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Weyl-van der Waerden-formalism for Helicity Amplitudes of Massive Particles

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Abstract:

We provide a new method for constructing helicity eigenstates both for massive spin-$\frac{1}{2}$- and spin-1-particles within the Weyl-van der Waerden-formalism. To show that our formulation is superior to the conventional method for the calculation of physical helicity amplitudes involving massive particles we treat the example $e^-\gamma \to e^-Z\gamma$.

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1 Introduction

The application of Weyl-van der Waerden (WvdW)-spinors [1] became more and more popular in recent years. Using WvdW-spinors the construction of transition matrix elements for massles external particles proves to be very simple. Therefore this formalism is e.g. well suited for the calculation of many gluon processes in QCD [2, 3]. As WvdW-spinors were originally introduced only for massless particles an extension to massive particles became necessary for applications in the electroweak standard model. One such possibility is described in [3] and used in [4] for the evaluation of the radiative process $e^+e^- \to W^+W^-\gamma$. For the sake of formal simplicity these authors did not use helicity states for the massive W-bosons. Although their formalism provides a method for calculating unpolarized cross sections it becomes complicated when applied to helicity amplitudes. Here we propose a different way to construct simple WvdW-spinors for massive spin-$\frac{1}{2}$- and spin-1-particles but with the additional advantage of describing helicity eigenstates. To illustrate the usefulness and the transparence of our procedure we compare it to the conventional method whenever possible. The treatment of the example $e^-\gamma \to e^-Z\gamma$ finally illustrates how the calculation of helicity amplitudes involving massive particles works in practice.

The paper is organized as follows: After fixing our notations and conventions in Sect. 2 we describe in Sect. 3 and Sect. 4 how to build helicity eigenstates for free spin-$\frac{1}{2}$- and spin-1-particles with WvdW-spinors, respectively. In Sect. 5 we give the rules for translating Feynman-rules into the WvdW-spinor technique which we apply to the example $e^-\gamma \to e^-Z\gamma$ in Sect. 6.

2 Notation and conventions

According to the two non-equivalent two-dimensional irreducible representations of the Lorentz-group we distinguish covariant and contravariant WvdW-spinors ψ_A and $\psi^{\dot{A}}$, respectively[1]. The transition between the different representations is achieved by complex conjugation indicated by dotting (undotting) of the spinor indices, i.e. $\psi_{\dot{A}} = (\psi_A)^*$ for instance. The indices are raised and lowered with the spinor metric ϵ^{AB}:

$$\epsilon^{AB} = \epsilon^{\dot{A}\dot{B}} = \epsilon_{AB} = \epsilon_{\dot{A}\dot{B}} = \begin{pmatrix} 0 & +1 \\ -1 & 0 \end{pmatrix}, \qquad \psi^A = \epsilon^{AB}\psi_B, \qquad \psi_{\dot{A}} = \epsilon_{\dot{A}\dot{B}}\psi^{\dot{B}}. \tag{2.1}$$

The antisymmetry of the spinor metric implies the antisymmetry of the spinorial product $\langle\phi\psi\rangle$ which is defined as

$$\langle\phi\psi\rangle = \phi_A\psi^A = \phi_A\epsilon^{AB}\psi_B, \qquad \langle\phi\psi\rangle^* = \phi_{\dot{A}}\psi^{\dot{A}} = \phi_{\dot{A}}\epsilon^{\dot{A}\dot{B}}\psi_{\dot{B}}. \tag{2.2}$$

$$\Rightarrow \quad \langle\phi\psi\rangle = -\langle\psi\phi\rangle, \qquad \langle\phi\phi\rangle = 0, \tag{2.3}$$

Moreover the Schouten-identity for the metric ϵ^{AB} implies that

$$\langle ab\rangle\langle cd\rangle + \langle ac\rangle\langle db\rangle + \langle ad\rangle\langle bc\rangle = 0. \tag{2.4}$$

[1] We denote the indices for WvdW-spinors with capital letters throughout.

The expression of a Minkowski four-vector $k^\mu = (k^0, \mathbf{k})$ in terms of WvdW-spinors is obtained by contraction with $\sigma^{\mu,\dot{A}B} = (\sigma^0, \sigma^k)$, where σ^0 denotes the 2×2-unit matrix and σ^k $(k = 1, 2, 3)$ the standard Pauli-matrices:

$$K_{\dot{A}B} = \sigma^\mu_{\dot{A}B} k_\mu = \begin{pmatrix} k^0 + k^3 & k^1 + ik^2 \\ k^1 - ik^2 & k^0 - k^3 \end{pmatrix}. \tag{2.5}$$

Note that there has to be taken care about the positions of the indices and their dots. For clarity we cite some relations for the σ-matrices:

$$\sigma^\mu_{\dot{A}B} = \epsilon_{\dot{A}\dot{C}}\, \epsilon_{BD}\, \sigma^{\mu,\dot{C}D}, \qquad \sigma^\mu_{A\dot{B}} = (\sigma^\mu_{\dot{A}B})^* = (\sigma^0, -\sigma^k), \tag{2.6}$$

$$\sigma^\mu_{\dot{A}B}\, \sigma^{\nu,\dot{A}B} = 2g^{\mu\nu}, \qquad \sigma^\mu_{\dot{A}B}\, \sigma_{\mu,C\dot{D}} = 2\epsilon_{\dot{A}\dot{D}}\, \epsilon_{BC}. \tag{2.7}$$

So the Minkowski inner product is translated into

$$2k_\mu p^\mu = k_\mu\, 2g^{\mu\nu}\, p_\nu = k_\mu\, \sigma^\mu_{\dot{A}B} \sigma^{\nu,\dot{A}B}\, p_\nu = K_{\dot{A}B} P^{\dot{A}B}. \tag{2.8}$$

Since the matrix $K_{\dot{A}B}$ is hermitian for a real four-vector k^μ it can be decomposed into its eigenvectors $n_{i,A}$ $(i = 1, 2)$ and eigenvalues λ_i as follows

$$K_{\dot{A}B} = \sum_{i=1,2} \lambda_i\, n_{i,\dot{A}}\, n_{i,B}, \qquad \lambda_{1,2} = k^0 \pm |\mathbf{k}|,$$

$$n_{1,A} = \begin{pmatrix} e^{-i\phi} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2} \end{pmatrix}, \qquad n_{2,A} = \delta_{A\dot{B}} n_1^{\dot{B}} = \begin{pmatrix} \sin\frac{\theta}{2} \\ -e^{+i\phi} \cos\frac{\theta}{2} \end{pmatrix} \tag{2.9}$$

with θ, ϕ denoting the polar and azimuthal angles of $\mathbf{k}$. The phases of the $n_{i,A}$ are chosen such that the orthonormality relations simply read

$$\langle n_i n_i \rangle = 0, \qquad \langle n_2 n_1 \rangle = -\langle n_1 n_2 \rangle = +1. \tag{2.10}$$

Note also the important relation for the eigenvalues λ_i:

$$\det(K_{\dot{A}B}) = \lambda_1 \lambda_2 = k_\mu k^\mu. \tag{2.11}$$

Let's have a closer look onto the special case of a light-like vector l^μ $(l_\mu l^\mu = 0)$. From (2.9) we deduce that

$$\lambda_1 = 2l^0, \qquad \lambda_2 = 0 \tag{2.12}$$

and consequently $L_{\dot{A}B}$ factorizes into a single product of two spinors

$$L_{\dot{A}B} = 2l^0\, n_{1,\dot{A}}\, n_{1,B} = l_{\dot{A}} l_B. \tag{2.13}$$

In this context $l_A = \sqrt{2l^0}\, n_{1,A}$ is often called a momentum spinor.

Although the representation (2.13) of light-like vectors was already applied by several authors, e.g. in [2, 3], their treatment of massive four-vectors k^μ was quite different. The authors of [3, 4] propose to split k^μ into two light-like vectors a^μ and b^μ as follows:

$$k^\mu = \alpha\, a^\mu + b^\mu, \qquad K_{\dot{A}B} = \alpha\, a_{\dot{A}} a_B + b_{\dot{A}} b_B. \tag{2.14}$$

Here a^μ can be chosen as an arbitrary light-like vector whereas the constant α and the vector b^μ (up to a phase) are determined by k^μ and a^μ:

$$b_B = \frac{K_{\dot{A}B} a^{\dot{A}}}{\sqrt{P_{\dot{C}D} a^{\dot{C}} a^D}}, \qquad \alpha = \frac{k_\mu k^\mu}{\langle ab\rangle \langle ab\rangle^*}. \tag{2.15}$$

Finally we remark that the special choice

$$a_A = \sqrt{\lambda_2}\, n_{2,A}, \qquad b_A = \sqrt{\lambda_1}\, n_{1,A}, \qquad \alpha = 1 \tag{2.16}$$

with the eigenvectors $n_{i,A}$ and eigenvalues λ_i of (2.9) reproduces all our results.

3 Spin-$\frac{1}{2}$-particles

We construct massive spinors within the WvdW-formalism by the decomposition of the Dirac-spinor Ψ into two WvdW-spinors ϕ_A, $\chi^{\dot{A}}$ and the explicit insertion of the chiral representation for the Dirac-matrices

$$\Psi = \begin{pmatrix} \phi_A \\ \chi^{\dot{A}} \end{pmatrix}, \qquad \gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu_{A\dot{B}} \\ \sigma^{\mu,\dot{A}B} & 0 \end{pmatrix}, \qquad \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} \sigma^0 & 0 \\ 0 & -\sigma^0 \end{pmatrix}. \tag{3.1}$$

Then the momentum space Dirac-equation is equivalent to

$$K_{A\dot{B}}\, \chi^{(\pm),\dot{B}} = \pm m\, \phi_A^{(\pm)}, \qquad K^{\dot{A}B}\, \phi_B^{(\pm)} = \pm m\, \chi^{(\pm),\dot{A}}, \qquad k_\mu k^\mu = m^2. \tag{3.2}$$

Remembering that in our case $K_{\dot{A}B} = \sum_{i=1,2} \lambda_i\, n_{i,\dot{A}}\, n_{i,B}$ and $\lambda_1\lambda_2 = m^2$ we can easily construct the particle/antiparticle solutions $\Psi_{k,i}^{(\pm)}$ with momentum $k^\mu = (k^0, \mathbf{k})$ in terms of the eigenvectors $n_{i,A}$:

$$\Psi_{k,1}^{(\pm)} = \begin{pmatrix} \sqrt{\lambda_1}\, n_{1,A} \\ \mp\sqrt{\lambda_2}\, n_2^{\dot{A}} \end{pmatrix}, \qquad \Psi_{k,2}^{(\pm)} = \begin{pmatrix} \pm\sqrt{\lambda_2}\, n_{2,A} \\ \sqrt{\lambda_1}\, n_1^{\dot{A}} \end{pmatrix}. \tag{3.3}$$

The $\Psi_{k,i}^{(\pm)}$ obey the standard normalization

$$\Psi_{k,i}^{(\pm)\dagger} \Psi_{k,j}^{(\tau)} = 2k^0\, \delta_{\tau\pm}\, \delta_{ij}, \qquad \Psi_{k,i}^{(\pm)\dagger} \gamma^0 \Psi_{k,j}^{(\tau)} = \pm 2m \delta_{\tau\pm} \delta_{ij}. \tag{3.4}$$

The important point is that they indeed form eigenstates of the helicity projector

$$\Sigma_k^\pm = \frac{1}{2}(1 \pm \gamma^5 \not{s}_k), \qquad s_k^\mu = \frac{k^0}{m}\frac{k^\mu}{|\mathbf{k}|} - g^{\mu 0}\frac{m}{|\mathbf{k}|}. \tag{3.5}$$

In particular Σ_k^+ projects onto $\Psi_{k,1}^{(+)}$ and $\Psi_{k,2}^{(-)}$, Σ_k^- onto $\Psi_{k,1}^{(-)}$ and $\Psi_{k,2}^{(+)}$.

In the conventional method mentioned above there can also be given free particle solutions $\widetilde{\Psi}_{k,i}^{(\pm)}$ in terms of the WvdW-spinors a_A and b_A, however, in general these do not form helicity eigenstates. We cite the $\widetilde{\Psi}_{k,i}^{(\pm)}$ from [3]:

$$\widetilde{\Psi}_{k,1}^{(\pm)} = \begin{pmatrix} b_A \\ \mp\sqrt{\alpha}\, a^{\dot{A}} \end{pmatrix}, \qquad \widetilde{\Psi}_{k,2}^{(\pm)} = \begin{pmatrix} \pm\sqrt{\alpha}\, a_A \\ b^{\dot{A}} \end{pmatrix}, \tag{3.6}$$

Finally we remark that in the case of massless fermions the two alternative formulations coincide and the helicity spinors simply consist of the momentum spinor k_A:

$$\Psi^{(\pm)}_{k,1} = \begin{pmatrix} k_A \\ 0 \end{pmatrix}, \qquad \Psi^{(\pm)}_{k,2} = \begin{pmatrix} 0 \\ k^{\dot{A}} \end{pmatrix}. \tag{3.7}$$

4 Spin-1-particles

The polarization vectors $\varepsilon^\mu_i(k)$ for a massive spin-1-particle with momentum k^μ fulfill the relations

$$k^\mu \varepsilon_{i,\mu}(k) = 0, \qquad \varepsilon^\mu_i(k)\,\varepsilon^*_{j,\mu}(k) = -\delta_{ij}, \quad i,j = 0,\pm, \qquad k^\mu k_\mu = m^2. \tag{4.1}$$

In terms of WvdW-spinors (4.1) reads

$$K^{B\dot{A}} \varepsilon_{i,\dot{A}B}(k) = 0, \qquad \varepsilon_{i,\dot{A}B}(k)\,\varepsilon^{B\dot{A}}_j(k) = -2\delta_{ij}, \quad i,j = 0,\pm, \tag{4.2}$$

which is solved by the following polarization spinors:

$$\begin{aligned} \varepsilon_{+,\dot{A}B}(k) &= \sqrt{2}\, n_{2,\dot{A}}\, n_{1,B}, \qquad \varepsilon_{-,\dot{A}B}(k) = \varepsilon_{+,B\dot{A}}(k) = \sqrt{2}\, n_{1,\dot{A}}\, n_{2,B}, \\ \varepsilon_{0,\dot{A}B}(k) &= \varepsilon_{0,B\dot{A}}(k) = \frac{1}{m}(\lambda_1\, n_{1,\dot{A}}\, n_{1,B} - \lambda_2\, n_{2,\dot{A}}\, n_{2,B}). \end{aligned} \tag{4.3}$$

After going back to Minkowski-vectors one can easily verify that these solutions correspond to the helicity vectors

$$\begin{aligned} \varepsilon^\mu_\pm &= \frac{e^{\mp i\phi}}{\sqrt{2}}(0, -\cos\theta\cos\phi \pm i\sin\phi, -\cos\theta\sin\phi \mp i\cos\phi, \sin\theta), \\ \varepsilon^\mu_0 &= s^\mu_k = \frac{k^0}{m}(\frac{|\mathbf{k}|}{k^0}, \cos\phi\sin\theta, \sin\phi\sin\theta, \cos\theta). \end{aligned} \tag{4.4}$$

On the other hand the polarization vectors $\tilde{\varepsilon}_i$ of [3]

$$\tilde{\varepsilon}_{+,\dot{A}B}(p) = \frac{\sqrt{2}\, a_{\dot{A}}\, b_B}{\langle ba\rangle^*}, \quad \tilde{\varepsilon}_{-,\dot{A}B}(p) = \frac{\sqrt{2}\, b_{\dot{A}}\, a_B}{\langle ba\rangle}, \quad \tilde{\varepsilon}_{0,\dot{A}B}(p) = \frac{1}{m}(b_{\dot{A}}\, b_B - \alpha\, a_{\dot{A}} a_B), \tag{4.5}$$

which are constructed in terms of a_A and b_A, in general do not form helicity eigenstates.

In contrast to the case of spin-$\frac{1}{2}$-particles the zero mass limit for vector bosons confronts us with a new physical situation. For $m \to 0$ the vector ε_0 does not exist since the longitudinal polarization yields no physical state for massless spin-1-particles. The corresponding degree of freedom manifests itself in the arbitrariness of a gauge for the remaining two polarization vectors $\varepsilon_\pm$. In the language of WvdW-spinors this fact can be expressed very elegantly:

$$\varepsilon_{+,\dot{A}B} = \frac{\sqrt{2}\, g_{+,\dot{A}}\, k_B}{\langle kg_+\rangle^*}, \qquad \varepsilon_{-,\dot{A}B} = \frac{\sqrt{2}\, k_{\dot{A}}\, g_{-,B}}{\langle kg_-\rangle} \tag{4.6}$$

with $g_{\pm,A}$ denoting arbitrary gauge spinors linearly independent of k_A. Note also that the transversality condition $k_\mu \varepsilon^\mu_\pm = 0$ is still fulfilled for any $g_{\pm,A}$, i.e. for the $\varepsilon_\pm$ of (4.6) the Lorentz-gauge is fixed. In the calculation of transition matrix elements suitable choices of the gauge spinors may simplify the explicit algebraic expressions drastically.

incoming f:	$\Psi^{(+)}_{k,i}$	outgoing f:	$\overline{\Psi}^{(+)}_{k,i}$
incoming anti-f:	$\overline{\Psi}^{(-)}_{k,i}$	outgoing anti-f:	$\Psi^{(-)}_{k,i}$
incoming V:	$\varepsilon_{i,\dot{A}B}(k)$	outgoing V:	$\varepsilon^{*}_{i,\dot{A}B}(k)$
$B \quad k \quad A$	f propagator:	$\frac{i}{(k^2-m_f^2)}\begin{pmatrix} m_f\,\delta^B_A & K_{A\dot{B}} \\ K^{\dot{A}B} & m_f\,\delta^{\dot{A}}_{\dot{B}} \end{pmatrix}$	
$AB \quad CD$	V propagator:	$\frac{2i\,\epsilon_{\dot{A}\dot{C}}\epsilon_{BD}}{(k^2-m_V^2)}$	
C, D, AB	$f_1 f_2 V$-vertex:	$i\,e\,g^{\tau}_{f_1 f_2 V}\begin{pmatrix} 0 & \delta_{\tau-}\delta^{\dot{A}}_{\dot{C}}\delta^B_D \\ \delta_{\tau+}\epsilon^{\dot{A}\dot{D}}\epsilon^{BC} & 0 \end{pmatrix}$	

Table 5.1: Some Feynman-rules for fermions (f) and vector bosons (V).

5 Feynman-rules for the WvdW-formalism

For completeness we briefly explain how to translate Feynman-rules given in terms of the usual Dirac-algebra and Minkowski-vectors into the WvdW-formalism. The procedure is straightforward and easily described:

(i) External particles: use the free spinors $\Psi^{(\pm)}_{p,i}$ and $\varepsilon_{i,\dot{A}B}$ of (3.3) and (4.3).

(ii) Fermion propagator: insert the chiral representation (3.1) for the Dirac-matrices.

(iii) Vector boson propagator: contract the metric tensor $g_{\mu\nu}$ with $\sigma^{\mu}_{A\dot{B}}\,\sigma^{\nu}_{C\dot{D}}$.

(iv) Vertices: insert the chiral representation (3.1) for the Dirac-matrices and contract each Lorentz-index μ with $\frac{1}{2}\sigma^{\dot{A}B}_{\mu}$.

In Tab. 5.1 we summarize the rules we need for the example of Sect. 6; the list could be easily completed with the Feynman-rules of [5].

6 The example $e^-\gamma \to e^-Z\gamma$

The WvdW-spinor technique is well suited for the calculation of real bremsstrahlung corrections to electroweak scattering processes. Here we turn to the amplitude of the radiative process $e^-\gamma \to e^-Z\gamma$ treating the Z boson with both different methods explained above. The full evaluation of this bremsstrahlung process, which has to be combined with the virtual radiative corrections to $e^-\gamma \to e^-Z$ [6], will be published elsewhere.

The Feynman diagrams corresponding to

$$e^-(p,\sigma) + \gamma(k,\lambda) \to e^-(p',\sigma') + Z(k_Z,\lambda_Z) + \gamma(k',\lambda')$$

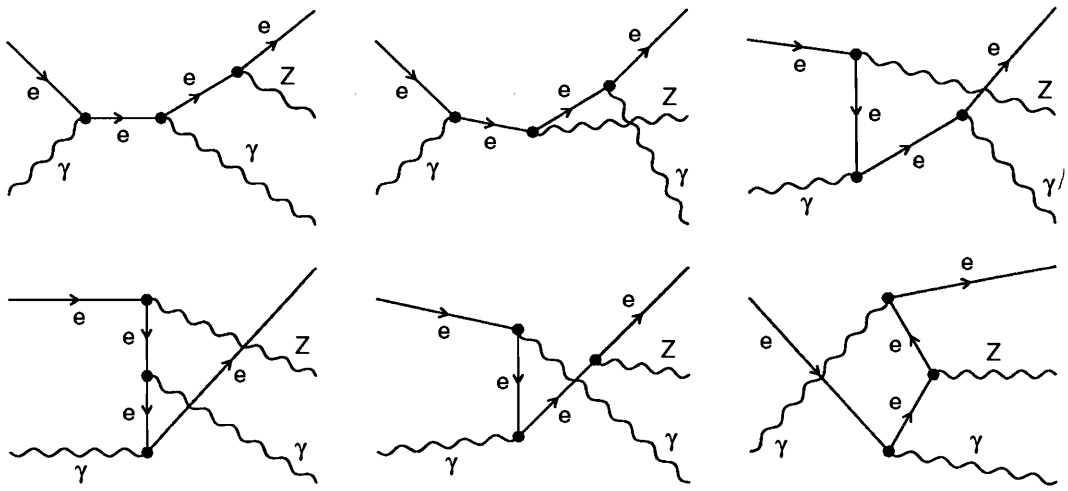

Figure 6.1: Feynman diagrams for $e^-\gamma \to e^-Z\gamma$.

in lowest order are shown in Fig. 6.1. For the relevant energies the electron mass m_e is negligible provided the direction of the photon momenta does not coincide with the ones of the electron. Since there is a standard method [7] for handling finite mass effects arising from collinear photon emission we can restrict our calculation to the case $m_e = 0$. Consequently the helicity of the electron is conserved and we set $\sigma' = \sigma$ in the following. Moreover the amplitudes $\mathcal{M}_{R,L}$ for right-handed and left-handed electrons are related by

$$g^-_{eeZ}\,\mathcal{M}_R(\lambda,\lambda',\lambda_Z) = -g^+_{eeZ}\,\mathcal{M}_L(-\lambda,-\lambda',-\lambda_Z)^* \tag{6.1}$$

so that it suffices to give $\mathcal{M}_L$. With the Feynman-rules of Sect. 5 and the spinors for massless particles of Sects. 3 and 4 we arrive at:

$$\begin{aligned}
\mathcal{M}_L(+1,-1,\lambda_Z) &= -2ie^3 g^-_{eeZ}\,\frac{p^{\dot A}K_{Z,\dot A B}\varepsilon_Z^{*,\dot C B}p_{\dot C}\,\langle pp'\rangle^*}{\langle pk\rangle^*\langle p'k\rangle^*\langle pk'\rangle^*\langle p'k'\rangle^*},\\
\mathcal{M}_L(-1,+1,\lambda_Z) &= -2ie^3 g^-_{eeZ}\,\frac{p'_A\varepsilon_Z^{*,\dot B A}K_{Z,\dot B C}p'^C\,\langle pp'\rangle}{\langle pk\rangle\langle p'k\rangle\langle pk'\rangle\langle p'k'\rangle},\\
\mathcal{M}_L(+1,+1,\lambda_Z) &= 2ie^3 g^-_{eeZ}\left[-\frac{k'_{\dot A}\varepsilon_Z^{*,\dot A B}p'_B}{\langle pk\rangle^*\langle pk'\rangle}\left(\frac{\langle pp'\rangle^*}{\langle p'k\rangle^*}-\frac{\langle pk'\rangle^*\langle kk'\rangle}{(p'+k_Z)^2}\right)+\frac{k'_{\dot A}\varepsilon_Z^{*,\dot A B}k_B}{\langle p'k\rangle^*\langle pk'\rangle}\right.\\
&\quad+\frac{p_{\dot A}\varepsilon_Z^{*,\dot A B}k_B}{\langle p'k\rangle^*\langle p'k'\rangle}\left(\frac{\langle pp'\rangle}{\langle pk'\rangle}-\frac{\langle p'k\rangle\langle kk'\rangle^*}{(p-k_Z)^2}\right)+\frac{p_{\dot A}\varepsilon_Z^{*,\dot A B}p'_B}{\langle pk\rangle^*\langle p'k\rangle^*\langle pk'\rangle\langle p'k'\rangle}\\
&\quad\left.\times\left(-\langle pp'\rangle^*\langle pp'\rangle+\frac{\langle pk\rangle\langle p'k\rangle^*\langle pk'\rangle^*\langle p'k'\rangle}{(p'+k_Z)^2}+\frac{\langle pk\rangle^*\langle p'k\rangle\langle pk'\rangle\langle p'k'\rangle^*}{(p-k_Z)^2}\right)\right],\\
\mathcal{M}_L(-1,-1,\lambda_Z) &= \left\{\mathcal{M}_L(+1,+1,\lambda_Z)\text{ with }k^\mu\leftrightarrow -k'^\mu,\text{ i.e. }k_{\dot A}k_B\leftrightarrow -k'_{\dot A}k'_B\right\}.
\end{aligned} \tag{6.2}$$

Note that we used beside momentum conservation also the Schouten-identity (2.4) to obtain compact expressions for $\lambda = -\lambda' = \pm 1$. Now we want to compare the two different methods of treating the massive Z-boson.

Helicity amplitudes are directly constructed by the insertion of (2.9) and (4.3) for $K_{Z,\dot{A}B}$ and $\varepsilon^*_{Z,\dot{A}B}$, respectively. For the sake of simplicity we merely give the terms of (6.2) involving $K_{Z,\dot{A}B}$ and $\varepsilon^*_{Z,\dot{A}B}$:

$$p^{\dot{A}} K_{Z,\dot{A}B} \varepsilon_Z^{*,\dot{C}B} p_{\dot{C}} = \begin{cases} +\sqrt{2}\lambda_1(\langle pn_1\rangle^*)^2 & \text{for } \lambda_Z = +1, \\ -\sqrt{2}\lambda_2(\langle pn_2\rangle^*)^2 & \text{for } \lambda_Z = -1, \\ -2M_Z\langle pn_1\rangle^*\langle pn_2\rangle^* & \text{for } \lambda_Z = 0, \end{cases}$$

$$p'_A \varepsilon_Z^{*,\dot{B}A} K_{Z,\dot{B}C} p'^C = \begin{cases} -\sqrt{2}\lambda_2(\langle p'n_2\rangle)^2 & \text{for } \lambda_Z = +1, \\ +\sqrt{2}\lambda_1(\langle p'n_1\rangle)^2 & \text{for } \lambda_Z = -1, \\ -2M_Z\langle p'n_1\rangle\langle p'n_2\rangle & \text{for } \lambda_Z = 0, \end{cases}$$

$$\phi_{\dot{A}} \varepsilon_Z^{*,\dot{A}B} \psi_B = \begin{cases} \sqrt{2}\langle \phi n_2\rangle^*\langle \psi n_1\rangle & \text{for } \lambda_Z = +1, \\ \sqrt{2}\langle \phi n_1\rangle^*\langle \psi n_2\rangle & \text{for } \lambda_Z = -1, \\ \left(\lambda_1\langle \phi n_1\rangle^*\langle \psi n_1\rangle - \lambda_2\langle \phi n_2\rangle^*\langle \psi n_2\rangle\right)/M_Z & \text{for } \lambda_Z = 0, \end{cases} \tag{6.3}$$

$$\text{with } \phi, \psi = p, p', k, k'.$$

Inspecting (6.2) with respect to the method of splitting $K_{Z,\dot{A}B}$ into a_A and b_A as outlined in Sect. 2 we perceive that there is no choice of a_A which simplifies the algebraic expressions considerably. Each of the possibilities $a_A = p_A, p'_A, k_A, k'_A$ forces only a few terms of (6.2) to vanish. On the other side the construction of helicity amplitudes would require the introduction of non-trivial linear combinations of the polarization vectors $\tilde{\varepsilon}_{Z,i}$ so that the corresponding analytical expressions become much more complicated. We refrain here from writing down the explicit expressions as it does not yield further insights and may easily be done by the interested reader.

Finally we point out that the example $e^-\gamma \to e^-Z\gamma$ is typical concerning the applicability of the constructed helicity spinors. In the same way we treated the more complicated radiative process $e^-\gamma \to W^-\nu_e\gamma$ which completes the discussion of the virtual corrections to $e^-\gamma \to W^-\nu_e$ [8].

7 Summary

Within the Weyl-van der Waerden-spinor technique we have provided a new method for the direct construction of helicity amplitudes for scattering processes involving massive spin-$\frac{1}{2}$- or spin-1-particles. With the help of the example $e^-\gamma \to e^-Z\gamma$ we have compared this method to the approved way of evaluating amplitudes without refering to helicity eigenstates, which was proposed and applied by other authors before. Although both alternatives prove to be suitable for the calculation of unpolarized cross sections our method turns out to be superior to the conventional treatment concerning the polarizations of massive particles. Because of their simplicity and clarity the stated helicity spinors may also represent a suitable form for the evaluation of polarized many particle cross sections by means of computer algebraic methods.

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