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SPACE, TIME, MOTION, AND GENERAL RELATIVITY

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December, 1994
Revised March 1995

Abstract

Space, time, motion, and general relativity are reviewed with its recent theoretical developments and experiments. Based on Einstein's original work, the theoretical framework of relativity together with implicit assumptions and unverified beliefs in current theory are identified by examining the later theoretical developments. It is concluded that the existence of the anti-gravity coupling is a necessary feature of general relativity. This implies that the energy conditions in the singularity theorems of Hawking and Penrose are not valid in physics. Consequently, these singularity theorems are irrelevant to theories on space-time. In contrast to the current belief, the principle of equivalence is actually a crucial requirement for a physical coordinate system. For example, the current solutions for electromagnetic plane waves are incompatible with this principle and consequently other physical principles. Then based on the principle of causality, it is proven that there is no physical solution unless an additional tensor with an anti-gravity coupling is included in the source term. Physical considerations and detailed calculations identify that such a tensor should be the energy-stress tensor of photons. In search for its general form, it is discovered that the real-complex "wave-duality" in electrodynamics has its origin from particle-wave duality. Thus, the existence of a connection between relativity and quantum theory is manifested. It is therefore conjectured that gravitational radiation is also associated with an anti-gravity coupling. Interestingly, this conjecture has actually been verified by the gravitational radiation of the binary pulsar PSR 1913+16, because Einstein's radiation formula implies that the Einstein equation must be modified with a gravitational energy-stress tensor added to the source term. Indeed, the energy-stress tensor for gravity must have an anti-gravity coupling. Moreover, the supposedly inevitable gravitational complete collapse of a super star in the current theory actually results from inadequate modeling due to the remnant influence of Newtonian gravity. In general relativity, a super star would end, in agreement with observations, as a supernova.

1. Introduction

In physics, the nature of matter is understood in terms of its motion. Since any motion is described in terms of space and time coordinates, the nature of space and time is a fundamental problem in physics. Since a motion invariably involves gravity, the understanding of motion and gravity historically goes hand in hand and theoretically is almost inseparable. Our understanding of the motion of bodies is improved with the depth of our observations. This can be dated back to Aristotle, Galileo and Newton.

Perhaps, the earliest recorded notion of motion, stated by Aristotle, is that the natural state of a body was to be at rest and that it moved only if driven by a force or impulse. Aristotle stated also that a heavy body should fall faster than a light one. Aristotle's statements are based on our crude daily experience without analyzing the resistence due to friction. The Aristotelian tradition also held that, based on some principles which may have been concluded from some observations by an authority, one could work out all the laws of nature by logical thinking which often involves dubious implicit assumptions. Such extrapolations need not to be checked by observation. Unfortunately, such a tradition often revived in the history of theories.

Galileo's studies of mechanics showed that a body does not come to a halt when the force propelling it is removed, but instead decelerates at a rate dependent on the amount of friction it encounters. Therefore, Galileo argued that all objects would fall at the same rate if there were no atmospheric frictoin. Also, as a body rolls down a constant slope, the same gravity force always makes it constantly speed up. This shows that the effect of a force is to change the speed of a body, rather than just to set it moving. Thus, whenever a body is not acted on by any force, it will keep on moving in a straight line at the same speed. Galileo's experiments were used by Newton as the basis of his three laws of motion. Newton's law implies that there is no unique standard of rest, and therefore one could not give an event an absolute position in space. Moreover, a speed is related to another object which serves as a reference frame of which a coordinate system is attached. But, both Aristotle and Newton believed in absolute time.

Newton, based on his laws of motion and Kepler's three laws of planetary motion, discovered a law to describe the force of gravity, which states that every body attracts every other body with a force that is proportional to the mass of each body. This explains why all bodies fall at the same rate. In the geometrical aspect, the gravity force between two bodies is inversely proportional to the square of their distance. This explains Kepler's law of planetary motion. Moreover, his law of gravity predicts the orbits of the earth, the moon, and the planets with great accuracy. Newtonian gravity is important in the formulation of relativity.

Newton's theory is based on observations of the mechanical slow speed phenomena. The limitations of his theory are exposed when electromagnetic phenomena are observed and analyzed. In spite of the most obstinate efforts, electromagnetism just cannot be explained in terms of mechanics. A new physical concept, "field" in space has to be accepted. Moreover, Maxwell's theory predicted that electromagnetic waves (or lights) travel at a certain fixed, but very high, speed. This speed, according to Newton's theory, would be measured relative to a certain reference frame; and ether was suggested. However, all attempts to measure the relative motion of a body to ether fail. In particular, the Michelson–Morley experiment showed that the speed of light is independent of the direction of the earth's motion.

There were several attempts, most notably by the Dutch physicist Hendrik Lorentz, to explain the result of the Michelson–Morley experiment in terms of objects contracting and clocks slowing down when moved through the ether. However, in 1905, Albert Einstein pointed out in his special theory of relativity that the whole idea of ether was unnecessary, but the idea of absolute time must be abandoned. A similar point was made a few weeks later by a French mathematician, Henri Poincaré. It is interesting to note that Einstein's arguments were physical; whereas Poincaré, being a mathematician, regarded this problem as mathematical.

Special relativity is based on inertial reference frames. This is, in principle, unsatisfactory since an inertial frame exists only in idealization. On the other hand, gravity cannot be made compatible with special relativity because of the experimental fact that all bodies have the same acceleration in a gravitational field [1]. Einstein solved both problems by realizing the equality of inertial and gravitational mass. Later, he developed this to the principle of equivalence, on which the general theory of relativity is formulated. From this exceedingly convincing theoretical framework, Einstein made three observable predictions: i) perihelion precession; ii) bending of lights; and iii) gravitational red shift. All three predictions are confirmed by experiments with high accuracy [2]. General relativity is also supported by subsequent tests [3].

In general relativity, matter curves the four-dimensional Reimannian space-time, and the metric plays the role of a gravitational potential. The equivalence principle implies that the geodesic equations are the equations of motion. However, as pointed out by Klein [4], there is no proof for the rigorous validity of Einstein's equation, which have been and will be further modified within the framework of general relativity.

Some relativists, who are confused over the differences between mathematics and physics, believed that the choice of physical coordinates is arbitrary. Obviously, this would not be compatible with the principle that "The justification for a physical concept lies exclusively in its clear and unambiguous relation to facts that can be experienced" [5]. Such a belief is based on their failure in recognizing that the equivalence principle actually limits the choice of valid physical coordinates [6]. (They thought that such a physical condition is satisfied automatically by any metric [3].) Since the principle of equivalence should be satisfied in a physical space, this mistake would not relate directly to observations. This would explain, in part, that the agreement between observation and theory remains excellent. However, it does hinder and mislead the theoretical developments of relativity. For instance, this incorrect belief is the root of the inadequate notion of gauge [7], and a reason of many unphysical solutions in the literature [8].

A well-known unsettled problem in general relativity is the notion of gravitational energy. Einstein proposed a gravitational energy-stress, in analogy to electromagnetism, to be essentially a quadratic form of the the metric's partial derivatives [9]. Based on linearized gravity, Einstein derived a formula for gravitational radiation. However, since such a gravity energy-stress is a pseudotensor, doubts have been raised by Lorentz [10], Levi-Civita [11], and Einstein [12] himself. Some theorists [13] have gone so far as to justify an alternative theory. Moreover, in his notion, gravitational energy cannot be localized since the principle of equivalence implies that such derivatives are always zero in a local Minkowski space. But, if gravitational energy is not localized, then how a particle can gain (or lost) energy in a gravitational field?

Nevertheless, the observation of Hulse and Taylor [14] supports Einstein's radiation formula although Wald [7] and Yu [15] find that its derivation is not self-consistent. Naturally, attempts [16] were made to justify the radiation formula by improving the approximation. But, this is a futile effort because, as Einstein [17] noted, the linearized field equation is incompatible with Einstein's equation. In fact, linearized gravity cannot be justified by mathematics alone, and the linearized field equation can be compatible only if the Einstein equation is modified with a gravitational energy-stress tensor subtracted from the source [18]. Also, his notion of gravitational energy-stress is actually an approximation of a localized tensor as required by physical considerations. But, Einstein's radiation formula remains valid [19]. Then, the current Einstein equation emerges as a static approximation, and the intrinsic difference from Newtonian gravity becomes clear.

Whereas the great advancement from the ideas of Aristotle to those of Galileo and Newton started from a better understanding of the simple fact, friction; the big step to Einstein's ideas started from the constancy of light speed, and the equality of inertial and gravitational mass. The Taylor-Hulse experiment would be another starting point for a great advancement because it verifies the existence of anti-gravity coupling.

However, since the exact form for a gravitational energy tensor remains to be found [19], general relativity is incomplete. Thus, Einstein's claim [20] of logical completeness of general relativity seems to be overstated. Moreover, in current theory, there are implicit assumptions and unverified beliefs [6,18]. (Another example is that the universal coupling for massive matter has been extended without sufficient justifications.) In view of these, it would be beneficial and necessary to clarify the theoretical framework and to identify the implicit assumptions and unverified beliefs. Then, it may become possible to see clearly that in general relativity what assumptions and to what extent have actually been confirmed by experiments. Another problem in general relativity is to identify the physical requirements for physical solutions.

Einstein [21] suggested that the appropriateness of a source tensor would be a problem. The source tensor for electromagnetic waves is investigated because existing solutions are incompatible with the equivalence principle. It is founded [22] that, based on causality, electromagnetic waves alone have no physical solution for gravity unless an energy-stress tensor with an anti-gravity coupling is added to the source. This tensor is identified as the energy-stress tensor for related photons. The existence of an anti-gravity coupling, which is established by both experiment and theory, means that the singularity theorems [7], on which the notion of black hole is based, are actually irrelevant to physics (see §6). Einstein's another suggestion is to consider the non-symmetric metric [23-25]. Another known direction of generalization is to increase the dimensionality of the Reimannian space [26-29] such that other physical interaction, e.g. electromagnetism, can be included. However, there is not yet a distinct experimental confirmation for any of the generalizations.

In this paper, the above problems in general relativity are addressed. In particular, the existence of the anti-gravity coupling is proven and the question of gravitational collapse is discussed. However, this review is by no means complete due to the limitation and prejudice of the author. It is hoped that this paper would be useful to those who wish to develop and achieve a deeper understanding in space, time, and motion.

2. Special Relativity, quivalence Principle, and Covariance.

The fundamental principle of relativity was that the laws of science should be the same for all freely moving observers, no matter what their speed. Based on experiments, the constancy of light speeds is also assumed. A consequence of special relativity is that it is impossible to increase the speed of a massive particle to the speed of light. The physical requirement that no event can propagate faster than the velocity of light in an empty space shall be called *relativistic causality* (see also §3). It becomes harder to increase the speed of an object as its speed gets faster. This effect is verified by modern high energy accelerators in numerous experiments [30] in which charged particles are accelerated.

However, special relativity is not exact. The best known consequence is perhaps

$$E = mc^2. \tag{2.1}$$

Eq. (2.1) implies $\Delta E = \Delta mc^2$ in connection with the mass-energy transformation in an interaction but not a mass-energy equivalence since the photon is massless. Note that general relativity implies that mass and energy are not equivalent since *gravity depends also on the energy form* (as illustrated by the Reissner-Nordstrom metric in §6). This is intrinsically different from Newtonian gravity.

In special relativity, the time coordinates of different reference frames may give different time separations for two events, and the distance of two points also depends on the reference frame. It should be noted, however, that the time coordinate is determined once the frame of reference (space coordinates) are chosen. (In physics, it is meaningless to use the space coordinate of one reference frame but the time of another reference frame although this is mathematically allowed.) This is why Einstein [31] said that "In physics, the body to which events are spatially referred is called the coordinate system."

The free moving frames of reference require that a coordinate transformation must be linear in time. However, the constancy of light speed implies that some linear coordinate transformations are not valid in physics. For instance, consider a Galilean transformation,

$$x' = x, \quad y' = y, \quad z' = z + vt, \quad \text{and} \quad t' = t, \tag{2.2}$$

where v is a constant. If the light speed $dz/dt = \pm c$, then the light speed in the z'-direction is

$$\frac{dz'}{dt'} = \frac{dz}{dt} + v = c + v, \quad \text{or} \quad \frac{dz'}{dt'} = -c + v. \tag{2.3}$$

Thus, transformation (2.2) leads to only a mathematical but not a physical coordinate system. In relativity, between inertial systems, only Lorentz transformations are physically valid. An example is,

$$x' = x, \quad y' = y, \quad z' = \gamma(z + vt), \quad \text{and} \quad t' = \gamma(t + zv/c^2), \tag{2.4}$$

where $\gamma = (1 - v^2/c^2)^{-1/2}$. Also, the Maxwell's equation is invariant under Lorentz transformations.

In special relativity, the flat metric η_{ab} $(+,-,-,-)$ is

$$ds^2 = c^2dt^2 - dz^2 - dx^2 - dy^2. \tag{2.5}$$

Therefore, the time coordinate and a space coordinate are distinct although there is no absolute time. A time coordinate ζ must be time-like ($d\zeta^2 > 0$); whereas a space coordinate ξ must be space-like ($d\xi^2 < 0$). Moreover, a physical coordinate system must satisfy relativistic causality. The Galilean transformation (2.2), which is valid in mathematics but not in physics, transforms the flat metric (2.5) to

$$ds^2 = [dz' + (c - v)dt'][-dz' + (c + v)dt'] - dx'^2 - dy'^2. \tag{2.6}$$

Then, all the space coordinates (x', y', z') are space-like, and coordinate t' is time-like if $c^2 > v^2$. However, metric (2.6) and the condition, $ds^2 = 0$ for a light ray in the z'-direction would produce eq. (2.2) which violates relativistic causality. Note that both metrices (2.5) and (2.6) satisfy the same Einstein equation ($G_{ab} = 0$) in an empty space. Thus, ***although a metric satisfies Einstein equation, the light cone condition is inadequate to ensure relativistic causality*** (see also §3).

An important link between special relativity and general relativity is Einstein's principle of equivalence. This principle implies that, in a nonrotating free falling coordinate system, special relativity is locally valid [2]. Thus, relativistic causality is guaranteed by Einstein's principle of equivalence. However, in calculations, both this principle and relativistic causality should be considered as physical requirements since Einstein equation allows solutions which are incompatible with relativistic causality (see also §3.2 & §3.3).

For a constant metric, there is no gravity since all Christoffel symbols $\Gamma^a{}_{bc}$ are zero (see §3.1). Then, in a nonrotating free falling, the velocity of an observer is a constant. According to special relativity, this observer carries with himself a new coordinate system which is obtained by a Lorentz transformation. But, a Lorentz transformation does not transform a constant metric (if it is not the flat metric) to a local Minkowski space. In other words, for a non-Minkowski constant metric, there is no physical free falling which relates to a local Minkowski space. Thus, ***the flat metric is the only physical constant metric*** and the principle of equivalence is incompatible with Galilean transformations. Moreover, there are manifolds for which there is no coordinate transformation such that the equivalence principle is globally satisfied in the transformed space (see §3). This means that it is not generally valid to consider light speeds in terms of local spaces.

The current notion of gauge is based on diffeomorphism (one-one, onto, infinitely differentiable C^∞ map with a C^∞ inverse between two manifolds) [7] in mathematics. Since no assurance is given for the validity of the equivalent principle, such a transformation may not lead to a ***physical*** coordinate system. The Galilean transformation is an example. Consequently, the covariance of physical laws should be considered only among physical coordinate systems but not all the mathematical coordinate systems (see also §3.3).

3. General Relativity and its Theoretical Framework.

In special relativity, laws of nature are described in terms of an inertial coordinate system, just as in Newton's theory. This is unsatisfactory in principle since, in reality, an inertial coordinate system exists only as an idealization. Therefore, laws of nature must be describable in terms of a non-inertial system and should be invariant under physical coordinate transformations (principle of general relativity).

On the other hand, special relativity is incompatible with Newton's gravity which requires action at a distance. To satisfy special relativity, as pointed out by Einstein [1], "the simplest thing was, of course, to retain the Laplacian scalar potential of gravity, and to complete the equation of Poisson in an obvious way by a term differentiated with respect to time." However, such a potential of gravity has difficulty with the experimental fact that all bodies have the same acceleration in a gravitational field. Then, Einstein realized that the equality of inertial and gravitational mass must lie the key to a deeper understanding of inertia and gravitation. Then, he formulated the principle of the equality of inertial and gravitational mass as follows: In a homogeneous gravitational field all motion takes place in the same way as in the absence of a gravitational field in relation to a uniformly accelerated coordinate system. Thus, gravity is related to a non-inertial frame.

These considerations lead to Einstein's field equation for gravity. It should be noted, however, its solution may not necessarily be compatible with the principle of equivalence (see §2). Moreover, for some cases, it is possible that an Einstein equation may *not* even have any *physical* solution (see §3.2 and §3.3).

3.1. Equivalence Principle, Metrics, and Equations of Motion

For a non-inertial system, due to non-linear transformations, equation (2.5) takes the general form,

$$ds^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu, \tag{3.1}$$

where metric $g_{\mu\nu}$ is a function of x^μ. Then, it follows from Einstein's principle of equivalence that the equation of motion for a particle should be associated with the geodesic equation in a Reimannian space,

$$\frac{d^2x^\alpha}{d\lambda^2} + \Gamma^\alpha{}_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} = 0, \quad \text{where } \Gamma^\alpha{}_{\mu\nu} = \frac{g^{\alpha\beta}}{2}\left(\frac{\partial g_{\beta\mu}}{\partial x^\nu} + \frac{\partial g_{\beta\nu}}{\partial x^\mu} - \frac{\partial g_{\mu\nu}}{\partial x^\beta}\right). \tag{3.2}$$

Eq. (3.2) satisfies the equality of inertial and gravitational mass, and can be compatible with the principle of equivalence since, at any point, the Christoffel symbols can be tranformed to zeros through a mathematical transformation to a local Minkowski space. However [32], "As far as the propositions of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality." Physically, the principle of equivalence requires that such a local Minkowski space can be obtained through a non-rotating free falling system [2]. (This principle also restores the constancy of light speed in a local system.) Thus, the satisfaction of the equivalence princple may not be *globally* valid for a manifold although the satisfaction of this principle is automatic for a local Minkowski space (see §2 and also §3.3).

3.2. Einstein's Field Equation and Gauge

Now, it is necessary to find a field equation to determine the metric. From eq. (3.2), for weak gravity and slow motion, g_{tt} is identified with the Newtonian potential of gravity. The correspondence principle and the principle of relativity indicate the ***approximate*** validity of the following equation,

$$\eta^{ab}\partial_a\partial_b\, g_{\mu\nu} = -\, 2K\; T(m)_{\mu\nu}, \tag{3.3a}$$

where $T(m)_{\mu\nu}$ is the energy-stress tensor of massive matter, and $K = 4\pi G$ (G is the Newtonian gravity coupling constant). Then, the principle of covariance suggests the following equation,

$$R_{\mu\nu} = -\, K\; T(m)_{\mu\nu}, \tag{3.3b}$$

where $R_{\mu\nu}$ is the Ricci curvature tensor. However, eq. (3.3b) is not compatible with the conservation law, $\nabla^\mu T(m)_{\mu\nu} = 0$. This physical requirement and $\nabla^\mu G_{\mu\nu} \equiv 0$ lead to the current Einstein equation,

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu}\, R = -\, K\; T_{\mu\nu}\ . \tag{3.4}$$

Note that if $T_{\mu\nu}$ has not been determined, any metric could be considered as a solution [8]. Obviously, the validity of eq. (3.4) depends on the appropriateness of $T_{\mu\nu}$ (see also §4 and §5). But, there is no principle to determine an exact $T_{\mu\nu}$ directly [21]. Thus, a solution should be examined with physical principles.

Hilbert [33] shows that the solutions of equation (3.4) are not unique. Thus, there are certain freedom in the choice of coordinate system. Before a choice restriction due to the principle of equivalence is recognized [6], it was believed that the choice of a coordinate system is completely arbitrary. The condition for such a choice is called a gauge. An often used gauge is the harmonic coordinate condition [34],

$$\frac{\partial}{\partial x^a}\left(|g|^{1/2} g^{ab}\right) = 0. \tag{3.5}$$

However, condition (3.5) may not be compatible with the principle of equivalence (see §3.3).

3.3. Velocity of Light, Relativistic Causality, and Physical Coordinates

Einstein [35] pointed out that "The principle of inertia and the principle of the constancy of the velocity of light are valid ***only*** with respect to an inertial system." A velocity component with respect to a space coordinate is that the difference of the space coordinate at different times is divided by the difference of time at different places. Then a light speed is determined by components of the space-time metric since the light-cone condition is invariant. To circumvent physical coordinates, some relativists "define" the light speed at any point of a manifold in terms of a local Minkowski space. However, such a "definition" is ***not well-***

defined since there are manifolds which cannot be transformed to a physical space. This "definition" is also ***misleading in physics*** since satisfying the equivalence principle must be implicitly assumed (see also §2). Moreover, a speed in a local coordinate system is not a speed in the physical space.

Since the equivalence principle implies relativistic causality, it can be used as a criterion. Obviously, that a speed of light is smaller than or equal to c (the light speed in a vacuum) may not be valid for some diffeomorphic manifolds. ***Nevertheless, the principle of covariance is compatible with relativistic causality since its violation means that the choice of coordinates is not valid in physics.***

It is not difficult to see that relativistic causality is satisfied by the Schwarzschild solution [2],

$$ds^2 = (1 - \frac{C}{r})dt^2 - (1 - \frac{C}{r})^{-1}dr^2 - r^2 d\Omega^2, \tag{3.6a}$$

where C (= 2M) is a positive constant, $d\Omega^2 = (d\theta^2 + \sin^2\theta\, d\phi^2)$, and (r,θ,ϕ) are spherical coordinates. Thus, the light speeds in the r-direction and θ-direction are respectively,

$$\frac{dr}{dt} = \pm (1 - \frac{C}{r}), \quad \text{and} \quad \frac{rd\theta}{dt} = \pm (1 - \frac{C}{r})^{1/2} . \tag{3.6b}$$

Eq. (3.6b) shows that, due to gravity, light speeds are slower.

One might argue on the ground that, based on the simultaneous distance and the local time, light speeds remain to be ± 1. However, from metric (2.6), for a light ray in the z'-direction, the simultaneous distance is determined by $ds^2 = -dz'^2$, and the local time is determined by $ds^2 = (c^2 - v^2)dt^2$. If such a z'-directional light speed were ± c, then one obtains $ds^2 = 2vdt'dz' \neq 0$. For a light ray in the x'-direction, althought light cone condition is not violated, one would still obtain the unphysical relation that $dx'^2/dt^2 = c^2 - v^2$.

In the literature [2,8], there are non-trivial metric solutions which do not satisfy relativistic causality and/or have other problems in physics. For example, an accepted metric [36] is as follows,

$$ds^2 = du\, dv + h_{ij}(u)x_i x_j\, du^2 - dx_i\, dx_i \,, \tag{3.7}$$

where $u = t-z$, $v = t+z$ (the light speed in a vacuum is denoted as 1), $h_{ii}(u) \geq 0$, and $h_{ij} = h_{ji}$ $(i,j = 1,2)$. This metric satisfies the harmonic gauge (3.5). It should be noted, however, that metric (3.7) depends on x (= x_1) and y (= x_2), and is not weak nor bounded although the cause of metric (3.7) can be an electro-magnetic z-directional plane wave. Moreover, metric (3.7) satisfies

$$\eta^{ab}\partial_a\partial_b\, \gamma_{tt} = -2[h_{xx}(u) + h_{yy}(u)], \quad \text{and} \quad \gamma = \eta^{cd}\gamma_{cd} = 0, \tag{3.8}$$

where γ_{ab} is the deviation from η_{ab}. Thus, metric (3.7) is rather arbitrary as the similar case derived by Peres and Bonnor [37]. It will be shown that metric (3.7) is ***not*** compatible with relativistic causality.

Metric (3.7) and similar metrices [37] have a general form as follows:

$$ds^2 = du\,dv + Hdu^2 - dx_i\,dx_i\,. \tag{3.9a}$$

A light trajectory satisfies $ds^2 = 0$. For a light in the z-direction (i.e. $dx = dy = 0$), one obtains

$$ds^2 = du\,dv + Hdu^2 = 0, \quad \text{and} \quad \frac{dz}{dt} = -\frac{1+H}{1-H} \tag{3.9b}$$

is the backward light speed while the forward light speed is 1. Then, ***relativistic causality*** requires,

$$H \leq 0. \tag{3.10}$$

Obviously, condition (3.10) is not satisfied by metric (3.7). Moreover, the gravitational force is related to $\Gamma^z{}_{tt} = (1/2)\partial H/\partial t$. There are arbitrary parameters (the choice of origin) which are not related to the cause (an electromagnetic plane wave). This violation of the principle of causality implies that ***it is impossible to transform metric (3.7) to a physical one.*** It will be shown in next section that there is no physical solution for an electromagnetic plane wave if the wave energy-stress tensor is the only source term.

3.4. Remarks on Theoretical Framework and Implicit Assumptions.

The general theory of relativity is based on the principle of equivalence. This principle, as a physical requirement, not only implies that the geodesic equation is a equation of motion, but determines the validity of a metric and possibly the source. Since this principle also implies relativistic causality, it can be used to examine the validity of a metric, and therefore a gauge. Another general physical requirement is the principle of causality (see Appendix A). In Einstein's field equation (3.4), only the left hand-side is determined. For massive matter, the right hand-side source term should be compatible with the principle of correspondence. These are the theoretical framework of relativity supported by experiments. However, since gravity depends on the form of the energy, the appropriateness of the source term may remain to be an issue (see §4 & §5).

The incorrect beliefs identified so far are: the arbitrariness of a coordinate system and consequently ***any*** metric with a proper signature would be incorrectly considered as physically valid. Einstein's postulate [38] is "Natural laws are to be formulated in such a way that their form is identical for coordinate systems of any kind of states of motion". Thus, arbitrariness is among ***physically realizable*** systems. Moreover, the coordinates in the Schwarzschild metric are measurable, since their validity has been verified by experiments [2,3,7]. An implicit assumption is the fact that the extension of Newtonian universal coupling has not been justified since general relativity implies that mass and energy are not equivalent (see §2). As pointed out by Pauli [9], in principle, general relativity allows to have different coupling constants even with ***different signs.*** The other implicit assumptions related to the source, shall be identified later (see §4, §5 and §6).

4. Gravity of Electromagnetic Plane Waves and Anti-Gravity Coupling.

The validity of an electromagnetic wave energy-stress tensor as the only tensor in the source should be examined because of duality. In general relativity, duality was used in the calculation of the star light deflection where light is considered as consisting of massless particles, photons [2]. Naturally, one may ask whether duality should be considered in Einstein's field equation, which would include the equation of motion for massive particles [39]. To be more specific, is there a tensor for photons as part of the source tensor?

Electromagnetic waves and photons are inseparable. It is therefore conjectured that for some cases, without a photon tensor, the Einstein's field equation may not have a *physical* solution. (It should be noted that although a variational principle assures mathematical compatibility [23], it does not ensure the existence of a physical solution.) Here, we consider a simple case when the source is the energy-stress tensor of an electromagnetic plane wave. Our conjecture is supported by the fact, as shown in §2, that the solutions obtained by Peres and Bonner [37] are not physical because the principle of equivalence is violated.

The principle of causality implies that an electromagnetic plane wave would generate an accompanying gravitational plane wave (see Appendix A). On the other hand, since the time average of the related G_{tt} is positive, there is no physical solution unless another tensor is subtracted from the source (see §4.3). This means that the ***anti-gravity coupling***, which was considered by Pauli [9] as a possibility, should be a necessary feature. Moreover, a physical solution requires that, in the flat metric approximation, this unknown tensor, on the time average, is the same as the electromagnetic energy-stress tensor. Thus, this unknown tensor would satisfy the condition for an energy-stress tensor for photons as required by quantum theories.

Based on that photons travel along a geodesic and other physical considerations, a photon tensor is obtained for monochromatic waves. Then, physical solutions are indeed obtained for different polarizations. Thus, the anti-gravity coupling for photons is confirmed theoretically. Naturally, this leads to the conjecture that a pure gravitational radiation (or gravitons) would also have an anti-gravity coupling. Surprisingly, this "conjecture" has actually been confirmed experimentally by Hulse and Taylor [14] 20 years ago (see §5)!

4.1. Duality and Causality

Let us consider a ray of electromagnetic waves propagating in the z-direction. Within the ray, one can assume a strong cylindrical condition. Thus, as in the literature [2,8], the electromagnetic potentials are:

$$A_k = A_k(t - z) \ . \tag{4.1}$$

Due to the principle of causality (see Appendix A), the metric g_{lk} is functions of u (= t−z), i.e.

$$g_{lk} = g_{lk}(u) \ . \tag{4.2}$$

Because the momentum of the photon P^k is in the z-direction and the photon is massless, one obtains

$$g_{tt} + 2g_{tz} + g_{zz} = 0, \quad g_{zx} + g_{tx} = 0, \text{ and } \quad g_{zy} + g_{ty} = 0 \tag{4.3a}$$

Eq. (4.3a) can be considered as a part of the harmonic gauge condition, and are equivalent to

$$g^{xt} - g^{xz} = 0\ , \quad g^{yt} - g^{yz} = 0, \text{ and } \quad g^{tt} - 2g^{tz} + g^{zz} = 0. \tag{4.3b}$$

Note that equation (4.3b) is equivalent to the transverse condition [22],

$$P^m g_{mk} = P_k = 0\ , \tag{4.3c}$$

for $k = x$, y, and v ($\equiv t + z$). The transversality of an electromagnetic wave would imply

$$P^m A_m = 0\ , \text{ or equivently } \qquad A_z + A_t = 0\ . \tag{4.4}$$

Eqs. (4.2) to (4.4) imply that not only the geodesic equation, the Lorentz gauge, but also Maxwell's equation are satisfied. Moreover, the Lorentz gauge becomes equivalent to a covariant expression.

The above analysis suggests also that an electromagnetic plane wave can be an exact solution. The scalar $\int P_m dx^m$ would equal to $P_m x^m$ in a coordinate system where P_m are constants. Then, $P_m x^m$ represents a scalar even though the space is not flat. Now, obviously eq. (4.3c) is necessary due to eq. (4.1).

4.2. The Reduced Field Equation

Then, eq. (4.2) and eq. (4.3) reduce the field equation to a single equation,

$$R_{tt} \equiv -\frac{\partial \Gamma^m{}_{tt}}{\partial x^m} + \frac{\partial \Gamma^m{}_{mt}}{\partial t} - \Gamma^m{}_{mn}\Gamma^n{}_{tt} + \Gamma^m{}_{nt}\Gamma^n{}_{mt} = -K\, T(E)_{tt} = K\, g^{mn} F_{mt} F_{nt}, \tag{4.5}$$

where F_{lk} is the electromagnetic field tensor. Note that eq. (4.5) is compatible to $R_{tt} = - R_{tz} = R_{zz}$, and the other components are zero. Then, eq. (4.5) is simplified to a differential equation of u as follows:

$$G'' - g_{xx}' g_{yy}' + (g_{xy}')^2 - G'\,[(G'/2G) + (g_t'/g_t)] = 2K\,(F_{xt}{}^2 g_{yy} + F_{yt}{}^2 g_{xx} - 2F_{xt}F_{yt}\, g_{xy}), \tag{4.6a}$$

where

$$G \equiv g_{xx}\, g_{yy} - g_{xy}{}^2, \quad \text{and} \quad g_t \equiv g_{tt} + g_{tz}\ . \tag{4.6b}$$

The metric elements are not necessarily independent since they are connected by the following relation:

$$-g = G\, g_t{}^2\ , \quad \text{where} \quad g = |g_{lk}| \tag{4.7}$$

is the determinant of the metric. If g is a constant, the metric shall be called *semi-unitary*.

Equations (4.3), (4.4), (4.6), and (4.7) allow A_t, g_{xt}, g_{yt}, and g_{zt} to be set to zero (or equivalently $g_{uk} = 0$ for k = x, y, u). These orthogonal conditions are valid because there is no physical reason to suggest otherwise. In any case, these assigned values have little effect in subsequent calculations. Note that equation (4.3) requires only $g_{tt} + g_{zz} = 0$. This allows g_{tt} to have a wave component.

Now, there are four metric elements (g_{xx}, g_{xy}, g_{yy}, and g_{tt}) to be determined. However, there is only one differential equation (since equation (4.7) is not really an equation if g is not specified by other means). Nevertheless, to show that there is no physical solution, equation (4.6) is sufficient.

4.3. Necessary Physical Conditions and the Source Tensor.

It will be shown that there is no physical solution for eq. (4.6) by using the required periodic nature of the metric due to causality (see Appendix A). Concurrently, it is shown that the curvature tensor, on the time average, is necessarily non-negative for a plane wave.

In the subsequent derivations, weak gravity is assumed. From the viewpoint of physics, this assumption is not a limitation because it is difficult to imagine that a strong gravitational wave can exist, but a weak one cannot. For simplicity, it is assumed also that the wave components are essentially monochromatic.

For clarity on the order of each term, the deviations γ_{lk} ($\equiv g_{lk}-\eta_{lk}$) are used. Let us consider

$$\gamma_{xx} + \gamma_{yy} \equiv - f = - (f_0 + f_1) \ . \tag{4.8a}$$

where f_0 is the time average of f over a multiple of periods. Then, one has

$$G' = f_1' - [f_1'\gamma_{xx} + f\gamma_{xx}' + 2\gamma_{xx}\gamma_{xx}' + 2\gamma_{xy}\gamma_{xy}'] \ , \tag{4.8b}$$

and

$$G'' - g_{xx}'g_{yy}' + (g_{xy}')^2 = f_1'' - f_1''\gamma_{xx} - (f_1\gamma_{xx}')'$$
$$- [(2\gamma_{xx} + f_0)\gamma_{xx}'' + (\gamma_{xx}')^2 + 2\gamma_{xy}\gamma_{xy}'' + (\gamma_{xy}')^2]. \tag{4.8c}$$

Now, it is clear that in R_{tt} the only term which can be of the first order of deviations is f_1''. Note that eq. (4.8c) includes all terms (from the curvature tensor) in the lowest order equation.

To give an approximation for eq. (4.6), a periodic function $A(\omega u)$ is represented by a sum

$$A = A_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega u + \alpha_n) \ . \tag{4.9a}$$

It follows that

$$(A')^2 + 2AA'' = - \omega^2 \sum_{n=1}^{\infty} n^2 A_n{}^2/2 + 2A''A_0 + \omega^2 [F(A) - 2F_1(A)]. \tag{4.9b}$$

where

$$F(A) = \sum_{m\neq n}^{\infty} [mn\, A_m A_n \sin(m\omega u + \alpha_m)\sin(n\omega u + \alpha_n)] - \frac{1}{2}\sum_{n=1}^{\infty} [n^2 A_n^2 \cos 2(n\omega u + \alpha_n)] ,$$

and

$$2F_1(A) = \sum_{m\neq n}^{\infty} [(m^2+n^2) A_m A_n \cos(m\omega u+\alpha_m)\cos(n\omega u+\alpha_n)] + \sum_{n=1}^{\infty} [n^2 A_n^2 \cos 2(n\omega u+\alpha_n)] .$$

Note that, in eq. (4.9b), the constant terms are negative, and the time average of other terms are zero.

Now, consider the case that f_1'' is of the first order of deviations. Then the first order equation is

$$f_1'' = - K\omega^2\{ \sum_{n=1}^{\infty} n^2 [A_{xn}^2 + A_{yn}^2] + 2[F(A_x) + F(A_y)]\}. \tag{4.10a}$$

Thus, it follows from equation (4.9b) that the solution of (4.10a) is not physical unless there is no electromagnetic wave. If f_1'' is of the second order of deviations, then the second order equation is

$$f_1'' = - K\omega^2\{ \sum_{n=1}^{\infty} n^2 [A_{xn}^2 + A_{yn}^2] + 2[F(A_x) + F(A_y)]\}$$

$$+ (2\gamma_{xx} + f_0)\gamma_{xx}'' + (\gamma_{xx}')^2 + 2\gamma_{xy}\gamma_{xy}'' + (\gamma_{xy}')^2 . \tag{4.10b}$$

From eqs. (4.9b), one can easily see that the constant terms in (4.10b) have the same sign. Consequently, f_1 cannot have a physical solution unless there are no electromagnetic waves.

The above calculation also gives a necessary condition for a physical solution. The time average of the source stress tensor must be negative and of the second order of deviations. If $(\gamma_{xx}+\gamma_{yy})'$ is of the second order, this is obvious. If $(\gamma_{xx}+\gamma_{yy})'$ is of the first order, the time average of the source stress tensor remains the same sign and order because the first order of the time average must be zero. Thus, the constant terms of the first order in the stress tensor for electromagnetic waves and in an unknown tensor must cancel each other. In terms of physics, this means, ***in the flat metric approximation, an electromagnetic wave and the unknown source tensor carry, on the time average, the same energy-momentum.***

4.4. Anti-Gravity Coupling and the Photon Tensor.

To verify the conjecture that a non-physical field equation is due to an inappropriate source tensor, one must find the photon tensor for electromagnetic plane waves. Let us assume that the source is

$$T_{\mu\nu} = T(E)_{\mu\nu} + CT(P)_{\mu\nu} , \tag{4.11a}$$

where $T(E)_{\mu\nu}$ and $T(P)_{\mu\nu}$ are the stress tensors for the electromagnetic wave and the related photons, and C is a constant. Since both $T(E)_{\mu\nu}$ and $T_{\mu\nu}$ are divergence free and traceless, $T(P)_{\mu\nu}$ is also divergence free and traceless. Moreover, the photons move along a geodesic. Since there is very little interaction, if any, among photons of the same ray, one may assume a dust-like model, $T^{ab}(P) = \rho P^a P^b$, for a monochromatic wave. The scalar ρ is a function of u, and should be a non-zero function of the electromagnetic potentials and/or fields. This implies $\rho(u) = \lambda\ A_m g^{mn} A_n$, where λ is a constant to be determined. Note that $\rho(u)$ is Lorentz gauge invariant because of eqs. (4.2) and (4.3). Also, the geodesic equation, $P^a \nabla_a P^b = 0$, is implied by $\nabla_a(\rho P^a) = 0$, and $\nabla_a T(P)^{ab} = 0$. In classical theory, light intensity is proportional to the square of the wave amplitude. Thus, ρ can be considered as the density function of photons if $\lambda = -1$. In anticipation of an anti-gravity coupling, one may assume $C = -1$ in eq. (4.11a), and obtain

$$T_{\mu\nu} = T(E)_{\mu\nu} - T(P)_{\mu\nu} = T(E)_{\mu\nu} - \lambda\ A_m g^{mn} A_n\ P_\mu P_\nu. \tag{4.11b}$$

Thus, a photon tensor changes nothing in the calculation, but only gives another term for eq. (4.6).

To determine λ, let us consider a circularly polarized monochromatic electromagnetic wave,

$$A_x = \frac{1}{\sqrt{2}} A_0 \cos \omega u\ , \text{ and } \ A_y = \pm\frac{1}{\sqrt{2}} A_0 \sin \omega u\ . \tag{4.12a}$$

Then, we have (see Appendix A)

$$T_{tt} = \frac{1}{2G}\omega^2 A_0{}^2 [(1 + C)(1 + \lambda) - (1 - \lambda) B_\alpha \cos\alpha] \leq 0. \tag{4.12b}$$

Eq. (4.12b) requires that $\lambda \leq -1$ because the constants C and B_α are much smaller than 1. As shown in §4.3, causality requires that, in a flat metric approximation, the time average of T_{tt} is zero. This implies that

$$\lambda = -1, \text{ and } \ T_{tt} = -\frac{1}{G}\omega^2 A_0{}^2 B_\alpha \cos\alpha \leq 0\ , \text{ and } \ B_\alpha = \frac{K}{2} A_0{}^2 \cos\alpha. \tag{4.13}$$

where α is the phase difference between the electromagnetic and the gravitational waves. Thus, ***pure electromagnetic waves can exist*** since $\cos\alpha = 0$ is possible.

To confirm the validity $\lambda = -1$, consider a wave linearly polarized in the x-direction,

$$A_x = A_0 \cos\omega(t - z)\ . \tag{4.14a}$$

Then, one has

$$T_{tt} = \frac{g_{yy}}{2G}\ \omega^2 A_0{}^2 \{(-\lambda - 1) + (1 - \lambda)\cos[2\omega(t - z)]\}. \tag{4.14b}$$

Thus, the flat metric approximation again requires that $\lambda = -1$. Then,

$$T_{tt} = \frac{g_{yy}}{G}\ \omega^2 A_0{}^2 \cos[2\omega(t - z)]. \tag{4.14c}$$

Eq. (4.14c) implies that the related Einstein tensor and $(g_{xx} + g_{yy})'$ are of first order of deviations. Thus, its polarization has to be different. One may expect that the time average of T_{tt} is also negative.

4.5. Unified Polarizations and Physical Solutions.

If a circularly polarized electromagnetic plane wave results in a circularly polarized gravitational wave, one may expect that a linearly polarized electromagnetic plane wave results in a linearly polarized gravitational wave. From the viewpoint of physics, it would be meaningful to require that, for an x-directional polarization, gravitational components related to the y-direction, remains the same. In other words,

$$g_{xy} = 0 \ , \quad \text{and} \quad g_{yy} = -1 \ . \tag{4.15a}$$

Mathematically, condition (4.15a) is compatible with semi-unitary (i.e. g is a constant, see Appendix B). Equation (4.15a) means that the gravitational wave is also linearly polarized. In the literature [2,8,36], there are other proposals. However, they all lead to unphysical solutions (see §3 and Appendix B).

It follows that equation (4.6) becomes

$$G'' = -2\,K\,G\,T_{tt}, \quad \text{and} \quad G = -g_{xx} \ . \tag{4.15b}$$

Then, the general solution for equation (4.15) is:

$$-g_{xx} = 1 + C_1 - \frac{K}{2} A_0{}^2 \cos[2\omega(t - z)], \quad \text{and} \quad g_{tt} = -g_{zz} = \sqrt{\frac{-g}{-g_{xx}}} \ , \tag{4.16}$$

where C_1 is a constant. Note that the frequency ratio is the same as the case of a circular polarization. For a polarization in the diagonal direction of the x-y plane, the solution is:

$$g_{xx} = g_{yy} = -1 - C_1/2 + \frac{K}{4} A_0{}^2 \cos[2\omega(t - z)] \ , \tag{4.17a}$$

$$g_{xy} = -C_1/2 + \frac{K}{4} A_0{}^2 \cos[2\omega(t - z)] \ , \tag{4.17b}$$

$$g_{tt} = -g_{zz} = -g/\{1 + C_1 - \frac{K}{2} A_0{}^2 \cos[2\omega(t - z)]\}^{1/2} \ , \tag{4.17c}$$

Note that for a perpendicular polarization, the metric element g_{xy} changes sign. Solutions (4.16) and (4.17) imply that linear superposition of electromagnetic waves is only approximately valid. The time averages of their T_{tt} are also negative as required. If $g = -1$, relativistic causality requires $C_1 \geq K A_0{}^2/2$.

If the photon tensor were absent (i.e. $\lambda = 0$), then the solution of equation (4.15) could have been

$$- g_{xx} = 1 + C_1 - \frac{K}{4} A_0^2\{2\omega^2(t - z)^2 + \cos[2\omega(t - z)] + C_2 (t - z) , \qquad (4.18)$$

where C_1 and C_2 are constants. Solution (4.18) is not physical because the term $(t - z)^2$ grows very large as time goes by. Thus, $T(E)_{tt}$ has a time limit zero, and therefore disagrees with special relativity.

For a circularly polarized electromagnetic wave, the phase difference controls the amplitude of the gravitational wave (see eq. (4.15)). This is different from the case of linearly polarized waves. Also, in both cases, there is a small constant C_1 to be determined. Nevertheless, this constant C_1 would not affect the gravitational force, and thus, the forces related to gravitational waves (4.16) and (4.17) can be compared with measurements. In particular, the frequency ratio would make such measurements easier. Thus, this calculation can be experimentally verified. Also, these formulas provide a theoretical basis to measure more directly the gravitational coupling constant for the electromagnetic energy-stress tensor.

Eq. (4.15a) can be interpreted as the transverse metric components are subjected to another constraint related to duality and invariance of polarization. Note that both the momentum P^k and the conjugate momentum P_k are in the z-direction. One may conjecture that an electromagnetic plane wave A_k and the corresponding contravariant electromagnetic potential A^k have the same plane of polarization. In other words, the ratios among their corresponding components are the same. Then one has the following additional equation:

$$(\alpha^2-\beta^2)g_{xy}=\alpha\beta(g_{xx}-g_{yy}), \quad \text{where } A_x=\alpha A,\ A_y=\beta A, \quad \text{and} \quad |\alpha|^2+|\beta|^2=1, \qquad (4.19)$$

where α and β are constants, and A is a periodic function of u. Equation (4.19) is equivalent to that, for an electromagnetic wave linearly polarized in the x-axis, $g_{xy} = 0$. For a circularly polarized wave, one could extend equation (4.19) to complex waves. Then, an electromagnetic and the accompanying gravitational wave have similar connection between real and complex waves.

A semi-unitary condition simplifies equation (4.6) considerably since eq. (4.7) implies that the last two terms on the left-hand side of equation (4.6a) cancel each other. Then equation (4.6a) is reduced into

$$G'' - g_{xx}'g_{yy}' + (g_{xy}')^2 = 2K\{(F_{xt}^2 g_{yy} + F_{yt}^2 g_{xx} - 2F_{xt}F_{yt} g_{xy}) + GT(P)_{tt}\}. \qquad (4.20)$$

Note that equation (4.20) includes only transverse metric elments, and G" can be of first order of deviations.

4.6. Real-Complex "Wave-Duality" and Duality.

Here, it will be shown that a general photon tensor can be obtained from duality considerations.

In classical electrodynamics, it is well-known that a real wave is the real part of a complex wave because Maxwell's equation is linear. These waves satisfy related Maxwell's equations in which the source term of the real equation is the real part of the complex source. It seems that such a "wave duality" is only a mathematical convenience and that complex waves are mathematical auxiliaries. ***However, QED (quantum electrodynamics) suggests that this wave-duality should have a physical origin, particle-wave duality*** because complex wave functions must be used in a hermitian field operator.

If "wave-duality" indeed has a physical origin from duality, then wave-duality should also be valid in general relativity. Then, QED (in which there is no photon tensor) would imply that the real part of the complex electromagnetic energy-stress tensor $T(\tilde{E})_{lk}$ is the modified tensor (4.11), i.e.

$$\mathrm{Re}[T_{lk}(\tilde{E})] = T_{lk}(E) - T_{lk}(P). \tag{4.21}$$

Eq. (4.21) implies that, for a static electromagnetic field, $T_{lk}(\tilde{E})$ is real, and there is no photon. The validity of eq. (4.21) is verified for monchromatic plane waves. Since the imaginary part, $\mathrm{Im}[T_{lk}(\tilde{E})]$ may not be zero, from the view point of physics, $T_{lk}(\tilde{E})$ should satisfy a complex Einstein equation, i.e.

$$G_{lk}(\tilde{g}) = -K\, T_{lk}(\tilde{E}); \tag{4.22}$$

and the real part of a complex gravitational wave satisfies a modified real Einstein equation. This would mean that, ***whereas particle-wave duality is explicitly manifested in a real Einstein equation; duality is implicitly included in a complex Einstein equation.***

Eq. (4.22) is supported by the facts that eqs. (4.2) and (4.3) are valid for complex functions and that the geodesic equation and the generalized Maxwell's equation can be extended to a complex metric. Since eq. (4.20) has only transverse metric elements, one may expect that wave-duality is valid for those elements. However, since the metric is semi-unitary, one may expect wave-duality to be only approximately valid for g_{tt} and g_{zz}. In short, wave-duality is valid at least for weak gravity.

It is not difficult to see that wave-duality is valid for directionally polarized electromagnetic waves. For example, consider the case of an electromagnetic wave linearly polarized in the x-direction. The complex wave related to the real wave in (4.14a) is

$$A_x = A = A_0 \exp\{-i\omega(t - z)\} . \tag{4.23a}$$

The complex gravitational metric elements are:

$$-g_{xx} = 1 + C - \frac{K}{2} A^2 , \quad \text{and} \qquad g_{tt} = -g_{zz} = \sqrt{\frac{-g}{-g_{xx}}} , \tag{4.23b}$$

where C is a complex constant. These metric elements satisfy equations (4.2), (4.3), (4.7), (4.20) and the following differential equation

$$G'' = -2K(A')^2, \quad \text{where} \quad G = -g_{xx}. \tag{4.24}$$

Equation (4.24) is a special case of equation (4.22).

To further support wave-duality, one can calculate the case of circularly polarized electromagnetic waves. A circularly polarized electromagnetic complex wave would be

$$A_x = \frac{1}{\sqrt{2}} A, \quad \text{and} \quad A_y = \pm \frac{i}{\sqrt{2}} A. \tag{4.25a}$$

The case of circularly polarized real wave suggests that the gravitational complex wave is also circularly polarized. This can be realized by using equation (4.19) which implies $T(E)_{tt}$ in (4.22) is zero. Then, eq. (4.19) and eq. (4.22) imply that the gravitational complex wave is circularly polarized as follows:

$$g_{xx} = -1 - C + B, \quad g_{yy} = -1 - C - B, \text{ and} \quad g_{xy} = \pm iB, \tag{4.25b}$$

where

$$B = B_\alpha \exp\{-i[2\omega(t - z) + \alpha]\}, \; C = C_1 \pm iB_\alpha,$$

and

$$G = (1 + C)^2 = (1 + C_1)^2 - B_\alpha^2 \pm i2(1 + C_1)B_\alpha. \tag{4.25c}$$

Formula (4.25) indeed further confirms wave-duality.

Thus, it is confirmed that wave-duality has its origin from particle-wave duality. Duality is implicitly included in complex waves. This manifests general relativity and quantum theory could be inextricably related. Now, the photon tensor can be calculated easily from eq. (4.21).

4.7. Some Theoretical Considerations.

To examine the appropriateness of a perceived source tensor, one should choose a simple situation and then identify the related physical requirements. For the case of an electromagnetic plane wave, relativistic causality, the equivalence principle, and the correspondence principle are essentially passive requirements which can be used to check the validity of a solution. But, the principle of causality is an active requirement since it implies that the metric is a plane wave. This is consistent with the principle of correspondence which requires the flat metric to be a valid approximation for weak waves. Then, duality and the Einstein tensor imply that, for a circularly polarized plane wave, the metric is also circularly polarized. This gives us added confidence to the Einstein tensor G_{ab}.

Both the principle of equivalence and the principle of causality have been used as physical requirements. However, since G_{tt} for a circularly polarized gravitational plane wave is positive, validity of the Einstein equation is impossible unless ***there is another source tensor with an anti-gravity coupling.***

Now, the remaining question is what is a required photon tensor? There are three physical conditions that a photon tensor should satisfy. They are: i) It produces the null geodesic equation for the photons; ii) In the flat metric approximation, on the time average it should equal to the electromagnetic tensor as required by quantum theories; iii) The polarization of the resulting gravitational wave matches the electromagnetic polarization. Condition i) makes a photon tensor distinct from an electromagnetic energy-stress tensor. But, condition ii) requires them to be intimately related. Nevertheless, calculation shows that, for a physical solution, condition ii) must be satisfied. This strong confirmation leads to a photon tensor which can also satisfy condition iii). One cannot help feeling that nature has a way to made things work.

The necessary inclusion of a photon tensor demonstrates a connection between relativity and quantum theory. It is interesting to note that both general relativity and the concept of photon were proposed by Einstein. As shown, these two seemingly unrelated theories may actually be inextricably related.

The modified source in Einstein equation implies that an energy-stress tensor of photons consists of two parts. One part is associated with the electromagnetic wave, and the other part provides for space-time curvatures of the gravitational wave components. However, for a circularly polarized wave, the gravitational wave component can be zero. ***Thus, in general relativity, some forms of energy do not generate gravity.*** For electromagnetic waves, gravity is generated by only a very small portion of the total energy. This shows that there are intrinsic differences between general relativity and Newtonian gravity.

This new source form indicates that the radiation would reduce gravity. This suggests that gravitational radiatioin would also have an anti-gravity coupling. Consequently, for an energy-stress tensor $T(m)_{\mu\nu}$ of massive matter, the Einstein equation should be modified to the following form (see also §5):

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu}R = - K \left[T(m)_{\mu\nu} - t(g)_{\mu\nu}\right], \qquad (4.26)$$

where $t(g)_{\mu\nu}$ is the energy-stress tensor for gravity. Then, $\nabla^{\mu}G_{\mu\nu} \equiv 0$ implies both conservation laws,

$$\nabla^{\mu}T(m)_{\mu\nu} = 0 \qquad \text{and} \qquad \nabla^{\mu}\, t(g)_{\mu\nu} = 0, \qquad (4.27)$$

because of the sign difference between them. Thus, eq. (4.26) remains compatible with the equivalence principle. If gravity is generated by massive matter, then $k\, t_{\mu\nu}$ is expected to be of second order.

Eq. (4.26) further manifests that there are mechanisms which would reduce gravity (see also §6). As the intensity of gravity increases, the gravity energy-stress tensor also increases. Then the anti-gravity coupling is a feed back mechanism which would restrict the intensity of gravity. One should note also that the Schwarzschild solution as well as Newtonian theory, excludes effects due to the radiation and other interactions. Therefore, for a contraction due to very strong internal gravity, the effective mass M in the schwarzschild solution may not be invariant. Moreover, the gravitational energy and high pressure would trigger interactions which may not be possible otherwise. For example, as discussed above, the intensity of gravity would be considerably reduced if large amount of high energy radiation could be generated.

5. Gravitational Radiation and Modifications in the Einstein Equation.

General relativity suggests the existence of gravitational waves whose existence is due to phyiscal considerations which are independent of Einstein equation [40]. Although gravity waves have never been directly observed, there is indirect evidence which supports energy loss by gravitational radiation [7,14]. While Einstein's radiation formula is supported by the observed data [14], one should not consider this as a verification of Einstein's gravitational radiation theory because his theory does not produce the radiation formula in a self-consistent manner [7,15,17]. Instead, one should first identify the problems in its derivation and understand their theoretical implications. Accordingly, one may develop a theory to support the formula.

Here, it will be shown that his radiation formula has important implications. It is concluded that, because of radiation, the source tensor is necessarily non-zero in a vacuum. The gravitational energy-stress, as conjectured in §4, is indeed a tensor with an anti-gravity coupling. And Einstein's notion is only an approximation. Moreover, Einstein's radiation formula can be supported within the theoretical framework of general relativity. Concurrently, it is founded that linearized gravity is not justifiable in terms of mathematics alone [18]. This supports Einstein's [17] observation that linearized gravity is not reliable.

Einstein's formula is based on a gravity pseudotensor [9]. As such, it has been proven by Denisov et. al. [13] that his formula is not an invariant; and the rate of energy emission, depending on the choice of the coordinate system, may be positive, negative or zero. Thus, it seems, only a covariant theory can be self-consistent. Although a covariant theory would not produce exactly the same radiation formula, as far as agreements with data, it is sufficient to show that the rate of energy loss, on the time average, are the same.

5.1. Einstein's Radiation Formula and the Problem of Self-Consistence.

To develop a supporting theory, let us first identify the causes of inconsistence in the derivation. In terms of the deviations γ_{ab} ($= g_{ab} - \eta_{ab}$), Einstein equation (3.4) and gauge (3.5) are linearized to

$$G_{ab}{}^{(1)} = - KT(m)_{ab}, \quad \text{where} \quad G_{ab}{}^{(1)} \equiv \frac{1}{2}\partial^c\partial_c\bar{\gamma}_{ab} + H_{ab}{}^{(1)}, \tag{5.1}$$

where

$$H_{ab}{}^{(1)} = - \frac{1}{2}\partial^c[\partial_a\bar{\gamma}_{bc} + \partial_b\bar{\gamma}_{ac}] + \frac{1}{2}\eta_{ab}\partial^c\partial^d\bar{\gamma}_{cd};$$

and

$$\partial^a\bar{\gamma}_{ab} = 0, \quad \text{where } \bar{\gamma}_{ab} \equiv \gamma_{ab} - \frac{1}{2}\eta_{ab}\gamma, \text{ and } \gamma \equiv \eta^{ab}\gamma_{ab}. \tag{5.2}$$

The Linearized "gauge" (5.2) sufficiently reduces (5.1) to the linear equation,

$$\frac{1}{2}\partial^c\partial_c\bar{\gamma}_{ab} = - K\, T(m)_{ab}. \tag{5.3}$$

Note that linear eq. (5.3) is similar to eq. (3.3a). Thus, eq. (5.3) can be justified on physical considerations which are independent of lineaized gravity. It follows from eq. (5.2) and eq. (5.3) that exactly

$$\partial^a T(m)_{ab} = 0 \ , \tag{5.4}$$

Note that the linearized conservation law (5.4) is also implied directly by eq. (5.1) since $\partial^a G_{ab}^{(1)} \equiv 0$.

On the other hand, the effective gravity pseudotensor [7], to second order, is equivalent to

$$t_{ab} = G_{ab}^{(2)} /K, \quad \text{where} \quad G_{ab}^{(2)} = G_{ab} - G_{ab}^{(1)}. \tag{5.5}$$

Then, the rate of energy loss due to radiation is [7]

$$-\frac{dE}{dt} = \int t_{k0} dS^k = \frac{1}{K} \int G_{k0}^{(2)} dS^k, \qquad \text{where} \quad E = \int t_{00} d^3x + \int T_{00} d^3x \ . \tag{5.6}$$

To evaluate formula (5.6), one solves eq. (5.3) without using eq. (5.2), and obtain

$$\bar{\gamma}_{ab}(x^i,t) = -\frac{K}{2\pi} \int \frac{1}{R} T_{ab}[y^i,(t-R)]\, d^3y, \quad \text{where} \quad R^2 = \sum_{i=1}^{3} (x^i - y^i)^2. \tag{5.7}$$

In the far field from the source, eq. (5.7) can be approximated by using the lineanized conservation law (5.4) to establish relationship between different components of T_{ab}, and obtain

$$\bar{\gamma}^{jk} = -\frac{K}{2\pi} \frac{1}{r} \frac{\partial^2}{\partial t^2} \int T^{00} x^k x^j \, d^3x \tag{5.8}$$

Based on eq. (5.8), the rate of energy loss formula (5.6) becomes

$$-\frac{dE}{dt} = \frac{G}{45} (\dddot{q}'_{kj}\, \dddot{q}'^{kj}) \geq 0 \tag{5.9}$$

where q_{jk} is the quadrupole moment. Eq. (5.9) is the famed "quadrupole radiation" formula.

But, Einstein's theory is not self-consistent. As pointed out by Wald [7] and Yu [15] that the linearized conservation law eq. (5.4) implies that "two stars would not orbit each other but would move on geodesics of the flat metric." This means $\ddot{q}'_{jk}$ is zero and therefore no gravitational radiation. The usual formula for the rate of change of orbital period has been derived by assuming eq. (5.9) without reference to eq. (5.4) and the analysis by Peters and Mathews [41] is based on Newtonian orbits. That derivation is illegitimate as eq. (5.9) has been derived from eq. (5.4).

However, since both Einstein's equation (3.4) and eq. (5.9) are supported by experiments, understandably one would conjecture that this could be a matter of improving the approximation on eq. (3.4) since the objection is based on an approximate equation. But, for the problem of gravitational radiation, eq. (3.4) and eq. (5.3) are actually not compatible as noted by Einstein [17] in 1936 (see §5.2). Inevitably, all efforts based on improving the approximation methods are proven to be futile, and Damour [16] remarked that "nearly all aspects of approximation methods need to be thoroughly re-investigated." Nevertheless, it is clear that the radiation formula is intimately related to eq. (3.4). Therefore, one may ask can eq. (3.4) be modified to accommodate the radiation formula?

To have radiation, as point out by Wald [7], one must obtain a gravitational acceleration. From

$$0 = \nabla_a T(m)^{ab} = \partial_a T^{ab} + \Gamma^b_{ac} T^{ac} + \Gamma^a_{ac} T^{cb}, \tag{5.10}$$

one can see that, for a first order approximation of the metric, the conservation law is accurate to the second order. Thus, a first order approximation of the metric, would describe the gravitational radiation. Moreover, due to weak gravity, eq. (5.10) can replace eq. (5.4) in obtaining eq. (5.8). In this alternative derivation, the accuracy of $G_{ab}{}^{(2)}$, up to second order of deviations, remains the same. Thus, the linearized conservation law (5.4) is indeed not needed to obtain the radiation formula (5.9).

However, eq. (5.4) is implies by the linearized eq. (5.1). Moreover, although his radiation formula is based on the subsequent eq. (5.3), "gauge" (5.2) still implies eq. (5.4). Thus, Einstein's formula is not only independent of, but inconsistent with linearized gravity. Therefore, eq. (5.3) should be justifiable without using eq. (5.2) and eq. (5.1) (see §5.4). Then, analysis by Peters and Mathews would become valid. In next subsection, it will be shown that, eq. (5.3) is, in fact, incompatible with eq. (3.4) because of radiation.

5.2. Validity of Linearized Gravity and Einstein's Radiation Formula.

Linearized gravity is actually based on implicit assumptions: i) an Einstein equation $G_{ab} = -KT_{ab}$, has a physical solution; ii) in $G_{ab,}$ the sum of first order terms has the lowest order; and iii) the linear gravity equation provides an approximation for Einstein equation; iv) the gauge is valid for any physical problems.

The invalidity of assumption iv) has been proven in §2. If one believes the linearized gauge because of its similarity with the Lorentz gauge, he should note that classical gauge invariance has been proven to be incompatible with experiments [42,43]. The static dust model, which provides no balance to gravity, actually does not have a physical solution [18]. Nevertheless, from linear eq. (5.3), Newtonian gravity is obtained with the static dust model. To understand this, one must realizes that the dust model is a ***linearization*** of the perfect fluid model. In other words, Einstein equation and its related linear equation may necessarily have different source tensors. Note that eq. (5.3) and Einstein equation have different physical meanings (see also §5.4). Also, there are exact solutions for which assumption ii) is not valid [6,18].

It will be shown that assumption iii) may not be valid even if ii) is valid. Concurrently, this will also show that Einstein's radiation formula is incompatible with the current Einstein equation (3.4). To determine whether (5.7) is a valid approximation, let us write Einstein equation (3.4) alternatively,

$$\frac{1}{2}\partial^c\partial_c\bar{\gamma}_{ab} = -K\breve{T}_{ab}, \qquad (5.11)$$

where

$$\breve{T}_{ab} = T_{ab} + T'_{ab}\,, \quad \text{where} \quad T'_{ab} = (H^{(1)}{}_{ab} + G^{(2)}{}_{ab})/K\,.$$

Then, a formal solution would be

$$\bar{\gamma}_{ab}(x^i,t) = -\frac{K}{2\pi}\int\frac{1}{R}\breve{T}_{ab}[y^i,(t-R)]\,d^3y, \qquad (5.12)$$

If T_{ab} is non-zero only in a finite region, then it is not clear whether the contribution of T'_{ab} (which may be non-zero almost everywhere) is negligible although, for a static case, T'_{ab} would be negligible.

However, for *radiation*, the contribution of T'_{ab} diverges if one assumes that solution (5.7) is a valid approximation. Due to radiation [2,7,19], at large r,

$$H^{(1)}{}_{tt} + G^{(2)}{}_{tt} \sim O(\frac{1}{r^2})\,. \qquad (5.13)$$

The contribution of T'_{ab} to $\bar{\gamma}_{tt}(x^i,t)$ would be

$$D_{tt}(x^i,t) = -\frac{1}{2\pi}\int\frac{1}{R}[H^{(1)}{}_{tt}+G^{(2)}{}_{tt}]\,d^3y = -\frac{1}{2\pi}\,[\int_{r\leq a} + \int_{r>a}]. \qquad (5.14a)$$

Then,

$$\int_{r>a} \sim \int_{r>a}\frac{d\Omega}{r}(\frac{1}{r^2})r^2dr = 4\pi\int_{r>a}\frac{dr}{r}, \qquad (5.14b)$$

for large a and x^i near the origin. Thus, (5.14) may not be negligible if the source has been emitting waves long enough [2], and ***divergence would occur so long as the source is non-zero only in a finite region.*** This is also a problem in an alternative theory by A. Logunov and M. Mestvirishvili [13].

The above considerations imply that this problem of divergence cannot be removed by improving the approximation of eq. (3.4). The divergent contribution must be canceled by an additional source tensor $Kt(g)_{ab}$, which must be of second order and ***non-zero*** almost everywhere in vacuum. From the viewpoint of physics, $t(g)_{ab}$ should be the energy-stress tensor for gravity. Moreover, since the term (5.14b) has nothing to do with the emmision process [2], physically this term should not appear in the solution. Thus, Einstein equation (3.4) must be modified since eq. (5.3) is justified by his radiation formula.

5.3. Observation and Modifications in the Einstein Equation.

Since the existence of gravitational waves is independent of Einstein's equation [40], its modification is feasible. Since such waves should carry energy-momentum [44], one may expect that the source tensor in a vacuum should be non-zero. This means that a source tensor due to gravity energy-stress tensor must exist. Moreover, such a tensor should have an anti-gravity coupling since gravity should not be self-generating. Fortunately, the radiation formula precisely confirms these. For simplicity, let us assume, for the moment, that eq. (5.2) were valid. Based on eq. (5.3), the modification steps are as follows [19]:

1) $t_{ab}(g)$, the effective stress-energy tensor of the gravitational field, is actually a tensor. The assumption that $G_{ab}{}^{(1)} = 0$ in vacuum is equivalent to

$$G_{ab} = G_{ab}{}^{(1)} + G_{ab}{}^{(2)} = Kt_{ab}(g) \ . \qquad (5.15a)$$

Therefore, t_{ab} is actually a tensor although its approximation appears in eq. (5.5a) as a pseudotensor. This means that the covariant nature of general relativity is maintained.

2) The coupling of $t_{ab}(g)$ is anti-gravity. Eq. (5.15a) means that the factual assumption in vacuum is

$$T_{ab} = -t_{ab}(g) \ . \qquad (5.15b)$$

Eq. (5.15b) means that the tensor t_{ab} has an anti-gravity coupling as conjectured in §4.

3) Eq. (5.15a) and eq. (5.15b) imply that Einstein equation must be extended to the following form,

$$R_{ab} - \frac{1}{2} Rg_{ab} = - K[T_{ab}(m) - t_{ab}(g)] = - KT_{ab} \qquad (5.16a)$$

where $T_{ab}(m)$ is the stress tensor for massive matter and $t_{ab}(g)$ is for the field energy.

4) Eq. (5.16a) implies

$$\nabla^a T_{ab}(m) = 0, \quad \text{and} \quad \nabla^a t_{ab}(g) = 0. \qquad (5.16b)$$

Because of the difference in coupling signs, energy-momentum conservation requires that $T_{ab}(m)$ and $t_{ab}(g)$ are conserved separately. Also, this would be demanded by the principle of equivalence.

Note that $t_{ab}(g)$, being an energy-stress tensor, is not a geometrical part. Due to different theoretical considerations, there are competing theories [25,45,46,47] of which a second order non-matter term is present in the source. But, there was no anti-gravity coupling. What is new is that ***both the presence of $t_{ab}(g)$ and its anti-gravity coupling are necessary due to the Taylor-Hulse experiment*** [14].

But, eq. (5.2) is actually not valid. Then, according to (5.3) (see also §5.4), one obtains

$$K\ t(g)_{ab} \simeq G_{ab}{}^{(2)} + H_{ab}{}^{(1)}, \qquad (5.16c)$$

Eq. (5.16c) implies that eq. (5.6) would be modified. However, if the motion is periodic, on the time average, the tensor component t_{k0} (k = x, y, z) remains essentially $G_{k0}^{(2)}/K$ as assumed earlier. It will be shown in next subsection that the radiation formula can be derived from eq. (5.3).

5.4. Maxwell–Newtonian Approximation and Einstein's Radiation Formula

Physically, eq. (5.3) gives the direct influence of the massive source to the field. Whereas the right hand-side of eq. (5.16c) represents the field self-interaction; $t(g)_{ab}$ is the field energy-stress. Eq. (5.3) implies also that a gravity wave propagates with the speed of light. Given a particle moving along a geodesic, eq. (5.3) would be the natural extension from Newtonian theory. For clarity, eq. (5.3) is rewritten as,

$$\partial^c \partial_c \bar{\gamma}_{ab} \simeq -2KT_{ab}(m). \tag{5.17}$$

Note that linear eq. (5.17) is now an approximation of eq. (5.16). Obviously, eq. (5.17) is not covariant with respect to all physical coordinate systems. It is an approximation after the coordinate system has been chosen. The asymptotic flatness of the metric is the implicit gauge. Mathematically, eq. (5.17) is due to the necessary approximate cancellation of the second order terms, and therefore is not a simple linearization (see §5.2). For the case of an electromagnetic plane wave, eq. (5.17) is exact since $T(m)_{ab}$ = 0.

Moreover, eq. (5.17) is justified on its agreements with experiments [48–50]. For a static mass distribution, it produces Newton's law of gravity. For non-static case, it produces Einstein's radiation formula. To be distinct from linearized gravity, eq. (5.17) shall be called the ***Maxwell-Newtonian approximation.*** The validity of this approximation will be further tested in the Stanford Gyroscope experiment [21].

Now, it remains to show that eq. (5.17) provides the required approximation as follows:

i) For self-consistency, it is necessary that according to eq. (5.16), eq. (5.17) gives indeed a first order approximation. (Thus, $\nabla_a T^{ad}(m) = 0$ is satisfied to second order.)

ii) To support a radiation formula, eq. (5.17) must imply, to second order, $\partial_a t^{ad} \simeq 0$.

It follows from eq. (5.17) that

$$\frac{1}{2}\partial^c \partial_c L_b \simeq -K\partial^a T_{ab}(m), \qquad \text{where} \quad L_b = \partial^a \bar{\gamma}_{ab}. \tag{5.18}$$

Since $\nabla^a T(m)_{ab} = 0$, $K\partial^a T(m)_{ab}$ are second order of deviations. It follows from eq. (5.18) (or solution (5.7)) that L_b are also second order. This implies, from eq. (5.16), that up to first order of deviations, $\partial^c \partial_c \bar{\gamma}_{ab} = -2K[T_{ab}(m) - t_{ab}]$. Due to the required compatibility between eq. (5.17) and eq. (5.16), $t(g)_{ab}$ must be of second order and essentially cancel the other second order terms. Thus, i) is proven.

From eq. (5.17), lengthy but straight forward calculation shows, up to second order,

$$\partial_c G^{cd} \simeq [\frac{1}{2}\partial_c \gamma^{ab}\eta^{cd} - \frac{1}{2}\partial^a \gamma \eta^{bd} - \partial^a \gamma^{bd}]\,[-KT(m)_{ab}]. \tag{5.19a}$$

It follows from eq. (5.16) that eq. (5.19a) implies, to second order,

$$\partial_a t^{ad} \simeq \partial_a T(m)^{ad} + \Gamma^d_{ab} T(m)^{ab} + \Gamma^a_{ab} T(m)^{db} = 0. \tag{5.19b}$$

Thus, ii) is proven. Therefore, as required, the Maxwell–Newtonian approximation is physically valid.

Since $\partial^a G_{ab}{}^{(1)} \equiv 0$, eq. (5.16) implies

$$G_{ab}{}^{(2)}/K + T_{ab}(m) = t_{ab}(g) - G_{ab}{}^{(1)}/K \tag{5.20a}$$

and

$$\partial^a G_{ab}{}^{(2)}/K + \partial^a T_{ab}(m) = \partial^a t_{ab}(g). \tag{5.20b}$$

From eq. (5.20b), owing to $\partial^a t_{ab} \simeq 0$ up to second order, $\partial^a G_{ab}{}^{(2)}$ would relate mainly to the energy-momentum of matter as its source while, in a vacuum, $\partial^a G_{ab}{}^{(2)}/K$ is equal to $\partial^a t_{ab}(g)$. In other words, gravity energy and the motion of particles influence each other mainly through geometry.

It follows from eq. (5.20) that approximately

$$-\frac{dE}{dt} \simeq \int t_{a0}(g) dS^a = \frac{1}{K}\int G_{a0}{}^{(2)} dS^a - \frac{1}{2K}\int (\partial_0 L_a + \partial_a L_0) dS^a. \tag{5.21}$$

The second integral comes from $\partial^a G_{ab}{}^{(1)} \equiv 0$. Since such a relation is independent of the physical process, from the viewpoint of physics, the second integral is irrelevant. In fact, based on solution (5.8), the time average of the second integral is zero. Then, eq. (5.21) is reduced to eq. (5.6). Also, since t_{ab} is of second order, eq. (5.16) would maintain the agreements with previous experiments. Thus, this modification process is self-consistent and valid, and Einstein's radiation formula is unequivocally due to general relativity.

5.5. Gravitational Energy–Stress Tensor and the Principle of Equivalence.

Now, the verification of Einstein's radiation formula settles that the current Einstein equation is only a static approximation. Also, the implicit assumption that the source is zero in a "vacuum", is actually invalid. One may ask, however, whether $t(g)_{ab}$ is compatible with the equivalence principle. This question has already been answered by Einstein himself in 1954. In his article, ***'Relativity and the Problem of Space'***, Einstein [35] added the crucial phrase, "at least to a first approximation" on the indistinguishability between gravity and acceleration. Note that whereas a geodesic equation requires only first order derivatives of the metric; the Einstein tensor, which equals to $K\, t(g)_{ab}$ in a vacuum, requires second order derivatives.

Also, it is interesting to note that eq. (5.16a) can be considered as a modification of the suggestion of Lorentz and Levi-Civita, which Einstein rightfully rejected [9]. The modified field eq. (5.16) is not exactly completed, since only an approximation of $t_{ab}(g)$ can be obtained through eq. (5.17). Nevertheless, owing to this ***anti-gravity coupling*** is verified by experiment, its implication on space-time is important (see §6).

6. Anti-Gravity Coupling, Motion, and Space and Time.

Defiance of gravity captures human imagination because everything, except light, is earth-bounded. This universal attractiveness is manifested in Newton's theory in terms of a universal gravitational coupling constant. Unlike charges in electrodynamics, a negative mass is against Newton's third law of motion because the resistence to acceleration is also due to mass. Thus, an anti-gravity coupling is not possible. But, light is not subjected to Newtonian gravity. From observation, lights seem to able to ignore gravity, and light up the sky. Then, Einstein told us that anything, including light, moves along a geodesic. But, Pauli [9] pointed out that in general relativity an anti-gravity coupling is allowed. Thus, light may still be an exception.

In general relativity, gravity may not always be attractive. A gravitational force can be attractive for long distances, but is replusive for short distances as indicated by the Reissner-Nordstrom metric [2,7],

$$ds^2 = \left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right)dt^2 - \left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right)^{-1}dr^2 - r^2 d\Omega^2, \qquad (6.1)$$

where q and M are the charge and mass of a particle. Note that the gravitational force changes sign at $r = q^2/M$. (Such a short distance is possible for the electron.)

Moreover, since motion is described by geodesics, a classification in term of attractiveness and repulsiveness may not always be meaningful. Although the motion of a massive particle manifests the characteristics of an attactive force, the motion of a massless particle does not show the same characteristics. As shown by the Schwarzschild solution, the motion of a massive particle bends toward the center of attraction; and its energy, momentum, and speed all increase (decrease) as the particle gets nearer to (farther from) the center. But, photons which are massless, are different. While the energy of a photon increases when it gets nearer the center, its momentum does not; and its speed actually decreases (see eq. (3.6)). Thus, if anything has an anti-gravity coupling, it should be the photons; and for the same reason, other massless particles. It has been shown in §4 that the coupling of photons, indeed, has a different sign.

Since the source determines the space-time structure, any seemingly natural assumption on the source should be carefully analyzed against experiments. Being a theory which abandoned naive visualizations, if it is not based on analysis of the agreement between observational data and mathematical quantities in a theoretical formalism, general relativity can be a victim of over extrapolation. Weisskopf [51] commented that "The existence of black holes follows from an extrapolation of Einstein's theory of gravity by many orders of magnitude beyond the range for which its validity has not yet been established beyond doubt".

In this section, the couplings of energy-stress tensors are considered from the theoretical framework of general relativity and experiments [14,48]. It is concluded that the anti-gravity coupling is crucial in understanding general relativity, and the singularity theorems [7] are based on invalid physical assumptions. The "theoretical existence" of black holes is actually based on over extrapolation of inadequate modeling. Moreover, not only is there no compelling reason which leads to black holes; but recent theoretical developments suggest that the rejection [52] by Eddington and Einstein would be correct.

6.1. Anti-Gravity Coupling and Duality.

Einstein eq. (3.4) requires only that the source $T_{\mu\nu}$ is divergence free. Since a divergence free tensor can be added to the source without changing the equation of motion, the appropriateness of the source can be determined only from its solutions. Moreover, the sign and the numerical value of a coupling constant remain to be determined by experiment [9]. Nevertheless, energy-momentum conservation requires that different matters which share the same equation of motion should have the same coupling sign. Then, it follows that

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu}R = - K\, T_{\mu\nu} = - K\, [T(N)_{\mu\nu} - T(A)_{\mu\nu}], \tag{6.2a}$$

where $T(N)_{\mu\nu}$ and $T(A)_{\mu\nu}$ denote the energy-stress tensors of matter and anti-gravity matter. Since

$$T_{tt} = T(N)_{tt} - T(A)_{tt}, \quad \text{where} \quad T(N)_{tt} > 0 \text{ and } T(A)_{tt} > 0, \tag{6.2b}$$

the time-time component of $T_{\mu\nu}$ is no longer always positive (see §4 and §5).

Due to energy-momentum conservation, these two tensors separately satisfy

$$\nabla^{\mu}T(N)_{\mu\nu} = 0, \quad \text{and} \quad \nabla^{\mu}T(A)_{\mu\nu} = 0. \tag{6.3a}$$

However, eq. (6.3a) does not mean that these two classes of matter do not interact. They interact through the geometry of the space-time as follows:

$$\partial^{\mu}[G_{\mu\nu} + K\, T(N)_{\mu\nu}] = K\, \partial^{\mu}T(A)_{\mu\nu} \tag{6.3b}$$

Thus, all matters interact with each other through possibly different forms of interaction.

For example, in principle, an electron can interact with photons through eq. (6.3). However, since each term conserves separately in a local minkowski space, one sees an electron interacts with the associated electromagnetic field. Thus, anti-gravity coupling implies the necessary existence of duality. It would be possible that gravitational wave and the gravity energy-stress tensor could be a similar type of duality.

6.2. Anti-Gravity Coupling and Gravitational Radiation.

Since all massive matters interact with an electromagnetic field, matter with an anti-gravity coupling must be massless and neutral. Since radiation is related to massless particles, ***anti-gravity coupling should be associated with radiation*** which may include the neutrino. Therefore, in general,

$$T(A) = g^{\mu\nu}T(A)_{\mu\nu} = 0. \tag{6.4a}$$

and

$$T = g^{\mu\nu}T_{\mu\nu} = g^{\mu\nu}T(N)_{\mu\nu} \geq 0. \tag{6.4b}$$

It follows that, in eq. (5.16) the trace of the gravitational energy-stress tensor,

$$t(g) = g^{\mu\nu}t(g)_{\mu\nu} = 0. \tag{6.5}$$

To verify this relation with a special case, let us consider a circularly polarized gravitational plane wave. From Appendix A, the components of the Einstein tensor satifies $G_{tt} = G_{zz} = -G_{tz} > 0$, and other components are zero. Thus, eq. (6.5) is satisfied due to eq. (4.3b). The requirement of anti-gravity coupling is also verified because the time-time component of an energy-stress tensor is positive. In the literature [8], there are wave solutions which do not relate to an anti-gravity coupling. However, these solutions violate causality as well as the principle of equivalence (see also §3 and §4).

Moreover, physical considerations imply that there should not be any physical gravitational wave which satisfies the Einstein equation, $R_{\mu\nu} = 0$ for an empty space. In the literature [8], there is no such propagating wave solution. Nevertheless, Au, Fang, and To [53] recently obtained some soliton wave solutions for $R_{\mu\nu} = 0$. Their related gravitational energy-stress pseudotensor is non-zero. Although, these waves are not connected to a source, they seem to support the incorrect belief that energy could possibly be transported in spite of the source of Einstein's equation being zero in a vacuum. But, analysis shows that these solutions also violate the principle of equivalence. This illustrates that ***the necessary association of the anti-gravity coupling with a radiation is another profound consequence of the equivalence principle.***

6.3. Anti-Gravity Coupling and Singularity.

In physics, a singularity (or infinite) is usually either a convenient idealization such as the δ-function or an indication of imperfectness of the theory such as an renormalizable infinite in QED. However, in current cosmology, a mathematical existence of singularities in space-time is considered as the evidence for the "inevitable" collapse of a super star to a black hole or its reverse, the big bang beginning of the universe. Moreover, it was claimed that general relativity would inevitably break down, and therefore quantum theory is the ultimate theory. It will be shown that these singularities are actually due to inadequate modeling.

In 1922, the Russian mathematician Alexander Friedman solved Einstein's equation and came up with an expanding universe model. His basic assumption is that the universe is isotropic and homogeneous. In spite of his questionable highly idealized assumption, Friedman's model gained creditability because Hubble's observations can be interpreted as supporting an expanding universe [7]. Also, its extrapolation would mean that there was a singular epoch in the past in which all the matter of the universe was concentrated into a single point. This creation of the universe was not taken seriously because the real universe contains irregularities which might grow large and cause the individual particles converging to miss each other. In fact, most people thought that there was no beginning [54]. But, the theorems of Hawking and Penrose, which show the necessary existence of singularities under some very broad assumptions, made the Big Bang and black holes not just plausible but even necessary. However, given the overall complexity of the universe and their drastic conclusions, the validity of their assumptions should be examined.

Historically, theorists often have a tendency of over extrapolating the valid range of physical assumptions. Hence, it would be reasonable to assume that the physical assumptions of Hawking and Penrose agree with theories then and are compatible with experiments previous to 1965. Theoretically, gravity of radiation was still on the wrong track as shown in the work of Peres (1960) and Bonnor (1969). It is known [17] that Einstien's equation does not give gravitational radiation. Experimentally, no evidence of gravitational radiation was observed until the experiment of Hulse and Taylor, performed in 1973. Moreover, the significances of this binary star experiment had not been fully appreciated [18,19]. Thus, their problem would be related to radiation. Indeed, their physical assumptions are not valid due to the existence of anti-gravity coupling. Thus, if one does not believe in sigularity, ***the singularity theorems of Hawking and Penrose can be interpreted as necessitating the existence of the anti-gravity coupling.***

For reference, the singularity theorems shall be listed below. In these theorems, the code for a singularity is an incomplete or inextendible geodesic. The relevant theorems [7] are:

Theorem 1. Let $(M, g_{\mu\nu})$ be a globally hyperbolic space-time with $R_{\mu\nu}\xi^\mu\xi^\nu \leq 0$ for all timelike ξ^μ, which will be the case if Einstein's equation is satisfied with the strong energy condition holding for matter. Suppose there exists a smooth (or at least C^2) spacelike Cauchy surface Σ for which the trace of the extrinsic curvature (for the past directed normal geodesic congruence) satisfies $K \leq C < 0$ everywhere, where C is a constant. Then no past directed timelike curve from Σ can gave length greater than $3/|C|$. In particular, all past directed timelike geodesics are incomplete.

Theorem 2. Let $(M, g_{\mu\nu})$ be a strongly causal spacetime with $R_{\mu\nu}\xi^\mu\xi^\nu \leq 0$ for all timelike ξ^μ as will be the case if Einstein's equation is satisfied with the strong energy condition holding for matter. Suppose there exists a compact, edgeless, achronal, smooth spacelike hypersurface S such that for the past directed normal geodesic congruence from S we have $K < 0$ every on S. Let C denote the maximum value of K, so $K \leq C < 0$ everywhere on S. Then at least one inextendible past directed timelike geodesic from S has length no greater than $3/|C|$.

Theorem 3. Let $(M, g_{\mu\nu})$ be a connected, globally hyperbolic spacetime with a noncompact Cauchy surface Σ. Suppose $R_{\mu\nu}k^\mu k^\nu \leq 0$ for all null k^μ, as will be the case if $(M, g_{\mu\nu})$ is a solution of Einstein's equation with matter satisfying the weak or strong energy condition. Suppose, further, that M contains a trapped surface T. Let $\theta_0 < 0$ denote the maximum value of θ for both sets of orthogonal geodesics on T. Then at least one inextendible future directed orthogonal null geodesic from T has affine length no greater than $2/|\theta_0|$.

Theorem 4. Suppose a spacetime $(M, g_{\mu\nu})$ satisfies the following four conditions. (1) $R_{\mu\nu}\zeta^\mu\zeta^\nu \leq 0$ for all timelike and null ζ^μ as will be the case if Einstein's equation is satisfied with the strong energy condition holding for matter. (2) The timelike and null generic conditions are satisfied. (3) No closed timelike curve exists. (4) At least one of the following three properties holds: (a) $(M, g_{\mu\nu})$ possesses a compact achronal set without edge [i.e. $(M, g_{\mu\nu})$ is a closed universe], (b) $(M, g_{\mu\nu})$ possesses a trapped surface, or (c) there exists a point $p \in M$ such that the expansion of the future (or past) directed null geodesics emanating from p becomes negative along each geodesic in this congruence. Then $(M, g_{\mu\nu})$ must contain at least one incomplete timelike or null geodesic.

The first two theorems established timelike geodesic incompleteness. The third theorem due to Penrose [55], proved null geodesic incompleteness in the context relevant to gravitational collapse. On the other hand, Theorem 1 and Theorem 3 could also be interpreted as showing the universe is not globally hyperbolic. In Theorem 4, which is due to Penrose and Hawking [56], the assumption of globally hyperbolic is entirely eliminated, and thus would have much wider applications. Also, Theorem 4. has been strengthened by adding a fourth alternative to condition 4 [57]. A common physical assumption in all these theorems is that the Einstein's equation satisfies either the weak or the strong energy condition.

However, these energy conditions are actually due to inadequate modeling in the current theory, and is not valid for general relativity due to radiation. To show this, let us rewrite Einstein's equation (3.4) as $R_{\mu\nu} = -K[T_{\mu\nu} - g_{\mu\nu}T/2]$. Then, for any timelike ξ^μ, one has

$$R_{\mu\nu}\xi^\mu\xi^\nu = -K[T_{\mu\nu}\xi^\mu\xi^\nu - T/2], \quad \text{and} \quad R_{\mu\nu}k^\mu k^\nu = -K[T_{\mu\nu}k^\mu k^\nu] \qquad (6.6)$$

for any null vector k^μ. A necessary condition for either expressions in (6.6) to be non-positive is that the time-time component, $T_{tt} \geq 0$. But, this is possible only if there is no anti-gravity coupling. Thus, these theorems can be interpreted as stating that, under some general assumptions on the universe, the existence of singularities is inevitable if there is no radiation. But, there are radiations in nature. Similarly, the supposedly inevitable complete gravitational collapse is also due to inadequate modeling. The perfect fluid model is crude and does not adequately account for the gravitational energy-stress tensor or radiation.

6.4. The Question of Black Hole and Gravitational Collapse.

On the strength of the singularity theorems, many cosmologists believed [58] that "The general theory of relativity has forced physicists to take black hole seriously. No one who accepts general relativity has found any way to overturn the prediction that black holes can form from the gravitational collapse of sufficiently massive objects and that they ought to exist in the universe." Now, it turns out that the supposedly strongest argument is, in fact, invalid. Moreover, if gravity can produce radiation and even cause an explosion, as suggested by Hawking, there is little reason to believe that these would not happen before a star reach a black hole state. If they do happen, why such a star must form a black hole first? Thus, one should ask "are there enough justifications to consider black holes as a possibility in reality?"

A rather interesting description of black holes is given by K.S. Thorne [58] as follows: "Of all the conceptions of the human mind from unicorns to gargoyles to hydrogen bomb, perhaps the most fantastic is the black hole: a hole in space with a definite edge over which anything can fall and nothing can escape; a hole with a gravitational field so strong that even light is caught and held in its grip; a hole that curves space and warps time." This may explain why Eddington, the leading authority on the structure of stars, exclaimed [52] that "I think that there should be a law of Nature to prevent a star from behaving in this absurd way!" Einstein also claimed that stars would not shrink to zero size. But, some followers were not convinced.

6.4.1. Black Hole and Newtonian Gravity.

The idea of a black hole was actually originated from Newtonian gravity. In 1783 the British natural philosopher John Michell theorized that there could be a strong enough gravitational attraction to recapture all the star's radiation, including light [52]. In 1796, the French natural philosopher Pierre Simon Laplace wrote "It is possible that the largest luminous bodies in the universe may actually be invisible."

If gravity is only attractive but nothing else, gravitational collapse is inevitable when there is insufficient energy of other forms to counter gravity. The life of a star would be described as essentially a tug-of-war between gravity and the outward-directed force of its heat and radiation which is maintained by energy supplied by other interactions taking place within the star. When all forms of energy supplies are exhausted, a star would be mainly supported by "cold matter" pressure. If the mass of the star is sufficiently small, the star simply cools down and remains in equilibrium. However, if the mass of the star were greater than the cold matter upper limit, the star would have to undergo a gravitational collapse. Once gravitational collapse started, what prevents it from continuing forever, and the star crushing itself down to an infinitesimal speck containing all its matter, a single point of infinite density? When the star collapsed to a single point, the gravity became infinitely strong in the neighborhood and naturally nothing can escape would be the conclusion.

This scenario includes two implicit assumptions: 1) Interactions are triggered through heat and pressure, but not by gravity directly; 2) Strong gravity would not generate resistence in the collapsing process. These two assumptions are explicitly included in Newtonian gravity, but not in general relativity. Nevertheless, the influence of Newtonian theory lingers. Although the cause of gravity is extended to all energy forms, it was incorrectly believed [52] that all energy has a mass equivalence in gravity (see also §2). Since the Schwarzschild solution, which is a result of ignoring effects due to other interactions, seems consistent with this picture, diverse gravitational effects due to different energy forms were inadequately investigated.

6.4.2. Gravitational Collapse and the Modified Einstein Equation.

The above two assumption could be valid in general relativity if gravitational energy-stress were not localized. However, as discussed in §5 and shown in eq. (5.16), gravitational energy-stress is localized. A localized gravitational energy would trigger interactions directly. Therefore, the inevitable complete gravitational collapse is actually a manifestation of the remnant influence of Newtonian Theory (see also §7).

Moreover, if an Einstein equation contains the equations of motion, its source term should have the information of the interactions involved. If gravity can trigger an interaction, there should be a gravitatioinal energy-stress tensor in the source. To prevent a runaway self-generating, this tensor should have an anti-gravity coupling (see §5). This is also a mechanism which reduces and suppresses gravitational intensity and thereby resists gravitational collapse (see also §4.7). An observable effect of this mechanism is that for agglomerations of gas and dust to shine as stars, the minimum mass would be considerably larger than that in the current theory. Now, the law of Nature that Edington looked for, seems just general relativity.

6.4.3. Gravitational Collapse and Supernova.

It is difficult to imagine that matter cannot be transformed under intensive gravity especially if one believes in the possibility of a complete gravitational collapse. As discussed earlier, different kind of matter may not generate the same effect on gravity. Under extreme pressure, strong gravity may trigger interactions which are otherwise impossible. Such interactions would transform massive matter to different forms, provide the pressure to resist "gravitational collapse" and also alleviate the intensity of gravity. Thus, massless matter, (i.e. photons, gravitons, and possibly neutrinos) would be generated since, as discussed in §6.1 and §6.2, only the creation of massless matter could reduce gravity; and radiation would provide pressure and heat. These considerations could lead to an alternative scenario slightly different from the current theory.

If the required conditions are high pressure and high intensity of gravity, these interactions would occur in between the center and the surface of a star. The resulting high temperature and high pressure would "melt" a middle shell and form a core which would collapse to neutrons inside the star since a neutron star is in a stable state [7]. On the other hand, the outer layer is getting weaker since its massive matter, due to gravity attraction, would keep joining the "melting" zone. The net effect of such interactions reduces gravity but increases pressure to the outer layer. Thus, such an "over heat" due to intense gravity cannot be stable, and would eventually lead to a star explosion. Such an explosion would likely occur when the core is collapsed to neutrons because of the large and sudden increment in pressure and reduction in gravity to the outer layer [59]. Such an explosion would be violent, but leave a core of neutrons intact. The high energy radiation and plasma will be released with the explosion and the star would become a supernova.

In conclusion, in general relativity, a black hole is not an inevitable end for a super star. Also, it is unlikely that the gravity of a star can be so strong that even light cannot escape. On the contrary, a super star would have a glorified death. This version of gravity collapse is more realistic and provides a plausible explanation for the creation of supernova type II. The idea that gravity, which is initially an attractive force, is the primary cause of a star explosion may seem strange. But, this is in perfect harmony with the time-tested principle that things, when carried to the extreme, would inevitably go to the opposite ( ).

6.5. On Expansion of the Universe.

Modern astronomy, started from N. Copernicus (1473-1543), T. Brahe, and J. Kepler, is based on detailed observation and analysis. They discovered that the earth and planets orbit around the sun, and their universe is not only orderly but peaceful. However, the supernova of July 1054 (whose remains is the Crab Nebula), was observed by the Sung imperial astronomers. Then, details of heavenly violences are observed with the telescope and more violences were discovered by radio-wave/X-ray observations.

Now, it is conclusively clear that the heaven is not static but dynamic. Stars are born and die sometimes violently. Thus, it is not surprising that we live in a part of the universe which is contracting or expanding. However, to conclude that the whole universe is expanding is a different matter. Moreover, based on an expanding state to conclude that the universe started from a pointed big bang, is an unprecedented dubious extrapolation in science (see also §6.3) since the singularity theorems have been proven to be irrelevant.

Currently, it seems, the strongest evidence to support global expansion of the universe is the Hubble's law. (This interpretation is rejected by Hubble himself [60].) However, one may not be sure that these red shifts of lights are due to speeds of the sources unless one can verify these velocities independently. For instance, one might attribute the red shifts as due to energy losses in the long travelling, and one would potentially get a new law in physics. Given the over all complexity, the extreme conditions, and the vast scale of the universe, it is difficult to assure that the laws of physics, which are discovered on earth, would cover all large scale problems in the universe. In short, there is no conclusive evidence for an expanding universe.

Nevertheless, the question of the structure and the origin of the universe is one of the most exciting topics for a scientist to deal with. It reaches far beyond its purely scientific significance, since it is related to human existence, to mythology, to philosophy, and to religion. All theories of the cosmos must be somewhat hypothetical because it is very hard to make empirical observations regarding the totality of the universe, and therefore one does not know whether the real facts have been caught. However, if scientists have to be right all the time, there would not be science. What required for a scientist is to be sufficiently objective.

7. Conclusions and Discussions.

In Newton's theory, time and space are independent. However, even before Newton, the close relationship between time and space is recognized because the motion of matter must be understood in terms of both. For example, the Chinese considers the universe as the space-time (Yu-Zhou). In special relativity, based on the constancy of light speed, space and time are understood in terms of a four-dimensional linear space with a constant indefinite metric [61]. Also, time is no longer unrelated to motion because the space-time coordinate system depends on motion. Nevertheless, the characteristic of space-time, the metric, was independent of the motion of matter. This deficiency is manifested by the fact that special relativity is incompatible with gravity. Based on the equality of inertial and gravitational mass, which was developed as the principle of equivalence (see also [46]), this deficiency is removed by Einstein in his general theory of relativity, in which gravity is a manifestation of the space-time curved by matter. Then, matter, motion, and space-time are different aspects of the reality, which are inextricably related by the geodesic equation and Einstein's field equation. In spite of this unprecedented revolutionary viewpoint in physics, general relativity was accepted because all its predictions, which are different from naive visualization, are verified by observations.

However, these great successes also lead to over confidence. The self-consistency of the theory has not been thoroughly examined, and this opens a door for incorrect theoretical developments later. To begin with, the possibility that the equivalence principle being a physical requirement may not be satisfied by a mathematical solution, is overlooked. This makes it possible to "establish" that any mathematical coordinate system was valid in physics. Also, in the name of abandoning naive visualization, physical principles are often ignored. Consequently, general relativity is effectively reduced to a branch of mathematics which is often unrelated to reality. This surrealistic theoretical development is culminated in the singularity theorems [55,56] which in turn serve as a foundation for the collapse of a super star to a single point and etc.. Ironically, this also proclaims the inevitable break down of general relativity.

Now, it has been proven that these singularity theorems are irrelevant in physics because, among others, the energy assumptions in these theorems are not valid due to the existence of anti-gravity coupling, which was anticipated by Pauli [9]. This conclusion is achieved by first recovering the equivalence principle as a physical requirement. This means that a mathematical coordinate system is not necessarily valid in physics. Then, based on the principle of causality, the necessity of the anti-gravity coupling is theoretically established in studying the gravity of electromagnetic plane waves. Finally, based on the observed radiation loss of the binary pulsar PSR 1913+16, the anti-gravity coupling is experimentally verified.

Upon examining the arguments for the supposed inevitable gravitational complete collapse, the improved understanding makes clearer that such a concept is actually due to the remnant influence of Newtonian gravity. But, outstanding scientists such as Einstein and Eddington [52], who can see beyond equations (Black holes just didn't smell right!), found immediately such a collapse unacceptable. In general relativity, instead of a complete collapse, a super star would end up as a supernova. This is feasible because of the existence of anti-gravity coupling. Also, the arbitrariness in the choice of coordinates has never been really established among theorists. For example, S.W. Hawking [62] inadvertently makes clear in *'The Arrow of Time'* of his book that a time coordinate must be distinct from a space coordinate.

Einstein [21] believes that, while his equation is essentially correct, the appropriateness of the source would be a major problem. He wrote "The phenomenological representation of the matter is, in fact, only a crude substitute for a representation which would do justice to all known properties of matter." The past and recent theoretical developments confirm his foresight. In this new theoretical development, not only general relativity does not break down, but the principle of causality leads to the discovery that particle-wave duality is necessary in general relativity. Thus, although "a clear connection between the general theory of relativity and quantum mechanics is not yet in sight [9]," a connection is clearly there. Also, Einstein's belief [17] that "physics of the future" would be based on general relativity, is supported. Therefore, the claim that there is nothing in general relativity to take into account the quantum behavior of subatomic particle, is groundless although such a claim could be a self-fulfilled prophecy if nobody works on that direction.

Einstein [23] wrote, "I do not see any reason to assume that the heuristic significance of the principle of general relativity is restricted to gravitation and that the rest of physics can be dealt with separately on the basis of special relativity, with the hope that later on the whole may be fitted consistently into a general relativistic scheme," and "The comparative smallness of what we know today as gravitational effects is not a conclusive reason for ignoring the principle of general relativity in theoretical investigations of a fundamental character." Unfortunately, it is precisely due to such implicit assumptions, which separate gravity from the rest of the physics, that the gravitational complete collapse became "inevitable".

Nevertheless, general relativity is only a step in the endless intellectual pursuance of human spirit. A limitation of general relativity, as pointed out by Einstein [63] and others [26], is that the electromagnetic field has not been deduced from the structure of the space. He considered "The idea that there exist two structures of space independent of each other, the metric-gravitational and the electromagnetic, was intolerable to the theoretical spirit." This philosophy leads to the flourishing of unified theories based on a higher dimensional space. However, because of the lack of a guiding physical principle, the progresses of such theories [26-29] have been essentially confined to formal mathematical manipulations, and no independent predictions has been verified by experiment. The necessity for such a unified theory actually has a far deeper reason than the theoretical spirit. First, the velocity of light itself is an electromagnetic phenomenon [25]. The structure of massive matter is essentially electromagnetic, and all the stable massive particles are charged. However, so far, successful unified theories [64-66] have not yet included gravity.

Pauli [9] pointed out that the most important aspect of Einstein's theory is his critical attitude, which abandoned naive visualizations in favour of a conceptual analysis of the correspondence between observational data and the mathematical quantities in a theoretical formalism. But, some theorists believe that mathematical consistency has been one of the most reliable guides to physicists in the last century. The development of general relativity shows that this belief is not supported. Einstein [32] pointed out "The propositions of mathematics referred to objects of our imagination, and not to reality." The logic of nature may be simple at times, but is often very subtle. The subtlety is revealed in experiment and observation. The developments in relativity and quantum theory support this view. Thus, although mathematics is an indispensable tool, the guidiance for physics must come from experiments and observations. As remarked by Pauli [9], the theory of relativity is "an example showing how a fundamental scientific discovery, sometimes even against the resistence of its creator, gives birth to further fruitful developments, following its own autonomous course."

8. Acknowledgements.

Appendix A: The Principle of Causality, Validity of an Equation, and Symmetry.

The concept of causality describes the ideas of cause and effect (which needs not be deterministic). There are two aspects in causality: its relevance and its time ordering. In time ordering, a cause event must happen before its effects. This is further restricted by relativistic causality. (see §2 & §3.3). The time-tested assumption that phenomena can be explained in terms of identifiable causes will be called the principle of causality. This principle is the foundation of scientific studies. Here, this principle will be elucidated first in connection with symmetries, and then in the validity of an equation in physics.

In practice, we assume certain properties (such as symmetries etc.) for a "normal" state whose existence is without any specific cause. Then, any deviation from the normal state must have physically identifiable cause(s). Since the principle of causality implies that symmetry breaking must have cause(s), a symmetry must be preserved if no cause breaks it. For example, in electrodynamics, the electromagnetic field is zero in a normal state. The implication of causality to symmetry is used in deriving the inverse square law from Gauss's law. Although a related potential may not be spherically symmetric, at least one is, the Coulomb potential. This shows that, at least, a gauge can be compatible with such symmetries.

In general relativity, matter is the cause of gravity. The normal state of a metric is the flat metric in special relativity. (This is a gauge choice.) The constant flat metric possesses all the symmetry allowed by special relativity. Thus, if a non-constant metric does not possess a certain symmetry, then there must be physical cause(s) which has broken such a symmetry. In other words, the metric should have at least the same symmetry as its physical cause(s). For example, in the Schwarzschild solution, causality requires that the metric is spherically symmetric and asymptotically flat. Thus, in agreement with the equivalence principle, the flat metric is the only solution for Einstein's equation in empty space.

However, the physical cause(s) should not be confused with the mathematical source term in the field equation. Such a confusion would be possible because, for some situations, such a distinction does not seem to be meaningful. For instance, in electrodynamics, the physical cause of an electromagnetic field and the source term in Maxwell's equation, are the same charged currents. In general relativity, the cause of gravity remains the physical matter, but not the source term in Einstein's field equation. The energy-stress tensors (for example the perfect fluid model) may explicitly depend on the metric. Since nothing should be a cause of itself, such a source tensor does not represent the cause of a metric. For the accompanying gravitational wave of an electromagnetic wave, the physical cause is the electromagnetic wave. In the Schwarzschild case, the cause is the mass distribution. Thus, it does not make sense, without directly using causality, to infer the symmetries of the metric from the source term although their symmetries are not unrelated.

Moreover, inferences based on the source term can be misleading. Sometimes, the source term may have higher symmetries than those of the cause and the metric. For instance, a transverse electromagnetic plane wave is not rotationally invariant with respect to the direction of propagation. But the related electromagnetic energy-stress tensor can be rotationally invariant and even be a constant [22]. In the literature (see [37], and also p. 961 of [2] and §13 & §21 of [8]), the metric is incorrectly assumed to be rotationally invariant. This assumption violates causality and results in theoretical difficulties (see §3.3).

Classical electrodynamics and experiments imply that the flat metric is an accurate approximation of the metric which is caused by the presence of a weak electromagnetic plane wave. This physical requirement is supported by the principle of causality which implies that such a metric is a bounded periodic function. However, this requirement is not satisfied by solutions in the literature, because they are not bounded, independent of how weak the electromagnetic plane waves are (see §3.3). Also, they violate causality.

This compatibility of symmetry due to causality is a physical requirement. On the other hand, since any field equation and its physical solutions must be compatible with the principle of causality, symmetry consideration can be used as a criterion, which is independent of the field strength.

For some mathematical equations, the symmetries of a solution can be very different from that of the source term (which may or may not be the physical cause). For example, consider the following equation,

$$\eta^{ab}\partial_a\partial_b F = f(u), \tag{A1}$$

where η_{ab} is the flat metric $(+---)$ and $u \equiv (t - z)$. If F is a function of only t and z, then the inhomogeneous solution of eq. (A1) is

$$F(t,z) = \frac{v}{4}\int^{u} f(t)\, dt, \tag{A2}$$

where $v \equiv (t + z)$. Solution (A2) depends not only on u, but also v.

Then, one may examine a field equation after the related physical cause is identified. The left-hand side of eq. (A1) can be considered as a Maxwell's equation or an equation in linearized gravity. For the case of Maxwell's equation, the principle of causality implies that the source term may not be in the form of plane waves. This restriction is satisfied physically because, in nature, a charged particle is invariably massive. For linearized gravity, Function F relates to the deviations from a flat metric. (An implicit assumption of weak gravity is that an empty space has a flat metric. This assumption is identical to the requirement of a normal state.) If the physical cause is an electromagnetic plane wave propagating in the z-direction, then the related source energy-stress tensor can be a function of u [2,8,37], and its lowest order approximation is a function of u, and thus the source term in linearized gravity would have the form f(u). Then, according to solution (A2), F(t,z) and therefore the metric has a factor v.

On the other hand, the principle of causality implies that the metric is a function of u only [2,8,22]. This contradiction suggests that, for gravitational waves, eq. (A1) is not an appropriate form. Thus, causality implies that there are weak gravity exact solutions, which cannot be approximated with linearized gravity. In other words, causality supports Einstein's observation that linearized gravity is not reliable [17]. One might argue that a solution could be a function of only u through a gauge transformation. This is not possible physically nor mathematically since a flat space-time has to remain flat.

In short, the principle of causality may appear to be questionable from the viewpoint of mathematics, but nature requires that this principle is satisfied. In classical electrodynamics, it is well-known that causality restricts the possible form for the radiation reaction force. Here, causality explains physically that, even for weak gravity, the field equation of gravity must go beyond a linear equation of Maxwell type because the physical causes of gravity include electromagnetic waves.

Moreover, the principle of causality supports the Einstein tensor $G_{\mu\nu}$. If one assumes that, for an electromagnetic plane wave, the metric is a function of only t and z, then the Einstein equation implies $g(u)_{\mu\nu}$ [22]. For some special cases, it can be shown that tensor $G(u)_{\mu\nu}$ implies that $g(u)_{\mu\nu}$ is periodic.

Let us consider a circularly polarized monochromatic electromagnetic plane wave,

$$A_x = \frac{1}{\sqrt{2}} A_0 \cos \omega u \ , \text{ and } \ A_y = \pm \frac{1}{\sqrt{2}} A_0 \sin \omega u \ . \tag{A3}$$

The rotational invariants with respect to the z-axis are constants. These invariants are: G, $(g_{xx} + g_{yy})$, R_{tt}, $T(E)_{tt}$, g_{tz}, g, g_{tt}, and etc. Let us assume the invariant,

$$g_{xx} + g_{yy} = -2 - 2C, \quad \text{then} \quad g_{xx} = -1 - C + B \ , \text{ and } \ g_{yy} = -1 - C - B. \tag{A4}$$

Then,

$$B^2 + g_{xy}{}^2 = (1+C)^2 - C \ , \text{ and } \ (B')^2 + (g_{xy}')^2 = 2CR_{tt} \geq 0 \tag{A5}$$

are constants. It follows that (A5) imply that

$$B = B_\alpha \cos(\omega_1 u + \alpha), \text{ and } \ g_{xy} = \pm B_\alpha \sin(\omega_1 u + \alpha) \ , \tag{A6a}$$

where

$$\omega_1{}^2 = 2R_{tt}C/B_\alpha{}^2, \quad \text{and} \quad B_\alpha{}^2 = (1+C)^2 - C \geq 0. \tag{A6b}$$

Thus, it is proven that the metric is a periodic functions. Also, as implied by causality, the metric is <u>not</u> an invariant under a rotation (since a transverse electromagnetic wave is not such an invariant).

Since $T(E)_{tt}$ is a constant, it is necessary to have

$$\omega_1 = 2\omega, \quad \text{and} \quad T(E)_{tt} = \frac{1}{2G} \omega^2 A_0{}^2 (C - B_\alpha \cos\alpha) > 0. \tag{A7}$$

Eq. (A6) implies that the metric is a circularly polarized wave with the same direction of polarization as (A3). However, if the photon tensor were zero, it is not possible to satisfy Einstein's equation because $T(E)_{tt}$ and R_{tt} have the same sign. (Note that $C > 0$; and the equation of motion of a charged particle does not allow changing the sign of the coupling constant K.) This also implies that it would be sufficient to modify the source tensor. The additional term should be a constant of different sign, and is larger in absolute value.

Appendix B: Compatibility of Polarizations.

Einstein's field equation shall be examined with different polarizations. Then, the relations between the polarizations of electromagnetic and gravitational wave components are established.

It has been established that for a circularly polarized electromagnetic wave (A3), the solution is also a circularly polarized wave (A4). Its curvature tensor and electromagnetic energy-stress tensor are:

$$R_{tt} = 2\omega^2 B_\alpha{}^2/G \,, \quad \text{where} \quad G = (1 + C_1)^2 - B_\alpha{}^2, \tag{B1}$$

and

$$T(E)_{tt} = \omega^2 A_0{}^2(1 + C_1 - B_\alpha \cos\alpha)/2G \tag{B2}$$

where 2ω and B_α are the frequency and the amplitude of the gravity wave. B_α and C_1 are small numbers. The determinant g and g_t are constants.

The frequency ratio suggests that $-K A_0{}^2$ is of first order of deviations. Thus, $\omega^2 A_0{}^2/2G$ in (B2) must be canceled by the photon tensor. (This means that, in the flat metric approximation, an electromagnetic wave and its photons carry, on the average, the same energy-momentum.) To support this, consider a linearly polarized electromagnetic wave

$$A_x = A = A_0 \cos[\omega(t - z)] \,. \tag{B3}$$

From equation (4.6), the equation of the lowest order terms is

$$f''/2 = -K(A')^2, \quad \text{where} \quad -f \equiv (g_{xx} + g_{yy}) + 2 \tag{B4}$$

Indeed, there is a constant $-K\omega^2 A_0{}^2/2$ to be canceled. To have a physical solution, the modified equation of the lowest order should be

$$f'' = -2K\,T(u), \tag{B5}$$

and the time average of T(u) is zero. Now, equation (4.6a) becomes

$$G'' - g_{xx}'g_{yy}' + (g_{xy}')^2 - G'(g'/2g) = 2K\,g_{yy}T(u). \tag{B6}$$

There are four unknowns in eq. (B6). The previous case suggests that eq. (B6) can be reduced.

From eqs. (4.7) and (B4), only one of g and g_t can be a constant. (The Schwartzchild solution suggests that g would likely be a constant.) Nevertheless, it is still possible to simplify (B6). To this end, define

$$F_1'' \equiv -2K\,T(u)\ , \quad \text{and} \quad f \equiv f_1 + F_1 + F_2\ , \tag{B7}$$

where F_2 is of second order and f_1 is the time average of f. Then, one obtains

$$G'' - g_{xx}'g_{yy}' + (g_{xy}')^2 = -(fg_{yy}' + F_2'g_{yy})' - F_1''g_{yy} + F_2'g_{yy}'$$

$$- \left[2(g_{yy}+1)g_{yy}'' + (g_{yy}')^2 + 2g_{xy}g_{xy}'' + (g_{xy}')^2\right]. \tag{B8}$$

Then (B6) is reduced to

$$G'(g'/2g) + (fg_{yy}' + F_2'g_{yy})' - F_2'g_{yy}' =$$

$$- \left[2(g_{yy}+1)g_{yy}'' + (g_{yy}')^2 + 2g_{xy}g_{xy}'' + (g_{xy}')^2\right]. \tag{B9}$$

where

$$G' = -(fg_{yy})' - 2\left[(g_{yy}+1)g_{yy}' + g_{xy}g_{xy}'\right].$$

For a physical solution, it requires that the time average of the term $(fg_{yy}' + F_2'g_{yy})'$ is zero.

If g is a constant, then eq. (B9) and eq. (4.9) imply

$$g_{yy}' = g_{xy}' = 0\ . \tag{B10}$$

These equations in turn imply that

$$F_2'' = 0\ , \tag{B11}$$

and consequently $F_2 = 0$. Thus, both g_{yy} and g_{xy} are constants, and equation (B6) is reduced to

$$g_{xx}'' = 2K\,T(u). \tag{B12}$$

If the constants are independent of the wave amplitude, then one has

$$g_{xy} = 0\ , \text{ and } \ g_{yy} = -1\ . \tag{B13}$$

On the other hand, in general, (B13) implies

$$G'(g'/2g) - F_2'' = 0 \tag{B14}$$

It follows that g_t cannot be a constant in a linear polarization. To illustrate this, let us assume

$$g_{tt} = 1 \ , \ g_{xy} = 0 \ , \text{ and } \ g = -1 \ . \tag{B15a}$$

as suggested by linearized gravity. Then equation (4.6) becomes

$$(g_{xx}')^2 = 2K g_{xx}[(A')^2 + g_{xx}T(P)_{tt}] \ . \tag{B15b}$$

Without a photon tensor ($\lambda = 0$), the solution for eq. (B15) is:

$$g_{xx} = -(1 + C \pm i\sqrt{\frac{K}{2}}A)^2 \simeq -1 - 2C \mp i\sqrt{2K}A \ , \tag{B16a}$$

$$g_{yy} = g_{xx}^{-1} \simeq -1 + 2C \pm i\sqrt{2K}A \ , \tag{B16b}$$

where C is a constant which is zero when $A = 0$. The imaginary sign comes from the fact that special relativity is an accurate approximation. Solution (B16) is not physical because it is essentially imaginary for a real electromagnetic wave. Moreover, the frequency ratio between gravitational and electromagnetic wave components should be two. Also, it is easy to see that no value of λ can make g_{xx} a real wave function.

If one assumes, as Misner et. al. [2], that

$$g_{tt} = 1 \ , \ g_{xy} = 0 \ , \text{ and } \ g_{xx} = g_{yy} \ , \tag{B17a}$$

then the resulting equation is

$$2LL'' = -K[(A')^2 + L^2T(P)_{tt}], \quad \text{where } (-g_{xx}) = L^2. \tag{B17b}$$

For $\lambda = 0$, equation (B17b) implies that L is not bounded. For $\lambda = -1$, equation (B17b) becomes

$$2LL'' = K\omega^2A_0^2\cos(2\omega u) \ . \tag{B18}$$

However, equation (B18) implies that L is not a periodic function of u. In fact, there is no λ which can make L a periodic function.

It should be noted that, in the above calculations, the sign of R_{tt} is crucial to the physical conclusions. Because R_{tt}, on the time average, is positive, it is necessary to have an additional energy-stress tensor with an anti-gravity coupling.

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