einstein's equation is problematic for questions related to radiation.

Moreover, if the metric is periodic, then a certain time average of $G_{a0}{}^{(1)}$ is zero. It follows from eq. (1a) that the time average of $-G_{a0}{}^{(2)}$ is the time average of KT_{a0}. Eq. (3c) implies that the time average of $G_{\check{r}0}{}^{(2)}$ (where $\check{r}$ is the radial direction) is non-zero in vacuum. This is supported by Einstein's own conclusion that based on exact solutions of his field equation, there is no radiation [14]. Thus, eq.(3c) implies that the source T_{ab} cannot be zero in vacuum. This is also supported by the fact that, in the literature [21], there is no exact physical solution of radiation when the source tensor in vacuum is assumed to be zero .

In conclusion, Einstein's radiation formula implies that his field equation must be modified such that the source tensor is non-zero in vacuum. The detailed theoretical implications are the following:

1) $t_{ab}(g)$, the effective stress-energy tensor of the gravitational field, is actually a <u>tensor</u>. The assumption that $G_{ab}^{(1)} = 0$ in vacuum is equivalent to

$$G_{ab} = G_{ab}^{(1)} + G_{ab}^{(2)} = Kt_{ab}(g) \ . \tag{10}$$

Therefore, $t(g)_{ab}$ is actually a tensor although its approximation appears in eq. (3a) as a pseudotensor. This means that the <u>covariant</u> nature of general relativity is maintained.

2) The coupling of $t_{ab}(g)$ is anti-gravity. Eq. (10) means that the <u>factual</u> assumption in vacuum is

$$T_{ab} = -t_{ab}(g) \ . \tag{11}$$

Eq. (11) means that the tensor t_{ab} has <u>anti-gravity</u> coupling. Pauli [6] pointed out that general relativity does not provide a physical interpretation for the sign of the gravitational coupling constant. Thus, in principle, anti-gravity coupling is allowed. Now, this is necessary due to the radiation formula. Moreover, anti-gravity coupling is supported by the fact that the Einstein tensor of a gravitational plane wave has, on the average, a different sign from that of the massive matter [15,22].

3) Eq. (10) and eq. (11) imply that, in general, Einstein equation must be extended to the following form,

$$R_{ab} - \frac{1}{2} Rg_{ab} = - K[T_{ab}(m) - t_{ab}(g)] = - KT_{ab} \qquad (12a)$$

where $T_{ab}(m)$ is the stress tensor for massive matter and $t_{ab}(g)$ is for the field energy. The term t_{ab}, as indicated in (9d), cancels the divergent integral. Now it is clear, if the stress tensor T_{ab} is zero in vacuum, then it is not possible to have gravitational radiation since $t_{ab} = 0$ (see also [12,14]). Note that eq. (12a) extends the suggestion of Lorentz [7] and Levi-Civita [8].

4) Due to energy-momentum conservation, eq. (12a) implies

$$\nabla^a T_{ab}(m) = 0, \qquad (12b)$$

and

$$\nabla^a t_{ab}(g) = 0. \qquad (12c)$$

because of their coupling sign difference. Thus, for a point-like particle, the equation of motion is still a geodesic equation.

Note that $t_{ab}(g)$, being an energy-stress tensor, is not a geometrical part. In the literature, based on different theoretical considerations, there are theories [12,23] in which a non-matter term is present in the source. But, there was no anti-gravity coupling. Note that *both the presence of the gravitational energy-stress $t_{ab}(g)$ and its anti-gravity coupling are necessary due to the Taylor-Hulse experiment.*

Since the linearized gauge (2b) is not valid, according to eq. (2c), one obtains approximately

$$Kt_{ab}(g) \simeq G^{(2)}{}_{ab} + H^{(1)}{}_{ab} , \tag{13}$$

Eq. (13) implies that eq. (3c) would be modified. However, if the motion is periodic, on the time average, the tensor component t_{0k} ($k = x, y, z$) remains essentially $G^{(2)}{}_{0k}/K$ as assumed earlier.

IV. Maxwell-Newtonian Approximation and Einstein's Radiation Formula

Now, it remains to justify eq. (2c). Physically, eq. (2c) gives the direct influence of the massive source to the field. Whereas the right-hand side of eq. (13) represents the gravity self-interaction; $t_{ab}(g)$ is the gravitational energy-stress. Eq. (2c) implies also that a gravitational wave propagates with the speed of light. Given a particle moving along a geodesic, eq. (2c) is the natural extension from Newtonian theory. For clarity, eq. (2c) is written as,

$$\partial^c\partial_c\overline{\gamma}_{ab}{}^{(1)} = -2KT_{ab}(m), \tag{14}$$

where $\overline{\gamma}_{ab}{}^{(1)}$ ($= \overline{\gamma}_{ab}-\overline{\gamma}_{ab}{}^{(2)}$) is an approximation of the first order deviations, $\overline{\gamma}_{ab}{}^{(2)}$ is of second order, and $T_{ab}(m)$ is the energy-stress tensor for massive matter. Note that eq. (14) is now an approximation of eq. (12). (In linearized gravity, T_{ab} instead of $T_{ab}(m)$ would be used.)

Obviously, eq. (14) is an approximation after the coordinate system

has been chosen. The asymptotic flatness of the metric is the implicit gauge. Mathematically, as shown above, eq. (14) manifests the necessary approximate cancellation of the second order terms, and therefore is not a simple linearization. For the case of an electromagnetic plane wave [15], eq. (14) is exact since $T_{ab}(m) = 0$.

Moreover, eq. (14) is justified on its agreements with experiments [11,24]. For a static mass distribution, it produces Newton's law of gravity. For non-static cases, it produces Einstein's radiation formula. To be distinct from linearized gravity, eq. (14) shall be called the Maxwell-Newtonian approximation. The validity of this approximation will be further tested in the Stanford Gyroscope experiment [20].

Having made clear the underline physics, it remains to show that such an approach provides the required approximation as follows:

i) For self-consistency, it is necessary that according to eq. (12a), eq. (14) gives indeed a first order approximation.

ii) To support a radiation formula, eq. (14) must imply, to second order, $\partial_a t^{ad} \simeq 0$. This is also required by eq. (12c).

Note that i) implies also that $\nabla_a T^{ad}(m) = 0$ is satisfied to second order.

It follows from eq. (14) that

$$\frac{1}{2}\partial^c\partial_c L_b = - K\partial^a T_{ab}(m), \qquad \text{where} \qquad L_b = \partial^a\bar{\gamma}_{ab}. \tag{15a}$$

Since $\nabla^a T(m)_{ab} = 0$, $K\partial^a T(m)_{ab}$ are second order of deviations. It follows from eq. (15a) or solution (4a) that L_b are also second order. This implies, from eq. (12a), that up to first order of deviations,

$$\frac{1}{2}\partial^c\partial_c\bar{\gamma}_{ab} = -K[T_{ab}(m) - t_{ab}]. \tag{15b}$$

Physically, since t_{ab} is induced by $T_{ab}(m)$, Kt_{ab} would be of 2nd order. If eq. (14) is a valid approximation, Kt_{ab} has to be of 2nd order. Thus, eq. (14), as a first order approximation, is consistent with eq. (12a).

It will be shown that $K\partial^a t_{ab}$ is of third order. Let us denote

$$R_{ab} = R_{ab}{}^{(1)} + R_{ab}{}^{(2)} \text{ and } \quad R^{(i)} = \eta^{cd} R_{cd}{}^{(i)}, \tag{16}$$

where

$$R_{ab}{}^{(1)} = \frac{1}{2}\partial^c\partial_c\gamma_{ab} - \frac{1}{2}[\partial^c\partial_b\bar{\gamma}_{ac} + \partial^c\partial_a\bar{\gamma}_{bc}]$$

and

$$R_{ab}{}^{(2)} \simeq \frac{1}{2}\partial^d\gamma^c{}_a(\partial_c\gamma_{db} - \partial_d\gamma_{bc}) - \frac{1}{2}\gamma^{cd}\partial_a\partial_b\gamma_{cd} - \frac{1}{4}\partial_b\gamma^{cd}\partial_a\gamma_{cd}$$

$$+ \frac{1}{2}\gamma^{dc}\partial_c(\partial_b\gamma_{da} + \partial_a\gamma_{db} - \partial_d\gamma_{ab})$$

up to second order deviations. It follows from eq. (16) that

$$R = R^{(1)} + R^{(2)} - \gamma^{cd}(R_{cd}{}^{(1)} + R_{cd}{}^{(2)}), \tag{17a}$$

$$G_{ab}{}^{(1)} = R_{ab}{}^{(1)} - \frac{1}{2}\eta_{ab}R^{(1)} \tag{17b}$$

and

$$G_{ab}{}^{(2)} = R_{ab}{}^{(2)} - \frac{1}{2}\eta_{ab}R^{(2)} + \frac{1}{2}\eta_{ab}\gamma^{cd}R_{ab}{}^{(1)} - \frac{1}{2}\gamma_{ab}R^{(1)}$$

$$+ \frac{1}{2}\{\eta_{ab}\gamma^{cd}R_{cd}{}^{(2)} - \gamma_{ab}[R^{(2)} - \gamma^{cd}(R_{cd}{}^{(1)} + R_{cd}{}^{(2)})]\} \quad (17c)$$

Lengthy but straight forward calculation shows, up to second order,

$$\partial_c G^{cd} \simeq [\partial^a G_{ab} - \gamma^{ca}\partial_c G_{ab}{}^{(1)} - \frac{1}{2}\partial^a\gamma G_{ab}{}^{(1)}]\eta^{bd} - G_{ab}{}^{(1)}\partial^a\gamma^{bd}$$

$$\simeq [\frac{1}{2}\partial_b\gamma^{ca}\frac{1}{2}\partial^k\partial_k\bar{\gamma}_{ac} - \frac{1}{2}\partial^a\gamma\frac{1}{2}\partial^k\partial_k\bar{\gamma}_{ab}]\eta^{bd} - \frac{1}{2}\partial^k\partial_k\bar{\gamma}_{ab}\partial^a\gamma^{bd}$$

$$\simeq [\frac{1}{2}\partial_c\gamma^{ab}\eta^{cd} - \frac{1}{2}\partial^a\gamma\eta^{bd} - \partial^a\gamma^{bd}][-KT(m)_{ab}]. \quad (18)$$

It follows from eq. (12a) that eq. (18) implies, to second order,

$$\partial_a T(m)^{ad} + \Gamma^d_{ab}T(m)^{ab} + \Gamma^a_{ab}T(m)^{db} \simeq \partial_a t^{ad}. \quad (19a)$$

and

$$\partial_a t^{ad} \simeq 0 \ . \quad (19b)$$

Thus, it has been proven that the Maxwell-Newtonian approximation is a self-consistent approach with valid physical justifications. In particular, it unequivocally supports that Einstein's radiation formula is a consequence of general relativity.

Since $\partial^a G_{ab}{}^{(1)} \equiv 0$, eq. (12) implies

$$G_{ab}{}^{(2)}/K + T_{ab}(m) = t_{ab}(g) - G_{ab}{}^{(1)}/K \quad (20a)$$

and

$$\partial^a G_{ab}{}^{(2)}/K + \partial^a T_{ab}(m) = \partial^a t_{ab}(g). \quad (20b)$$

For weak gravity, owing to $\partial^a t_{ab} \simeq 0$ up to second order, $\partial^a G_{ab}{}^{(2)}$ would relate mainly to the energy-momentum of matter as its source while, in vacuum, $\partial^a G_{ab}{}^{(2)}/K$ is equal to $\partial^a t_{ab}(g)$. In other words, gravity energy and the motion of particles influence each other mainly through geometry.

It follows from eq. (20) that approximately

$$-\frac{dE}{dt} = \int t_{a0} dS^a = \frac{1}{K}\int G_{a0}{}^{(2)} dS^a - \frac{1}{2K}\int (\partial_0 L_a + \partial_a L_0) dS^a. \qquad (21)$$

Note that the second integral comes from $\partial^a G_{ab}{}^{(1)} \equiv 0$. Since such a relation is independent of the physical process, from the viewpoint of physics, the second integral is irrelevant. In fact, based on solution (6), calculation shows that the time average of the second integral is zero. Then, eq. (21) is reduced back to eq. (3c).

In summary, assuming Einstein's radiation formula is valid, as if predicted by Einstein's remark, modifications on the source tensor are necessary. Remarkably, his radiation formula means that in vacuum the source tensor is not zero, and that its coupling is anti-gravity. Then, after the necessary modifications, the radiation formula remains the same. Thus, this modification process is self-consistent.

V. Conclusion and Discussion.

It is interesting that a self-consistent covariant theory can be developed to support Einstein's radiation formula. Some colleagues may have linked the validity of "gauge" and linearized gravity with his radia-

tion formula. However, the fact is that both linearized gravity and the "gauge" is inconsistent with his formula. The most important conclusion is, however, that Einstein's radiation formula requires modifications of his field equation. A crucial starting point of this analysis is the observation that the gauge may not be applicable for gravitational waves. (In electrodynamics, classical gauge is also not valid [25,26].) The inadequacy of this notion is recently proven [15] because it can be inconsistent with relativistic causality and the principle of equivalence.

The source tensor in vacuum was controversial [12]. Now, based on Einstein's radiation formula, T_{ab} is determined to be non-zero in a vacuum. Consequently, T_{ab} must consist of two parts: a massive tensor $T_{ab}(m)$ and a gravity energy-stress tensor $t_{ab}(g)$. Now, it is clear that $t_{ab}(g)$ only appears to be a pseudotensor due to approximation. Physically, since $t_{ab}(g)$ is induced by $T_{ab}(m)$, $t_{ab}(g)$ should, in agreement with this analysis, have a coupling sign which is opposite to the coupling sign of $T_{ab}(m)$. Such an anti-gravity coupling is further confirmed by gravitational plane waves [15,22].

The main eq. (12) is within the theoretical framework of general relativity since general relativity does not specify the coupling constant nor the form of an energy-stress tensor [6]. (One may recall that the initial field equation was $R_{ab} = -KT_{ab}$ in general relativity.) Moreover, as an Einstein's field equation, eq. (12) is covariant and can be derived from a Lagrange function with the variational principle.

It should be pointed out that $t_{ab}(g)$, being an energy-stress tensor,

is not a geometrical part although $t_{ab}(g)$ involves only gravity. Moreover, tensor $t_{ab}(g)$ is not a physical cause of the metric while $Kt_{ab}(g)$ is a source term in Einstein's equation. A physical cause and a mathematical source term can be different [15] although they are the same in electrodynamics. In general relativity, an energy-stress tensor involves the metric, and therefore cannot be a cause of the metric. Thus, $t_{ab}(g)$ is a valid source term in Einstein's field equation. Note that eq. (12a) extends the proposal made by Lorentz [7] and Levi-Civita [8].

In short, this modification is necessary if Einstein's radiation formula is valid. Also, this modification is self-consistent.

The Maxwell-Newtonian approximation eq. (14) is not general covariant and is an approximation after the coordinate system has been chosen. Eq. (14), is based on physics instead of the notion of gauge. From the viewpoint of physics, this is appropriate. As pointed out by Wheeler [1], radiation is due to the fact that gravity travels at a finite speed; and should not be inextricably related to a notion in pure mathematics. It seems, once eq. (14) is accepted, in this new derivation, problems related to the linearized conservation law no longer exist. Damour [18] has concluded that nearly all aspects of approximation methods need to be thoroughly re-investigated. Without $t_{ab}(g)$, as shown in § III, Einstein's field equation is inconsistent with Einstein's radiation formula. Thus, this analysis also supports Damour's conclusion.

For the static case, the Maxwell-Newtonian approximation (MNA) coincides with linearized gravity. Thus, MNA produces Newton's law and is

supported by experiments [24]. The verification of Einstein's radiation formula can be considered as the strongest evidence which supports the MNA. Moreover, there would be one more evidence to establish MNA directly. Because of the superficial mathematical similarity between MNA and the "linearized" Einstein's equation (due to the cancellation effect of two errors), existing theories to test linearized gravity can be used to check MNA. For instance, magnetic gravitational effects would provide a good test. An experiment in the near future is the Gravity Probe B gyroscopes [20], which measures the change of the spinning direction.

In comparison with the massive tensor $T_{ab}(m)$, the field tensor $t_{ab}(g)$ is of a higher order. For static cases, this analysis implies that a zero $t_{ab}(g)$ in vacuum would be a good approximation. This would mean that the effects of this modification will not be normally observed under most circumstances. Therefore, this modification would not noticably change the existing agreements between theory and previous observations. However, further investigations on strong gravity can be done only after the exact form of $t_{ab}(g)$ is determined. Since Einstein's radiation formula is well supported by observations, it is expected that eq. (12) would give a more accurate physical description for verified predictions and may provide a more complete picture for unverified predictions because there is a gravity energy-stress tensor $t_{ab}(g)$.

Now, it seems, much of the severe criticism [13] of general relativity has become meaningless. They are as follows:

i) Einstein's quadrupole formula for gravitational radiation is not

a corollary of general relativity.

ii) When a gravitational field and matter are taken in conjunction, the general theory of relativity has not, and cannot have, energy momentum conservation laws.

iii) It does not follow from general relativity, in principle, that a double star loses its energy by gravitational radiation.

iv) General relativity does not have the classical Newtonian limit and, hence, it does not satisfy a fundamental physical principle, that is the correspondence principle.

However, this theory is not exactly complete since eq. (14) is only an approximation. It remains to find an exact expression for gravity tensor $t_{ab}(g)$. Nevertheless, this theory does justify Einstein's radiation formula. In view of the absence of a commonly accepted theory, the present analysis may serve as an interim theory to be completed.

The focus of this paper is limited to the necessary implications of the Tayulor-Hulse experiment. To address the possible exact forms of the gravitational energy-stress $t(g)_{ab}$, requires extensive and thorough considerations. This is beyond the scope of this paper. This analysis has established a new criterion for the validity of a gravity theory if it includes the Einstein tensor in its field equation. This means that the existing theories of Brans and Dicke [23], of Yu [12], of Yilmaz [27], and etc. can be further examined with this criterion. I believe that at present such considerations should be left to those authors themselves. Such discussions would be more appropriate when the possible exact forms of $t(g)_{ab}$ are considered.

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Appendix A: The Gravitational Energy-stress Tensor.

In a field theory, a central problem is the exchange of energy between a particle and the field where the particle is located. Because a particle gains energy from or loss energy to the field, the field energy-stress has to be *localized.* This is independent of the specific form of an energy conservation law.

The energy-momentum conservation law is usually related to the equation of motion for a particle. In the spirit of Faraday and Maxwell, the conservation law would be written in the following form:

$$\partial_\mu [T(p)^{\mu\nu} + T(g)^{\mu\nu} + T(E)^{\mu\nu} + \ldots] = 0. \qquad \text{(A1)}$$

where $T(p)^{\mu\nu}$, $T(g)^{\mu\nu}$, and $T(E)^{\mu\nu}$ are respectively the energy-stress tensor of the particle, of gravity, and of electromagnetism. In general relativity, the partial derivative ∂_μ is supposedly just replaced by the covariance derivative ∇_μ. This is, indeed, the case for electromagnetism. However, the terms $\partial_\mu[T(p)^{\mu\nu} + T(g)^{\mu\nu}]$ are actually replaced by only $\nabla_\mu T(P)^{\mu\nu}$. Thus, due to geometry is involved in gravity, the question of a gravitational energy-stress tensor is not a straight forward matter.

The equivalence principle implies that gravity is a manifestation of the metric and a neutral particle follows a geodesic. The geodesic equation is,

$$\frac{d^2x^\alpha}{d\lambda^2} + \Gamma^\alpha{}_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} = 0, \qquad \text{(A2)}$$

where

$$\Gamma^{\alpha}{}_{\mu\nu} = \frac{g^{\alpha\beta}}{2}\left(\frac{\partial g_{\beta\mu}}{\partial x^{\nu}} + \frac{\partial g_{\beta\nu}}{\partial x^{\mu}} - \frac{\partial g_{\mu\nu}}{\partial x^{\beta}}\right)$$

are the christoffel symbols, and λ is a parameter. The fact that $\Gamma^{\alpha}{}_{\mu\nu}$ is a pseudotensor leads to the viewpoint that the gravitational energy-stress is not localizable [2] or even there is no energy conservation law in general relativity [13]. While the latter view explicitly proclaims the invalidity of general relativity, the former view attempts to defense relativity. However, since physics requires a field energy-stress to be localized, the effect of this defence is actually destructive.

The non-localizable argument is as follows: The equivalence principle implies that all the Christoffel symbols are zero in a local Minkowski space. This implies no gravitational force and no force would means no energy. First of all, in general relativity, there is no gravitational force. When all the Christoffel symbols are zero, the observed effect is no acceleration. But, no acceleration does not mean a non-localized energy-stress. For example, consider an electron in an electromagnetic field. The field energy-stress is localized; but, in the reference frame which the electron carries, the electron has no acceleration. Thus, non-localizable argument based on the equivalence principle is not valid.

Moreover, in *Relativity and Problems of Space (1954)*, Einstein [28] added the crucial phrase, "at least to a first approximation" on the indistinguishability between gravity and acceleration. Note that whereas a geodesic equation requires only first order derivatives of the metric; the Einstein tensor requires second order derivatives. Eddington [29] also

pointed out the limitation of the equivalence principle. He wrote,

> "...there are more complex phenomena governed by equations in which the curvatures of the world are involved; terms containing these curvatures will vanish in the equations summarising experiments made in a flat region, and would have to be reinstated in passing to the general equations. Clearly there must be some phenomena of this kind which discriminate between a flat world and a curved world; otherwise we could have no knowledge of world-curvature. For these the principle of Equivalence breaks down."

In short, both Einstein and Eddington recognize that the equivalence principle can be compatible with a localized energy-stress tensor.

Since Einstein's notion of gravitational energy-stress is a pseudotensor, it can only be an approximation for some coordinate systems. Einstein's radiation formula is expressed in terms of the quadrupole moments of a Cartesian coordinate system. Moreover, the agreement with data is only on the time average. Thus, experiment supports only the time average of an integral which is based on Einstein's notion. In other words, the gravitational energy-stress can be different from his.

Einstein believed that the gravitational field together with matter must obey a conservation law of some kind. However, his derivation is based on intuition rather than a physical principle. (Einstein did not realize that, because geometry is involved in gravity, such a conservation law may not be as straight forward as in eq. (A1).) Moreover, from his derivation, his notion of gravitational energy-stress is by no means unique. The Einstein tensor can be written as

$$gG^{ab} = \partial_c h^{abc} + Kg\tau^{ab}, \qquad \text{where} \qquad h^{abc} = -h^{acb}. \tag{A3}$$

and g is the determinant of the metric. Then, $G_{ab} = -KT_{ab}$, is equivalence to

$$-g(T^{ab} + \tau^{ab}) = \partial_c h^{abc}, \tag{A4}$$

which implies

$$\partial_a[g(T^{ab} + \tau^{ab})] = 0. \tag{A5}$$

Eq. (A4) can be written alternatively as

$$\sqrt{-g}(T^a{}_b + \tau^a{}_b) = \partial_c \sigma^{ca}{}_b, \tag{A6}$$

Obviously, τ^{ab} depends on the choice of h^{abc} (or $\sigma^{ca}{}_b$). If

$$\sigma^{na}{}_b = \frac{g_{bm}}{\sqrt{-g}}\partial_l[-g(g^{ma}g^{nl} - g^{mn}g^{al})]/2, \tag{A7a}$$

then we arrive at the Einstein pseudotensor,

$$K\tau^a{}_b = \sqrt{-g}\{-2\Gamma^a{}_{ml}\Gamma^l{}_{bp}g^{mp} + \Gamma^l{}_{ml}\Gamma^m{}_{bp}g^{ap} + \Gamma^l{}_{lm}\Gamma^a{}_{bp}g^{mp} + \Gamma^l{}_{bl}\Gamma^a{}_{mp}g^{mp}$$

$$- \Gamma^l{}_{bl}\Gamma^p{}_{mp}g^{ma} - \delta^a{}_b[g^{mp}\Gamma^l{}_{mp}\Gamma^n{}_{ln} - g^{nl}\Gamma^p{}_{ml}\Gamma^m{}_{pn}]\}/2. \tag{A7b}$$

If

$$h^{abc} = \partial_m[-g(g^{ab}g^{mc} - g^{ac}g^{mb})]/2, \tag{A8a}$$

then we arrive at the Landau-Lifshitz symmetric pseudotensor,

$$K\tau^{ab} = \{(2\Gamma^n{}_{ml}\Gamma^p{}_{np} - \Gamma^n{}_{ml}\Gamma^p{}_{np} - \Gamma^n{}_{ml}\Gamma^p{}_{np})(g^{al}g^{mb} - g^{ab}g^{ml})$$

$$+ g^{al}g^{mn}(\Gamma^b{}_{lp}\Gamma^p{}_{mn} + \Gamma^b{}_{mn}\Gamma^p{}_{lp} - \Gamma^b{}_{np}\Gamma^p{}_{ml} - \Gamma^b{}_{ml}\Gamma^p{}_{np})$$

$$+ g^{bl}g^{mn}(\Gamma^a{}_{pl}\Gamma^p{}_{mn} + \Gamma^a{}_{mn}\Gamma^p{}_{pl} - \Gamma^a{}_{np}\Gamma^p{}_{ml} - \Gamma^a{}_{ml}\Gamma^p{}_{np})$$

$$+ g^{ml}g^{np}(\Gamma^a{}_{nl}\Gamma^b{}_{mp} - \Gamma^a{}_{ml}\Gamma^b{}_{np})\}/2. \qquad \text{(A8b)}$$

If

$$\sigma^{na}{}_b = \sqrt{-g}\; g^{ma}g^{nl}[\partial_l g_{bm} - \partial_m g_{bl}]/2, \qquad \text{(A9a)}$$

we arrive at the Lorentz's pseudotensor,

$$K\tau^a{}_b = \sqrt{-g}[\partial_b\Gamma^l{}_{pl}g^{pa} - \partial_b\Gamma^a{}_{mp}g^{mp} - \delta^a{}_bR]/2. \qquad \text{(A9b)}$$

Note that there is no physical criterion to determine the gravitational energy-stress pseudotensor t_{ab} although they all give the same integral conservation law. It seems that considering the energy-stress without considering the geometry, one can not get a valid and complete physical picture of the energy-momentum transfer (see eq. (20)).

In general relativity, as Hilbert [30] pointed out, there are simply no ordinary conservation laws for energy and momentum. Since experiments have proved that the current Einstein equation must be modified, the validity of these pseudotensors has become even more groundless.