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The confined-deconfined interface tension, wetting, and the spectrum of the transfer matrix

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The reduced tension σ_{cd} of the interface between the confined and the deconfined phase of $SU(3)$ pure gauge theory is determined from numerical simulations of the first transfer matrix eigenvalues. At $T_c = 1/L_t$ we find $\sigma_{cd} = 0.139(4)T_c^2$ for $L_t = 2$. The interfaces show universal behavior because the deconfined-deconfined interfaces are completely wet by the confined phase. The critical exponents of complete wetting follow from the analytic interface solutions of a $\mathbb{Z}(3)$-symmetric Φ^4 model in three dimensions. We find numerical evidence that the confined-deconfined interface is rough.

The tension of the interface between the low temperature hadron phase and the high temperature quark-gluon plasma phase is an important parameter of the QCD phase transition. In the early universe the interface tension determines the nucleation rate of hadronic bubbles from the high temperature plasma. Thus, it sets the scale for spatial inhomogeneities in the baryon density, which may influence the primordial nucleosynthesis of light elements. To obtain an estimate for the interface tension in QCD we neglect the quarks and we concentrate on an $SU(3)$ pure glue theory. Then the phase transition is first order, i.e. the confined and the deconfined phase coexist at the critical temperature T_c, and the interface tension is nonzero. It is convenient to define the reduced interface tension

$$\sigma_{cd} = \frac{F}{AT_c}, \tag{1}$$

where F is the free energy of the interface and A is its area.

In the pure glue theory the deconfinement phase transition is associated with the spontaneous breakdown of the $\mathbb{Z}(3)$ center symmetry. Hence, as opposed to full QCD, there are three distinct high temperature phases, which coexist at temperatures $T > T_c$. Two different deconfined phases are separated by deconfined-deconfined interfaces with a reduced interface tension $\sigma_{dd}(T)$. At T_c all four phases (three deconfined and one confined) coexist with each other. Thermodynamical stability requires $\sigma_{dd}(T_c) \leq 2\sigma_{cd}$. Frei and Patkós [1] have suggested that

$$\sigma_{dd}(T_c) = 2\sigma_{cd}. \tag{2}$$

Then a deconfined-deconfined interface consists of two confined-deconfined interfaces with a macroscopic layer of confined phase in between. One says that the deconfined-deconfined interfaces are completely wet by the confined phase. Complete wetting is a critical phenomenon of interfaces (hence T_c is really a critical temperature although the bulk transition is first order). As one approaches the phase transition from above the thickness of the confined wetting layer

$$z_0 \propto (T - T_c)^{-\psi} \tag{3}$$

diverges with a critical exponent ψ, while the order parameter at the interface

$$\Phi_1(0) \propto (T - T_c)^{\beta} \tag{4}$$

vanishes with another critical exponent β.

In our case the order parameter is the Polyakov loop

$$\Phi(\vec{x}) = \mathrm{Tr}[\mathcal{P} \exp \int_0^{L_t} dt A_4(\vec{x}, t)], \tag{5}$$

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where $A_\mu(\vec{x}, t)$ is the anti-hermitean $SU(3)$ gauge potential. The Polyakov loop $\Phi(\vec{x}) = \Phi_1(\vec{x}) + i\Phi_2(\vec{x})$ is a complex scalar field in three dimensions. To compute the values of the critical exponents one may construct an effective $\mathbb{Z}(3)$ symmetric Φ^4 theory with an action [2]

$$S[\Phi] = \int d^3x [\frac{1}{2}\partial_i \Phi^* \partial_i \Phi + V(\Phi)],$$
$$V(\Phi) = a|\Phi|^2 + b\Phi_1(\Phi_1^2 - 3\Phi_2^2) + c|\Phi|^4. \tag{6}$$

The bare interfaces are solutions of the corresponding classical equations of motion. Fig.1 shows the shapes $\Phi_1(z)$ and $\Phi_2(z)$ of a planar deconfined-deconfined interface perpendicular to the z-direction deep in the deconfined phase (a), in a region where wetting sets in (b), and very close to the phase transition (c) where interface splitting and hence complete wetting is clearly visible. Close to the phase transition an analytic interface solution yields $z_0 \propto \log(T - T_c)$ and $\Phi_1(0) \propto \sqrt{T - T_c}$. In particular, the thickness z_0 of the confined complete wetting layer diverges only logarithmically. The corresponding critical exponents are [2]

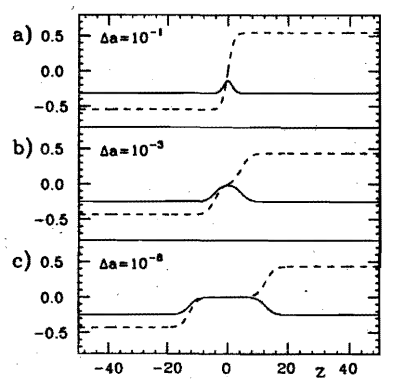

Figure 1. The deconfined-deconfined interface. The solid line is Φ_1 and the dashed line is Φ_2.

$$\psi = 0, \ \beta = \frac{1}{2}. \tag{7}$$

Complete wetting is a peculiarity of the pure glue theory. In full QCD it does not occur because the quarks break the $\mathbb{Z}(3)$ symmetry explicitly, thus eliminating two of the three deconfined phases. In numerical simulations of the pure glue theory wetting complicates the situation and makes the determination of the reduced interface tension σ_{cd} more difficult. To avoid the complications it is advantageous to impose C-periodic boundary conditions [3]. Just like quarks they break the center symmetry and they reduce the number of deconfined phases from three to one [4]. With C-periodic boundary conditions the Polyakov loop is replaced by its charge conjugate when it is shifted a distance L_i in the direction of the spatial unit vector $\vec{e}_i$

$$\Phi(\vec{x} + L_i\vec{e}_i) = {}^C\Phi(\vec{x}) = \Phi^*(\vec{x}). \tag{8}$$

Hence, the two deconfined phases with non-real expectation values of the Polyakov loop are not consistent with C-periodic boundary conditions. Of course, one must use periodic boundary conditions in the euclidean time direction.

To extract the value of σ_{cd} we investigate the system in a cylindrical spatial volume with two short x- and y-directions of lengths L_x and L_y and a much longer z-direction. Close to T_c typical configurations then consist of several blocks of confined and deconfined phases, aligned along the z-direction, and separated by confined-deconfined interfaces of area $A = L_x L_y$ spanned in the short x- and y-directions. The interface shape $\Phi_1(z)$ (averaged over the short x- and y-directions) is shown in fig.2 for a typical configuration on an $8 \times 8 \times 128 \times 2$ lattice with C-periodic boundary conditions. The interfaces influence the spectrum of the transfer matrix in the z-direction. In particular, the first two transfer matrix eigenvalues $t_{0,1}(z) = \exp(-E_{0,1}z)$ can be obtained from a 2×2 matrix of transition amplitudes

$$t(z) = \begin{pmatrix} t_{dd}(z) & t_{cd}(z) \\ t_{cd}(z) & t_{cc}(z) \end{pmatrix}. \tag{9}$$

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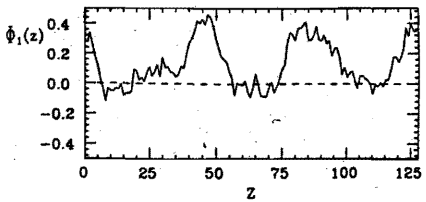

Figure 2. A typical interface shape.

To lowest order of the dilute interface approximation the amplitude for transitions between the confined and the deconfined phase is given by the one-interface contribution

$$t_{cd}(z) = \int_0^z dz_0 \exp(-xz_0)\delta\exp(-\sigma_{cd}A)\exp(x(z-z_0)). \quad (10)$$

One integrates over the position z_0 of the interface. The Boltzmann factors $\exp(-xz_0)$ and $\exp(x(z-z_0))$ represent a block of confined phase of thickness z_0 and a block of deconfined phase of thickness $z - z_0$, where $x = \frac{1}{2}(f_c - f_d)A/T$ depends on the free energy densities f_c and f_d of the confined and the deconfined phase. The interface contribution $\delta\exp(-\sigma_{cd}A)$ is suppressed by the interface free energy $F/T = \sigma_{cd}A$, and it also contains a factor δ which describes capillary wave fluctuations of the interface. A rigid interface without capillary wave fluctuations has $\delta = 1$. Using similar expressions for the amplitudes $t_{dd}(z)$ and $t_{cc}(z)$ and working to all orders of the dilute interface approximation one finds an energy difference $E_1 - E_0 = 2\sqrt{x^2 + \delta^2\exp(-2\sigma_{cd}A)}$, which is minimal for $x = 0$ [5]. Putting $E_0 = 0$ one obtains at the minimum

$$E_1 = 2\delta\exp(-\sigma_{cd}A). \quad (11)$$

In the numerical simulations the energy difference is determined from the exponential decay of a correlation function of Polyakov loops separated in the z-direction. We have used lattices $4\times4\times64$, $4\times6\times64$, $6\times6\times64$, $6\times8\times96$ and $8\times8\times128$ always with $L_t = 2$ lattice points in the euclidean time direction, and with several values of the Wilson coupling very close to $\beta_c = 5.0933(7)$. Fig.3a shows the energy E_1 for simulations on a $4\times6\times64\times2$ lattice with C-periodic boundary conditions. As expected the minimum of E_1 occurs close to β_c, where $f_c = f_d$ and hence $x = 0$. Fig.4

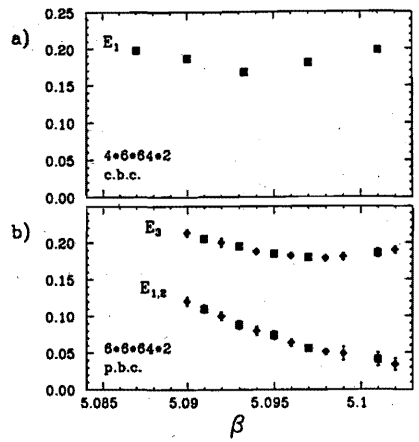

Figure 3. The spectra of the lowest energies.

shows a fit of $\log E_1$ as a function of the interface area $A = L_xL_y$. According to eq.(11) the slope of the curve determines the value of the reduced interface tension $\sigma_{cd} = 0.035(1) = 0.140(4)T_c^2$. The fluctuation factor $\delta = 0.196(5) \neq 1$ indicates that the confined-deconfined interface is rough as it should be in the continuum limit. Note that rigid interfaces are lattice artifacts.

When periodic boundary conditions are used the situation becomes more complicated, because then three deconfined phases are present and wetting may arise. In particular, there are four energies E_0, E_1, E_2 and E_3 which become degenerate at T_c in the infinite volume limit. In case of complete wetting (putting again $E_0 = 0$) we find $E_{1,2} = \sqrt{x^2 + 3\delta^2\exp(-2\sigma_{cd}A)} - x$ and $E_3 = 2\sqrt{x^2 + 3\delta^2\exp(-2\sigma_{cd}A)}$ [5]. The energies

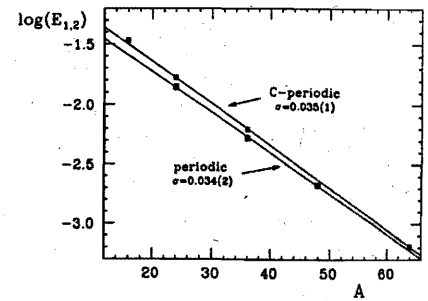

Figure 4. The dependence of the energies on the interface area A.

E_1 and E_2 are degenerate because of $Z(3)$ symmetry and charge conjugation invariance. The energy E_3 has its minimum at $x = 0$ where

$$E_3 = 2\sqrt{3}\delta\exp(-\sigma_{cd}A) = 2E_{1,2}. \quad (12)$$

On the other hand, if wetting is incomplete (i.e. if $\sigma_{dd}(T_c) < 2\sigma_{cd}$) the minimum of E_3 occurs at $x = -\gamma\exp(-\sigma_{dd}A) < 0$, while the point where $E_3 = 2E_{1,2}$ is shifted to $x = 2\gamma\exp(-\sigma_{dd}A) > 0$ [5]. The transfer matrix spectrum is then qualitatively different. Fig.3b shows the spectrum on a $6\times6\times64\times2$ lattice with periodic boundary conditions. The minimum of E_3 and the point where $E_3 = 2E_{1,2}$ are only slightly shifted away from each other. In case of incomplete wetting one would expect a shift, but it would go exactly in the opposite direction. Therefore, our data are consistent with complete and not with incomplete wetting. In fact, using eq.(12) for an independent fit which is also shown in fig.4, we obtain $\sigma_{cd} = 0.034(2) = 0.136(8)T_c^2$ and $\delta = 0.202(6)$ in agreement with the C-periodic results.

To summarize we have found evidence that the deconfined-deconfined interfaces are completely wet by the confined phase. Combining the C-periodic with the periodic results we obtain

$$\sigma_{cd} = 0.139(4)T_c^2 \quad (13)$$

for the reduced confined-deconfined interface tension. Our result is consistent with the one of the Boston group $\sigma_{cd} = 0.12(2)T_c^2$ [6] and larger than the one of the Helsinki group $\sigma_{cd} = 0.08(2)T_c^2$ [7]. As opposed to these methods we use the necessarily present finite size effects to extract the value of σ_{cd}, instead of trying to avoid finite size effects by going to larger lattices. In particular, a reliable extrapolation to the infinite volume limit is then possible, because we find agreement with our analytic finite size formulae. The fluctuation factor $\delta = 0.199(4)$ indicates that the confined-deconfined interface is rough, as it should be in the continuum limit. Of course, our results are obtained with $L_t = 2$ and it would be interesting to get closer to the continuum limit, e.g. to $L_t = 4$. This is clearly feasible using our finite size technology, but it requires larger lattices like e.g. $16\times16\times256\times4$ because the phase transition is then more weakly first order. Also high statistics is needed to be able to determine reliably the spectrum of the transfer matrix. The existing data already indicate that the interface tension is small at least in the pure glue theory. If this remains true in full QCD, only small spatial inhomogeneities are produced at the QCD phase transition and primordial nucleosynthesis is likely to take place under spatially homogeneous conditions.

REFERENCES

1 Z. Frei and A. Patkós, Phys. Lett. B229 (1989) 102.
2 T. Trappenberg and U.-J. Wiese, Nucl. Phys. B372 (1992) 703.
3 A. S. Kronfeld and U.-J. Wiese, Nucl. Phys. B357 (1991) 521.
4 U.-J. Wiese, Nucl. Phys. B375 (1992) 45.
5 B. Grossmann, M. L. Laursen, T. Trappenberg and U.-J. Wiese, HLRZ Jülich preprint 92-47, Bern preprint BUTP-92/34 (1992).
6 S. Huang, J. Potvin, C. Rebbi and S. Sanielevici, Phys. Rev. D42 (1990) 2864.
7 K. Kajantie, L. Kärkkäinen and R. Rummukainen, Nucl. Phys. B333 (1990) 100; Nucl. Phys. B357 (1991) 693.