Heavy Quark Spin Symmetry and Old Formalism

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Heavy quark spin symmetry in the context of old formalism of $\tilde{U}(12)$ and $SU(4)$ current algebra is reviewed.

1. Introduction

"Human existence depends upon compassion and curiosity. Curiosity without compassion is inhuman, compassion without curiosity is ineffectual". So said Prof. V. Weisskopf. It was curiosity which led Prof. Abdus Salam to seek " unity in Nature", the crowning achievement here was his unified theory of electroweak interactions for which he shared the 1979 Nobel Prize. It was compassion which led him to seek unity in " Mankind", the crowning achievement here was the creation and subsequent development of International Center for Theoretical Physics at Trieste for which he was justly awarded the Atom's for peace prize in 1967. Thus, Prof. Abdus Salam has the rare distinction of being an outstanding physicist and a brilliant administrator, concerned with the organization and development of science, especially in the third world countries. His concern for human welfare through science and technology is as great as his love for physics. He is equally at home with solving intricate problems of physics and those of isolation of scientists working in developing countries.

In 1960's, there was a great deal of interest in spin-flavor symmetry of light quarks[1], the so called static $SU(6)$. However, the derivation of several "no-go" theorems showed that non-trivial extensions of mixing a Lie global internal symmetry (e.g. flavor $SU(3)$) with the Poincare's group in order to achieve a dynamical theory of the strong interaction were impossible [2]. The spin-flavor symmetry of light quarks was recently extended [3] to heavy quarks interacting through gluons

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where we have defined $\not{v} = -i\gamma.v$ so that $\not{v}^2 = -v^2 = 1$.

Since in the rest frame $\frac{1}{2}(1 \pm \gamma_4)$ projects out the upper and lower two components of the Dirac spinor, it follows that in an arbitrary frame, they are given by

$$h_\pm = \frac{1}{2}(1 \pm \not{v})h_v \tag{7}$$

with the properties

$$\not{v}h_\pm = \pm h_\pm. \tag{8}$$

When we substitute Eq. (7) in Eq.(5) and make use of Eq.(8), we obtain

$$\begin{aligned}\{(\mp iv.D - \frac{D^2}{2M} - \frac{g_s}{4M}\sigma_{\mu\nu}G_{\mu\nu})h_\pm \\ \pm \frac{g_s}{4M}(v_\mu\gamma_\nu - v_\nu\gamma_\mu)G_{\mu\nu}h_v\} = 0.\end{aligned} \tag{9}$$

Due to the last term, the equations of motion for h_+ and h_- would be coupled. However, it is easy to see that the last term is $O(\frac{1}{M^2})$. This is obvious in the frame $\mathbf{v} = 0$, since then the last term involves "relativistic" matrices γ_i. Thus to order $\frac{1}{M}$, the equations of motion for $h_\pm$ are

$$[\mp iv.D - \frac{D^2}{2M} - \frac{g_s}{4M}\sigma_{\mu\nu}G_{\mu\nu}]h_\pm = 0. \tag{10}$$

Here h_+ annihilates a heavy quark, while h_- creates a heavy antiquark.

The corresponding Lagrangian for $h_\pm$ is [9]

$$\mathcal{L}_\pm = \bar{h}_\pm(\pm iv.D + \frac{D^2}{2M} + \frac{g_s}{4M}\sigma_{\mu\nu}G_{\mu\nu}]h_\pm. \tag{11}$$

Note that the Lagrangian (6) or (11) contains leading $\frac{1}{M}$ corrections to the kinetic energy term, namely the first term.

Now from the relation

$$i\partial_\mu\Psi(x) = [\hat{P}_\mu, \Psi(x)], \tag{12}$$

it follows through the transformation (3) that

$$i\partial_\mu h_v = M\ \not{v}v_\mu h_v + [\hat{P}_\mu, h_v], \tag{13}$$

so that

$$i\partial_\mu h_\pm = \pm M v_\mu h_\pm + [\hat{P}_\mu, h_\pm]. \tag{14}$$

The third term in the Lagrangian (6) or (11) undergoes short - distance QCD corrections and as such is multiplied by the renormalization factor

$$Z_Q(\mu) = [\alpha_s(\mu)/\alpha_s(M_Q)]^{-9/25}$$

with $Z_Q(\mu = M_Q) = 1$.

2.Spin Symmetry of Heavy Quark

In the limit $M \to \infty$, the Lagrangian $\mathcal{L}_+$ (or $\mathcal{L}_-$) given in Eq. (11) has additional symmetries not present in the full QCD Lagrangian. One such symmetry, namely, the spin symmetry of heavy quark, is reflected in the fact that the first term in the Lagrangian (11) giving the gluon interaction vertex as $\pm ig_s\frac{\lambda_A}{2}v_\mu$, makes no reference to Dirac structure at all which can couple to the spin degrees of freedom of $h_\pm$. More explicitly we can define the spin operator

$$S_{\pm i}(v) = \int h_\pm^\dagger s_i h_\pm d^3x, \tag{20a}$$

where

$$s_i = -i\gamma_5 \not{v}\gamma.\epsilon_i \tag{20b}$$

with $\epsilon_i.v = \epsilon_{i\mu}v_\mu = 0, \epsilon_i^2 = \epsilon_{i\mu}\epsilon_{i\mu} = 1, \epsilon_{j\mu}\epsilon_{k\mu} = \delta_{jk}$ [note that in the frame $\mathbf{v} = 0,\ \epsilon_{i\mu} = \delta_{i\mu}, s_i = \sigma_i$]. We note that

$$[S_{+i}, h_+] = -s_i h_+. \tag{21}$$

Hence

$$\delta h_+ = i\theta.\mathrm{s}h_+$$

$$\delta\bar{h}_+ = -i\bar{h}_+\theta.\mathrm{s} \tag{22}$$

(θ being an infinitesimal parameter), showing that the Lagrangian $\mathcal{L}_+$ in Eq. (11) is invariant under $SU(2)_+$ of heavy quark spin symmetry. Similarly $\mathcal{L}_-$ is invariant under $SU(2)_-$ of heavy antiquark spin symmetry. It means that pseudoscalar and vector mason states $|P(v)>$ and $|V(v,\epsilon)>$, containing the same heavy quark, with momentum $P_\mu^B = M_B v_\mu$ can be related to each other

$$S_+(v)|P(v)>= \eta|V(v,\epsilon)>, \tag{23}$$

where η is a phase factor [which in our metric and convention of γ-matrices, can be conveniently chosen to be $\eta = -i$]. Their masses are degenerate in this limit. This

Thus in $HQET$,

$$f_- = 0, f_+ = \xi_0(-v.v') \tag{27}$$

with

$$f_+[(m_B - m_D)^2] = 1. \tag{28}$$

Now using the relations (20), (21) and (23) for b quark we have

$$\begin{aligned} &< D^0(v')|i\bar{c}\gamma_\mu(1+\gamma_5)i\gamma_5 \not{b}\gamma.\epsilon b|B^-(v) > \\ &=< D^0(v')|i\bar{c}\gamma_\mu(1+\gamma_5)[S_+, b_+]|B^-(v) > \\ &= i < D^0(v')|i\bar{c}\gamma_\mu(1+\gamma_5)b|B^{*-}(v) > . \end{aligned} \tag{29}$$

By the use of the identity

$$\begin{aligned} \gamma_\mu\gamma_\nu\gamma_\lambda = \delta_{\mu\nu}\gamma_\lambda - \delta_{\mu\lambda}\gamma_\nu + \delta_{\nu\lambda}\gamma_\mu \\ + \epsilon_{\mu\nu\lambda\rho}\gamma_5\gamma_\rho, \end{aligned} \tag{30}$$

it is easy to see that term involving Dirac matrices on the left hand side of Eq.(29) can be written as

$$-B_{\mu\nu}\gamma_\nu(1+\gamma_5), \tag{31}$$

where

$$B_{\mu\nu} = v_\mu\epsilon_\nu - \epsilon_\mu v_\nu + \epsilon_{\mu\nu\rho\lambda}v_\rho\epsilon_\lambda. \tag{32}$$

Then using Eq. (25), we obtain

$$\begin{aligned} &< D^0(v')|i\bar{c}\gamma_\mu(1-\gamma_5)b|B^{*-}(v) > \\ &= -iB_{\mu\nu} < D^0(v')|i\bar{c}\gamma_\nu(1+\gamma_5)b|B^-(v) > \\ &= \frac{-i}{\sqrt{v_0 v_0'}} B_{\mu\nu}\xi_0(-v.v')(v+v')_\nu . \end{aligned} \tag{33}$$

Thus the four form factors in the matrix elements $< D^0(v')|i\bar{c}\gamma_\mu(1+\gamma_5)b|B^{*-}(v) >$ get related to the one universal form factor $\xi_0(-v.v')$.

As another application if we replace in eq.(29), $|D^0 >$ by $|0 >$ and c by q where $q = u, d, s$, then we obtain, on using Eqs. (31) and (32) and first part of Eq.(33),

$$\begin{aligned} &< 0|i\bar{q}\gamma_\mu(1+\gamma_5)b|B_q^*(v) > \\ &= -iB_{\mu\nu} < 0|i\bar{q}\gamma_\nu(1+\gamma_5)b|B_q(v) > . \end{aligned} \tag{34}$$

one can express

$$\Phi^{\alpha}_{\beta} = \frac{1}{4}\sum_{A}(\gamma_A)^{\alpha}_{\beta}\phi_A. \tag{41}$$

Here α, β etc. are spinor indices $\gamma_A = 1, \gamma_5, \gamma_\mu, i\gamma_\mu\gamma_5, \sigma_{\mu\sigma}$ and ϕ_A are meson fields. As we have discussed earlier, for mesons with one heavy quark one can replace h_v by $h_+ = \frac{1+\not{v}}{2}h_v$ (we will not write the suffix v explicitly). Hence in this case using

$$\sum_{A}(\gamma_A)^{\rho}_{\sigma}(1+\not{v})^{\alpha}_{\gamma}(\gamma_A)^{\gamma}_{\beta} = 4(1+\not{v})^{\alpha}_{\gamma}\delta^{\rho}_{\beta}\delta^{\gamma}_{\sigma}, \tag{42}$$

we can write for heavy vector and pseudoscalar meson fields [c.f.Eq(41)] [10]

$$H_a = \frac{1+\not{v}}{2}[\gamma_\mu P^*_{a\mu} + \gamma_5 P_a], \tag{43}$$

where $a = 1, 2, 3$ for $u, d,$ and s quarks and $P^*_{a\mu}$ and P_a are annihilation operators normalized as

$$< 0|P^*_{a\mu}|Q\bar{q}_a(1^-) >= \epsilon_\mu \tag{44}$$

$$< 0|P_a|Q\bar{q}_a(0^-) >= 1. \tag{45}$$

We define the adjoint field

$$\bar{H}_a = \gamma_4 H^\dagger_a \gamma_4 = [-\bar{P}^*_{a\mu}\gamma_\mu - \bar{P}_a\gamma_5]\frac{1+\not{v}}{2}. \tag{46}$$

We note that

$$\not{v}H_a = H_a$$

$$H_a \not{v} = -H_a. \tag{47}$$

The Lorentz symmetry and $SU(2)$ spin symmetry of heavy quark are now obvious in view of Eqs. (20) and (21). The spin symmetry which relates P_a and $P^*_{a\mu}$ is automatically incorporated in Eq.(43).

In case of spinor field $\Psi(x)$, the wave function can be written

$$< 0|\Psi(x)|p >\sim e^{ipx}u(p). \tag{48}$$

Thus in view of Eqs. (43), (44) and (45), we can write for mesons containing a heavy quark ($P = B$ or D)

$$< 0|H_a|P_a(v) >= \frac{1+\not{v}}{2}\gamma_5$$

Thus for $v.v' = -1$ which coresponds to $t = (m_B^* - m_B)^2 = \delta_B^2 \rightarrow 0$, we get in the heavy quark limit

$$F_0(0) = 4m_B^2 F_+(0) = 4m_B^2 F_-(0) = 2m_B A_1(1). \tag{55}$$

Note that unlike $A(1)$ the normalization for $A_1(1)$ is not fixed. We shall make use of the result (55) in the next section

3. Heavy Quark Limit and Chiral Symmetry

There are some intersting consequences of heavy quark limit when it is used in conjunction with the chiral symmetry consisting of partial conservation of axial vector current $(PCAC)$ and current algebra. As an example, let us consider the $SU(4)$ algebra generated by the axial charges [11,12]

$$[S_i^{(+)}, S_j^{(-)}] = 2\delta_{ij} I_3, \tag{56}$$

where

$$S_i^{(+)} = -\int A_i^{(+)}(\mathrm{x}, 0) d^3x, \tag{57}$$

$$A_\mu^{(+)} = i\bar{u}\gamma_\mu\gamma_5 d, \;\; A_i^{(+)} = -u^\dagger \sigma_i d, \sigma_i = -i\gamma_4\gamma_i\gamma_5 \tag{58}$$

and

$$I_3 = -i\int V_{34}(\mathrm{x}, 0) d^3x, V_{i4} = \frac{1}{2} q^\dagger \tau_3 q. \tag{59}$$

Taking the matrix elements of Eq.(56) between $|B^+(p)>$, we get

$$\begin{aligned} &< B^+(p)|[S_i^{(+)}, S_j^{(-)}]|B^+(p)> \\ &= 2\delta_{ij} < B^+(p)|I_3|B^+(p)> \\ &= \delta_{ij}. \end{aligned} \tag{60}$$

In order to evaluate the left hand side, we introduce a complete set of allowed states, the lightest of which is 1^- state $|B^{*o}(p')>$, a member of the isopin multiplet (B^{*+}, B^{*o}). We take $i = j = 3$ and obtain

$$\int d^3p' | < B^+(p)|S_3^{(+)}|B^{*o}(p')> |^2 + \sum_{n \neq \vec{B}^o} | < B^+(p)|S_3^{(+)}|n> |^2 = 1. \tag{61}$$

Note that the left hand side is positive definite. We work in the rest frame of B^+ meson, $\mathbf{p} = 0$. Then using the Eq. (52), we obtain $[t = \delta_B^2 \rightarrow 0]$

$$\frac{1}{4m_B m_B^*} F_0^2(0) = 1 - \delta^2, \tag{62}$$

where δ^2 is the contribution from the second term on the left hand of Eq.(61). In this conection it may be noted that the possible contributions of p-wave states $D_J(J = 0,1,2)$ to δ^2 vanish in the static limit in the frame $\mathbf{p} = 0$, the couplings involved being momentum dependent. PCAC applied to the axial vector current appearing on the left hand side of Eq. (52) gives

$$2f_\pi g_{B^*B\pi} = [F_0(0) + (m_{B^*}^2 - m_B^2)F_+(0)], \tag{63}$$

where $g_{B^*B\pi}$ is defined by

$$< B^+(p')|j_\pi|B^{*o}(p) >= \frac{1}{\sqrt{4p_0p_0'}} g_{B^*B\pi}(2p' - p).\epsilon \tag{64}$$

In the normalization used above $f_\pi = 130$ MeV. In the heavy quark limit using Eqs. (55)

$$F_+(0)/F_0(0) \sim \frac{1}{m_B^2}, (m_{B^*}^2 - m_B^2) \sim 2m_B\delta_B \quad \text{i.e.}$$

is constant. In this limit, Eqs. (62) and (63) give [12]

$$\begin{aligned} g_{B^*B\pi} &\simeq \frac{(m_B m_B^*)^{1/2}}{f_\pi}(1-\delta^2)^{1/2} \\ &\leq \frac{(m_B m_B^*)^{1/2}}{f_\pi}. \end{aligned} \tag{65}$$

Thus we can write

$$g_{B^*B\pi} = \lambda \frac{m_B}{f_\pi}, \tag{66}$$

where $\lambda = (1-\delta^2)^{1/2}[1 + 0(\frac{\delta_B}{m_B})] \leq 1$. In other words

$$g_{B^*B\pi} = \frac{m_B}{f_\pi} \tag{67}$$

gives an upper limit on $g_{B^*B\pi}$. The following remarks are in order. For ρ and K^* meson, the $SU(6)$ relation $g_{\rho\pi\pi} = \frac{\sqrt{2}m_\rho}{f_\pi}, g_{K^*K\pi} = m_{K^*}/\sqrt{2}f_\pi$ were known since 1966 [13,14]. These relations differ from $KSRF$ value [15], $g_{\rho\pi\pi} = \frac{m_\rho}{f_\pi}$ and $g_{K^*K\pi} = \frac{m_{K^*}}{2f_\pi}$, by a factor of $\sqrt{2}$. Experimetally $SU(6)$ relations for ρ and K^* are not good. But for the heavy quarks the relation (67) is expected to hold for the following resons: (i) For $SU(4)$ algebra considered here, the natural frame, is $\mathbf{p} = 0$ frame, a good frame for heavy mesons, for which static limit is a good approximation. The spin of B or D meson is flipped with the emission of a pion with negligihle recoil (ii) The $PCAC$ relation (63) reduces to $F_0 = 2f_\pi g_{B^*B\pi}$ in the heavy quark limit. This relation is exact in this limit ($m_B \to \infty$) and is crucial in deriving the relation (48). This is not the case for ρ and K^* meson (iii) Eq.(67) was dervied in reference [16], using $PCAC$ and non-relativistic bound state picture for mesons with a heavy quark. The above derivation of Eq.(67) thus shows a close relation between bound state picture and the algebraic approach followed above . The relation (66) was also obtained in ref [17], using Adler- Weisberger type sum rule with only pole contribution in the sum rule. Note for this type of algebra, the natural frame is $\mathbf{p} \to \infty$ frame. How does the relation (66) compare with experiment?. There is no experimental information for $g_{B^*B\pi}$. However, for D meson there are estimates from the analysis of D^* radiative decay [17] and give $\lambda \sim 0..5 - 0.8$ consistant with $\lambda \leq 1$ found in Eq. (66).

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