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Primitive Quantization of *Zitterbewegung*

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Abstract

Simple quantization of the free circular motion of a spinning point-particle around its center of mass, without further postulates, reveals a complex structure of partons, mass-energy levels, and accurate mass-ratios.

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The center of mass of a spinning point-particle is generally displaced from it, so that the momentum is not necessarily proportional to the velocity of the particle. In the quantum theory momentum and velocity are therefore represented by different operators, and classically the usual expression for momentum is supplemented by another term proportional to the spin. In particular, the velocity can be different from zero when the momentum is zero, leading to the *zitterbewegung* of the quantum theory and its classical counterpart, the free motion of a spinning particle in a circle around its own fixed center of mass.

In this paper we show how to quantize this motion, developing for the relativistic theory of a particle with spin a method similar to that used so many years ago by de Broglie and Bohr for the nonrelativistic theory of a particle without spin.

According to relativistic classical mechanics, the kinetic momentum $\mathbf{P}$ and the relativistically increased mass-energy E of a particle with spin $\boldsymbol{\sigma}$ are[1]

$$\mathbf{P} = \mathbf{P}_m + \mathbf{P}_\sigma , \quad E = E_m + E_\sigma$$

where

$$\mathbf{P}_m = m\gamma\mathbf{v} , \quad \mathbf{P}_\sigma = -\frac{\gamma^2}{c^2}\boldsymbol{\sigma} \times d\mathbf{v}/dt \tag{1}$$

$$E_m = m\gamma c^2, \quad E_\sigma = -\frac{\gamma^2}{c^2}\, \mathbf{v} \cdot (\boldsymbol{\sigma} \times d\mathbf{v}/dt)$$

For any circular motion with angular velocity $\boldsymbol{\omega}$ parallel or antiparallel to $\boldsymbol{\sigma}$, it follows that

$$\mathbf{P}_\sigma = \gamma^2 \frac{\boldsymbol{\sigma}\cdot\boldsymbol{\omega}}{c^2}\, \mathbf{v} , \quad E_\sigma = \gamma^2\beta^2\boldsymbol{\sigma}\cdot\boldsymbol{\omega}$$

and the total angular momentum is

$$\mathbf{J} \equiv \mathbf{r} \times \mathbf{P} + \boldsymbol{\sigma} = \mathbf{L}_m + \mathbf{L}_\sigma$$

with $\quad \mathbf{L}_m = m\gamma\mathbf{r} \times \mathbf{v} , \quad \mathbf{L}_\sigma = \gamma^2\boldsymbol{\sigma}$

For a particle with center of mass at rest ($\mathbf{P} = 0$, rest energy Mc^2) it follows that

$$
\begin{aligned}
&- \boldsymbol{\sigma}\cdot\boldsymbol{\omega} = mc^2/\gamma = E = Mc^2 = E_m + E_\sigma \\
&E_m = m\gamma c^2, \quad E_\sigma = -\, m\gamma\beta^2 c^2 \qquad (2)\\
&\mathbf{L}_m = -\,\beta^2\gamma^2\boldsymbol{\sigma}, \quad \mathbf{L}_\sigma = \gamma^2\boldsymbol{\sigma}, \quad \mathbf{J} = \boldsymbol{\sigma} \\
&\mathbf{r} = (\boldsymbol{\sigma}\times\mathbf{v})\,(Mc^2)^{-1}
\end{aligned}
$$

This expression for $\mathbf{r}$ describes the separation of the particle from the center of mass.[2]

Thus $\quad E_m^2 - P_m^2c^2 \equiv m^2c^4, \quad E_\sigma^2 - P_\sigma^2c^2 \equiv \mu^2c^4$

with

$$M^2 = m^2 + \mu^2, \quad \mu^2 = -m^2\beta^2 \qquad (3)$$

and

$$\frac{P_m c}{E_m} = \frac{v}{c} = \beta; \qquad \frac{P_\sigma c}{E_\sigma} = \frac{1}{\beta} \equiv \frac{V}{c} \qquad (4)$$

The single spinning particle therefore behaves <u>as if</u> it is two correlated spinless particles with equal and opposite momenta, total orbital angular momentum equal to $\boldsymbol{\sigma}$, one moving on radius r with rest mass m, velocity $\mathbf{v}$, and the other, with velocity $\mathbf{V} = \boldsymbol{\omega}\times\mathbf{R}$, moving on an orbit of radius $R = \beta^{-2}r$, with $V > c$, $R > r$ if $v < c$ and $V < c$, $r > R$ if $v > c$. Despite their different origins, these two orbits possess a remarkable symmetry. With $\Gamma^2 = (1 - B^2)^{-1}$, $B = \beta^{-1}$, $\Gamma = i\gamma\beta$:

$$
\begin{aligned}
&\mathbf{P}_\sigma = \mu\Gamma\mathbf{V} = -\, m\gamma\mathbf{v} = -\,\mathbf{P}_m \\
&E_\sigma = \mu\Gamma c^2 = -\, m\gamma\beta^2c^2 \\
&E_m = m\gamma c^2 = -\,\mu\Gamma B^2c^2 \qquad (5)\\
&\mathbf{L}_\sigma = \gamma^2\boldsymbol{\sigma} = -\,\Gamma^2B^2\boldsymbol{\sigma} \\
&\mathbf{L}_m = \Gamma^2\boldsymbol{\sigma} = -\,\gamma^2\beta^2\boldsymbol{\sigma}
\end{aligned}
$$

We define j_z, N, n:

$$\sigma_z \equiv j_z\hbar\,; \quad (\mathbf{L}_\sigma)_z \equiv N\hbar\,; \quad (\mathbf{L}_m)_z \equiv -\,n\hbar$$

or

$$N = \gamma^2 j_z \; ; \quad n = \gamma^2\beta^2 j_z \; ; \quad N - n = j_z \tag{6}$$

so that

$$\gamma^2 = N/j_z \, , \quad \beta^2 = n/N = \frac{|E_\sigma|}{E_m} \tag{7}$$

and

$$\begin{aligned} &E_m = N\hbar\omega, \quad E_\sigma = -n\hbar\omega, \quad Mc^2 = j_z\hbar\omega \\ &cP_m = cP_\sigma = (nN)^{1/2}\hbar\omega \, , \quad mc^2 = (j_z N)^{1/2}\hbar\omega \\ &r = n\lambda\!\!\!^{-} \; ; \quad R = N\lambda\!\!\!^{-} \end{aligned} \tag{8}$$

where $\quad \lambda\!\!\!^{-} = \hbar/P_m = \hbar/P_\sigma \, .$ (9)

Up to this point we have introduced some definitions, but no assumptions of any kind have been made. We now assume that n and N are integers or half integers. According to (9), the de Broglie relations therefore apply to the individual orbits, with $\lambda\!\!\!^{-}$ their common de Broglie wavelength (divided by 2π). The velocities and therefore the energies become quantized, and in particular equation (7) gives the ratio of the energies associated with the outer and inner orbits:

$$\frac{|E_\sigma|}{E_m} = \frac{n}{N} = \begin{Bmatrix} \dfrac{2n}{2n+1} & \left(j_z = \dfrac{1}{2}\right) \\ \dfrac{2n}{2n+3} & \left(j_z = \dfrac{3}{2}\right) \end{Bmatrix} \tag{7'}$$

Because E_σ is negative, this representation of a spinning particle describes a particle of energy E_m created on the inner orbit (for $v < c$) and a particle of energy $|E_\sigma|$ annihilated on the outer orbit, leaving the energy Mc^2. With N and n positive, the sign of j_z is determined by their relative magnitude and is unrelated to any external axis. Change of sign of j_z interchanges N and n and thus interchanges the orbits, leaving the ratio of the energy on the outer orbit to that on the inner orbit unchanged, and giving us two representations of the same spin and mass.

The center of mass of E_σ on $\mathbf{R}$ and E_m on $\mathbf{r}$ is at the position

$$- m\gamma\beta^2\mathbf{R} + m\gamma\mathbf{r} = 0$$

i.e., it remains fixed at the origin, as required. No forces are postulated to keep the particles in orbit, but if they are considered separately, their motion can be understood only in terms of an effective force acting on them. In fact, particle m moves <u>as if</u> it had a potential energy

$$V = m\gamma c^2 = mc^2(r^2 + r_0^2)^{1/2}r_0^{-1} \quad (r_0 = \sigma/mc) \tag{10}$$

which would produce an inward force, for fixed σ, m,

$$F = \frac{mc^2 r}{r_0(r^2 + r_0^2)^{1/2}} = \frac{m\gamma v^2}{r}$$

since $r = \gamma\beta r_0$. For $r \gg r_0$, $V = mc^2 r/r_0$ and for $r \ll r_0$, $V = mc^2$, i.e., V is constant for small distances and it increases indefinitely with r as r increases. These are the characteristics of postulated "asymptotic freedom" and "infrared slavery."

In order to test the applicability of equation (7') to the particle spectrum, examples given in Tables 1 and 2 show that there is a number of cases in which this very simple ratio is verified with remarkable accuracy, mostly within experimental error, which is sometimes as low as one part in 10^4. Since this accuracy is much greater than the splitting of isospin components, individual members of multiplets are listed in the tables. The mass parameter m is constant under any electromagnetic force. For the $n = 5/2$ state of the p-ω mass difference (see Table 1), m has the value 383.05 MeV --the lowest value of m for the two tables. The corresponding value of $|\mu|$ is 350 MeV. For the $\Sigma^-(n = 2)$, m = 535.5 MeV. These and other particles, partons or quark candidates appear naturally even at this elementary level of theory. For the case $M = m_p$, the mass of the proton, it follows that

$$E_m = (2n + 1)m_p c^2, \quad E_\sigma = -2nm_p c^2$$

as if n pairs are created on the inner orbits r_n and annihilated on the outer orbits R_n.

In Tables 1 and 2, experimental values are used in the "baryon" column (which also includes one meson). Since a number of states are repeated, both Tables may be constructed using only the mass of the proton to fix the scale. The relation $\eta'^2 = 3\omega^2/2$ emerges from the analysis of the $n = 1\Delta$ state, determining the mass of the η' as 957.62 MeV (well within experimental error) and then leaving the masses of the other states within experimental error, or less than 0.2% outside of it. The ratios between the masses of the various states then become simple ratios between their angular velocities.

In summary, the classical-quantum theory of a spinning particle with its center of mass at rest reveals a complexity of energy and mass levels that appear to be related to the $C = 0$ particle spectrum, without the need of postulating free parameters or arbitrary potentials.

n	j_z	N $=n+j_z$	Baryon[3]	Meson	Ratio n/N	Meson Mass (calculated)	Meson Mass (Experiment)[3]
3/2	3/2	3	Ξ^- 1535±0.6	$\rho(1^-)$	1/2	767.5±0.3	768.3±0.5
3/2	1/2	2	Σ^+ 1189.37±0.06	$K^{*\pm}(1^-)$	3/4	892.03±0.05	891.83±0.24
2	1/2	5/2	Λ 1115.63±0.05	$K^{*\pm}(1^-)$	4/5	892.50±0.04	891.83±0.24
5/2	1/2	3	p 938.2723 ±0.0003	$\omega(1^-)$	5/6	781.8936 ±0.0003	781.95±0.14
5/2	3/2	4	Δ 1232±2	$\rho(1^-)$	5/8	770±1	768.3±0.5
3	1/2	7/2	Σ^+ 1189.37 ±0.06	$\phi(1^-)$	6/7	1019.46 ±0.05	1019.412±0.008

Table 1. Baryons and $J^P = 1^-$ mesons.

n	j_z	N =n+j_z	Baryon[3]	Meson	Ratio n/N	Meson Mass (calculated)	Meson Mass (Experiment)[3]
1	3/2	5/2	Δ1232±2	$K^{\pm}(0^-)$	2/5	492.8±0.8	493.646 ±0.0009
2	1/2	5/2	Σ^- 1197.43 ±0.06	$\eta'(0^-)$	4/5	957.94±0.05	957.50±0.24
5/2	3/2	4	Ξ^0 1531.8±0.3	$\eta'(0^-)$	5/8	957.38±0.2	957.50±0.24
5/2	1	7/2	$\begin{pmatrix}\rho(1^-) \\ 767.5 \pm 0.3\end{pmatrix}$	$\eta(0^-)$	5/7	548.2±0.2	548.8±0.6
3	1/2	7/2	Λ1115.63 ±0.05	$\eta'(0^-)$	6/7	956.23±0.4	957.50±0.24
2	3/2	7/2	Ω1672.43 ±0.32	$\eta'(0^-)$	4/7	955.67±0.18	957.50±0.24

Table 2. Baryons and some $J^P = 0^-$ mesons.

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