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Derivation of Thermal Mass In Finite-temperature Field Theory Under the Renormalization-Group Constraint.

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Abstract

The renormalization group equation in finite-temperature field theory is discussed. It is shown that the mass-independent temperature-independent form is satisfied to two loops for ϕ^4 theory. This form is applied to show the invalidity of the usual derivation of the expression for the thermal mass and the large-T expansion of the finite-temperature effective potential.. Relevance to high-temperature symmetry restoration is pointed out.

Investigation of symmetry restoration in a renormalizable finite-temperature field theory (FTFT) requires knowledge of the asymptotic behaviour of its effective potential at high temperature[1,2]. Such behaviour is usually obtained from the truncated loop-expansion of the effective potential. Although the loop-expansion is known to break down near the critical temperature T_c [2], it is common practice to extend the asymptotic expression down to $T=T_c$, to obrain a rough estimate of T_c. To partially circumvent this difficulty and to improve the large-temperature behaviour, modifications of the loop expanssion have been proposed. These may, for example, involve the addition of a temperature-dependent mass term in the orignial lagraugian density, the replacement of the free propagator by the full propagator or the resummation of a sub-class of Feynman graphs [1-8].

In this letter we show that an important check on the validity of asymptotic forms of the finite temperature effective potential (FTEP) is provided by the constraint of the renormalization group equation (RGE). Our analysis clearly indicates the invalidity of the usual practice of deriving the expression for the thermal mass, and deducing the restoration of symmetry, from the term-by-term high-temperature behaviour of the truncated loop expansion.

We restrict our considerations to ϕ^4 theory and use the mass-independent temperature-independent renormalization scheme with dimensional regularization [9-10] In this scheme the renormalization group (RG) functions and the running couplings are temperature-independent. The RGE for the FTEP $V(\phi,T)$ is

$$DV(\phi,T) = 0 , \tag{1}$$

where D is the RG differential operator

$$D = \mu\frac{\partial}{\partial\mu} + \beta\frac{\partial}{\partial\lambda} + \gamma_m m\frac{\partial}{\partial m} + \gamma\phi\frac{\partial}{\partial\phi} \quad , \tag{2}$$

and $\beta(\lambda), \gamma_m, \gamma(\lambda)$ are the RG functions.

It has been suggested by some authors [11] that, in addition to the usual RGE based on the arbitrariness of the renormalization scale in momentum, one must also consider an independent RGE obtained from an arbitrary renormalization scale in temperature. More recently it has been proposed that adequate treatment of the renormalization constraint in FTFT requires the consideration of temperature-dependent renormalization schemes [12]. It is however clear that if equation (1) is valid, then it fully embodies the condition of the renormalizability of the theory and must therefore contain all information that may be extracted from the renormalization constraint. There is thus no need to consider an independent arbitrary temperature scale, as in ref.[11]. It is also not necessary to consider temperature-dependent RG functions in the usual RGE, as in ref.[12].

We now explicitly check that the RGE (1) is satisfied by the loop expansion of the effective potential up to two loops.

One may write

$$V(\phi,T) = V(\phi) + V^{(T)}(\phi,T) \quad , \tag{3}$$

where $V(\phi)$ is the zero-temperature effective potential. Since $V(\phi)$ satisfies the RGE(1), one has

$$DV^{(T)}(\phi,T) = 0 \quad . \tag{4}$$

The loop expansion gives power series in $\hbar$:

$$\left.\begin{aligned} &D = \mu\frac{\partial}{\partial\mu} + \sum_{\ell=1}^{\infty} D_\ell \hbar^\ell, V^{(T)}(\phi,T) = \sum_{\ell=1}^{\infty} V_\ell^{(T)}(\phi,T)\hbar^\ell, \\ &D_\ell = \beta^{(\ell)}\frac{\partial}{\partial\lambda} + \gamma_m^{(\ell)} m\frac{\partial}{\partial m} + \gamma^{(\ell)}\phi\frac{\partial}{\partial\phi}, \quad \ell = 1,2,\ldots, \end{aligned}\right\} \tag{5}$$

where, e.g., $\beta^{(\ell)}(\lambda)$ is the ℓ-loop contribution to $\beta(\lambda)$.

Substituting from equ.(5) into equ.(4) one obrains the following RG equations for $V_\ell^{(T)}(\phi,T)$:

$$\mu\frac{\partial V_1^{(T)}(\phi,T)}{\partial\mu}=0 \tag{6}$$

$$\mu\frac{\partial V_2^{(T)}}{\partial\mu}(\phi,T)+D_1V_1^{(T)}(\phi,T)=0 \tag{7}$$

Equ.(6) asserts the μ-independence of $V_1^{(T)}(\phi,T)$ and is therefore satisfied by the well-known expression

$$V_1^{(T)}(\phi,T)=\frac{T^4}{2\pi^2}\int_o^\infty x^2\log[1-e^{-(x^2+\frac{\sigma^2}{T^2})^{\frac{1}{2}}}]dx, \tag{8}$$

where $\sigma^2=m^2+\frac{1}{2}\lambda\phi^2$.

For equ.(7) one only needs the μ-dependent part of $V_2^T(\phi,T)$ [10]:

$$V_2^{(T)}(\phi,T)=\frac{-\lambda}{128\pi^4}\int^\sigma d\sigma\sigma\{(2m^2-3\sigma^2)B(o;\frac{\sigma}{T})+6\sigma TB(1;\frac{\sigma}{T})\}\log\frac{\mu^2}{\sigma^2}+\ldots \tag{9}$$

where

$$B(\nu;x)=\sum_{n=1}^\infty\frac{1}{n^\nu}K_\nu(nx)\ ,\qquad \nu>-\frac{1}{2}, \tag{10}$$

and $K_\nu(x)$ is the modified Bessel function of the second kind. Substituting from equs.(9) and (10) into equ.(7) one finds:

#1 Minor corrections to equ.(A.12) of ref.[10] have been made to obtain the corrected expression in equ.(9).

$$m^2(\frac{1}{\lambda}\beta^{(1)}+2\gamma^{(1)}-2\gamma_m^{(1)})B(0;\frac{\sigma}{T})+\sigma^2(\frac{1}{\lambda}\beta^{(1)}+2\gamma^{(1)})\{\sum_{n=1}^{\infty}K_2(\frac{n\sigma}{T})-2B(0;\frac{\sigma}{T})\}$$
$$=\frac{\lambda}{16\pi^2}[(2m^2-6\sigma^2)B(0;\frac{\sigma}{T})+3\sigma^2\sum_{n-1}^{\infty}K_2(\frac{n\sigma}{T})]. \qquad (11)$$

This condition is identically satisfied for all σ,m and T by the usual one-loop RG functions:

$$\beta^{(1)}=\frac{3\lambda^2}{16\pi^2},\quad \gamma^{(1)}\ =\ 0,\quad \gamma_m^{(1)}=\frac{\lambda}{32\pi^2}\ , \qquad (12)$$

which confirms that the RGE's for $V_\ell^{(T)}(\phi,T)$ are exactly satified. Thus the RGE (1) is perturbatively valid for all T.

It is our purpose now to apply these equations to check the validity of the derivation of the usual expression of the induced thermal mass in massless ϕ^4-theory, $m_{th}^2=\frac{\lambda}{24}T^2$. This is normally obtained from the large-T expression of the one-loop FTEP,

$$V(\phi,T)=\frac{-\pi^2}{90}T^4+\frac{1}{48}\lambda\phi^2T^2+\ldots \qquad . \qquad (13)$$

In general the induced thermal mass in massless ϕ^4 theory is given by

$$m_{th}^2=(\frac{\partial^2V^{(T)}(\phi,T)}{\partial\phi^2})_{\phi=0} \quad . \qquad (14)$$

One therefore requires the behaviour of $V^{(T)}(\phi,T)$ near ϕ=0. It is known that, for T=0, this behaviour may be obtained from the RGE [13] and is not directly given by the perturbative loop expansion.

We now consider the small-ϕ behaviour of $V^{(T)}(\phi,T)$ as determined by the RGE. The general form of the loop expansion for massless:

$$V^{(T}(\phi,T) = \hbar T^4 F(\frac{\lambda\phi^2}{T^2},\frac{\lambda\phi^2}{\mu^2},\hbar\lambda). \quad . \tag{15}$$

The solution of the RGE (1) gives

$$V^{(T)}(\phi,T) = \hbar T^4 F(\frac{\lambda(t)\phi^2(t)}{T^2},\frac{\lambda(t)\phi^2(t)}{\mu^2}e^{-2t},\hbar\lambda(t)), \tag{16}$$

where t is an arbitrary parameter. To obtain a form of $V^{(T)}(\phi,T)$ in which we may usefully employ the loop expansion when ϕ is small we take for the arbitrary parameter t the value $t = t_1(\frac{\lambda\phi^2}{\mu^2})$ where

$$\frac{\lambda(t_1)\phi^2(t_1)}{\mu^2} = e^{2t_1} \tag{17}$$

Using the one-loop RG functions, this condition yields

$$t_1 = \frac{1}{2}\log\frac{32\pi^2\phi^2}{\hbar\mu^2} - \frac{1}{2}\log\log\left\{\frac{32\pi^2}{\hbar\mu^2}\exp(\frac{32\pi^2}{3\hbar\lambda}\right\}+... \tag{18}$$

for small ϕ. This implies

$$\lambda(t_1) = \frac{\lambda}{1+\frac{3\hbar\lambda}{32\pi^2}\log\frac{3\hbar\mu^2}{32\pi^2\phi^2}}+... \tag{19}$$

Using these express ons in equ. (16) one obtains, for small ϕ,

$$V^{(T)}(\phi,T)=\hbar T^4 F(\xi\frac{\lambda\phi^2}{T^2},1,\xi\hbar\lambda), \tag{20}$$

where

$$\xi=\left\{1+\frac{3\hbar\lambda}{32\pi^2}\log\frac{3\hbar\mu^2}{32\pi^2\phi^2}\right\}^{-1}+\ldots \quad . \tag{21}$$

We now use the loop expansion for $V^{(T)}(\phi,T)$:

$$F(\frac{\lambda\phi^2}{T^2},\frac{\lambda\phi^2}{\mu^2},\hbar\lambda)=F_1(\frac{\lambda\phi^2}{T^2})+\hbar\lambda F_2(\frac{\lambda\phi^2}{T^2},\frac{\lambda\phi^2}{\mu^2})+\ldots \tag{22}$$

The integrals [10] defining the one-and two-loop contributions have the following behaviour for small x:

$$\begin{aligned} F_1(x)&=\frac{\pi^2}{90}+\frac{1}{48}x-\frac{1}{12\pi}(\frac{x}{2})^{\frac{3}{2}}+O(x^2),\\ F_2(x,1)&=O(x^2). \end{aligned} \tag{23}$$

Thus working to order $(\frac{\lambda\phi^2}{T^2})$, for small ϕ, we obtain from equations (20) and (23):

$$V^{(T)}(\phi,T)=\hbar T^4\left\{\frac{-\pi}{90}+\frac{2\pi^2}{9\hbar}\frac{\phi^2}{T^2\left|\log\frac{32\pi^2\phi^2}{3\hbar\mu^2}\right|}-\frac{1}{12\pi}\left(\frac{16\pi^2}{3\hbar}\frac{\phi^2}{T^2\left|\log\frac{32\pi^2\phi^2}{3\hbar\mu^2}\right|}\right)^{\frac{3}{2}}+\ldots\right\} \tag{24}$$

This equation given the small-ϕ expanrion of $V^{(T)}(\phi,T)$. One cannot obtain it directly from the loop expansion (22). This imphes that the loop expantion for $V^{(T)}(\phi,T)$ is

not a valid representation near $\phi = o$, which is already known to be the case at zero temperature [13].

The difference between the $\hbar\lambda$ and small ϕ expantions lies in the form of the function ζ in equ.(21):

$$\xi \to \frac{32\pi^2}{3\hbar\lambda \log \frac{3\hbar\mu^2}{32\pi^2\mu^2}}$$ for small ϕ at fixed $\hbar\lambda$ at fixed ϕ, giving (24), while

$$\xi \to 1 - \frac{3\hbar\lambda}{32\pi^2} \log \frac{3\hbar\mu^2}{32\pi^2\phi^2}$$ for small $\hbar\lambda$ at fixed ϕ, giving (22).

We also note that this RG analysis yields the large.T behaviour of $V^{(T)}(\phi, T)$, at fixed ϕ, giving

$$V^{(T)}(\phi, T) = \hbar T^4 \left\{ \frac{-\pi^2}{90} + \frac{1}{48} \frac{\lambda\phi^2}{T^2} \left(1 + \frac{3\hbar\lambda}{32\pi^2} \log \frac{3\hbar\mu^2}{32\pi^2\phi^2} \right)^{-1} + \ldots \right\} \tag{25}$$

It folows that the usual procedure of deriving the large-T behaviour equ.(13), directly from the loop expaurion is not valid.

If one uses the correct exprerion near $\phi = o$, one finds

$$\frac{\partial^2 V^{(T)}(\phi, T)}{\partial\phi^2} \approx \frac{4\pi^2}{9} \frac{T^2}{\left| \log \frac{32\pi^2\phi^2}{3\hbar\mu^2} \right|} \tag{26}$$

which contradicts the result $m_{th}^2 = \frac{1}{24}\lambda T^2$ obtained from equ.(13).

Further, our analysis casts doubt on the usual derivation of the restoration of symmetry at high temperature, since this also involves use of the loop expansion up to $\phi = o$, where it is not valid.

References

(1) L. Dolan and R. Jackiw, Phys. Rev. D9 (1974) 3320.

(2) S. Weinberg, Phys. Rev. D9 (1974) 3357.

(3) T. Altherer, Phys. Lett. B238 (1990) 360.

(4) D.E. Brahm and S.D.H. Hsu, CALT-68-1705/HUTP-91.

(5) N. Banerjee and S. Mallik, Phys. Rev. D 43 (1991) 3368.

(6) M.E. Carrington, Phys. Rev. D45 (1992) 2933.

(7) H. Meyer-Ortmanns and A. Patko's, Phys. Lett. B297 (1992) 331.

(8) P. Arnold and O. Espinosa, Phys. Rev. D47 (1993) 3546.

(9) M.B. Kislinger and P.D. Morley, Phys. Rev. D13 (1975) 2771.

(10) A.J. Niemi and G.W. Semenoff, Nucl. Phys. B230, (1984) 181.

(11) H. Matsumoto, Y. Nakano and H. Umezawa, Phys. Rev. D29 (1984) 1116.

(12) D. O'Connor, C.R. Stephens and F. Freire, Imperial/DIAS/Utrecht preprint; F. Freire, C.R. Stephens, Z. Phys. C60 (1993) 127.

(13) M.O. Taha, Phys. Rev. D26 (1982) 2766.