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Evidence for quantum statistical coherence from experimental data on higher order Bose-Einstein correlations

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Abstract

We prove that the empirical relation between "radii" and order of the Bose-Einstein correlations found by the UA1 minimum bias collaboration is a consequence of quantum statistics. It constitutes thus evidence for coherence and the standard form of the density matrix.

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Bose-Einstein correlations (BEC) constitute an object of intensive study both from the experimental and theoretical point of view (for a review cf. eg.[1]). One of the unsolved questions in this field relates to the experimental evidence for coherence. According to quantum statistics [2] the presence of coherence manifests itself in the second order Bose-Einstein correlation through the appearance of two correlation terms instead of one as would be the case if the fields would be completely chaotic. Consider the correlator of the chaotic fields

$$< \pi^+_{ch}(k_1)\pi_{ch}(k_2) >= F(k_1,k_2) = \sqrt{F(k_1,k_1)F(k_2,k_2)}\; d_{12} \tag{1}$$

where, in the assumption of "stationarity" in k space, d_{12} is a function of $q^2_{12} \equiv -(k_1-k_2)^2$ only. Here $F(k,k) =< n_{ch}(k) >$ is the mean number of chaotic particles of momentum k. For a superposition of coherent and chaotic fields[1],

$$\pi \;=\; \pi_c \;+\; \pi_{ch}, \tag{2}$$

the second order correlation function reads then

$$C_2(1,2) \;\equiv\; \frac{\frac{1}{\sigma}\frac{d^2\sigma}{d^3k_1 d^3k_2}}{\frac{1}{\sigma}\frac{d\sigma}{d^3k_1}\frac{1}{\sigma}\frac{d\sigma}{d^3k_2}}$$

$$C_2(1,2) \;=\; 1 + 2\,p(1-p)\;d_{12} + p^2\;d^2_{12} \tag{3}$$

where p is the chaoticity defined by

$$p = \frac{< n_{ch} >}{< n >} \tag{4}$$

and where $< n >$ is the total mean multiplicity. The linear superposition in eq.(2) is the usual way to introduce coherence in quantum optics. It differs from the Landau-Ginzburg type ansatz proposed in [4]. In eqs. (3), (4) we assume for simplicity that p is

[1]It should be stressed that π is a bona fide field, to be distinguished from e.g. phenomenological "fields" as introduced in refs.[3] which are real quantities in momentum space.

independent of momentum. The form of the function d_{12} is not determined by quantum statistics. The only constraint d has to satisfy on general physical grounds is that it be a decreasing function of its argument. In quantum optics d is usually parametrized by an exponential $d^{exp}_{12} = exp(-Rq_{12})$, while in particle physics a Gaussian parametrization $d^G_{12} = exp(-R^2q^2_{12})$ is more frequently used, although in the few cases where exponentials have also been tried, they proved to be at least as good as Gaussians. Eq. (3) then reads [2] in these cases

$$C_2^{Gauss} = 1 + 2p(1-p)\exp(-q^2R^2) + p^2\exp(-2q^2R^2) \tag{5}$$

and

$$C_2^{exp} = 1 + 2p(1-p)\exp(-qR) + p^2\exp(-2qR) \tag{6}$$

As emphasized already in [2] there is nothing fundamental in any of these parametrizations and it is up to the experiment to determine which form is more adequate. (Obviously a microscopic theory like QCD should eventually be able to derive the "true" form of d.) In most experimental studies however a one exponent correlation formula for C_2 was used, containing again two parameters, the meaning of which is not obvious:

$$C_{12} = 1 + \lambda \begin{cases} \exp(-2q^2R^2) & : \text{ for a Gaussian} \\ \exp(-2qR) & : \text{ for an exponential} \end{cases} \tag{7}$$

Here λ is an empirical parameter which can be identified with either $2p(1-p)$ or p^2 in eq. (3) only if $p \ll 1$ or $p \simeq 1$, respectively. Only then λ is related to the chaoticity. If one rewrites eqs. (5),(6) in the form

$$C_2^{Gauss} = 1 + \lambda^G_1\exp(-q^2R^2_{1G}) + \lambda^G_2\exp(-q^2R^2_{2G}) \tag{8}$$

and

$$C_2^{exp} = 1 + \lambda^{exp}_1\exp(-qR_{1exp}) + \lambda^{exp}_2\exp(-qR_{2exp}) \tag{9}$$

then the characteristic QS feature of eqs. (5), (6) is the fact that the two correlation terms are in a well defined relationship, namely the coefficients λ_1, λ_2 are related by

$$\lambda_1 = 2\sqrt{\lambda_2}(1-\sqrt{\lambda_2}) \tag{10}$$

and the radii appear in the ratio

$$\frac{R_1^G}{R_2^G} = \frac{1}{\sqrt{2}} \quad \text{and} \quad \frac{R_1^{exp}}{R_2^{exp}} = \frac{1}{2} \tag{11}$$

Recently [5, 6] there have been attempts to test this prediction of QS in the second order correlation function, but because of the limited statistics no clearcut distinction between the forms (5), (6) on the one hand and (7) on the other hand could be established. There exists however another possibility to check the presence of coherence in BEC[2] by studying higher order particle (intensity) correlations. Indeed, as a consequence of the standard Gaussian form of the QS density matrix (cf. below), these higher order correlations reduce to second order field correlations[3] $F(k_1, k_2)$, and in [2] $C_3^{Gaussian}$ was also derived. The important point here is that relations analogous to (10),(11) are obtained for the higher order correlations but the only independent parameters are still p and R. In [9] correlation functions up to and including C_5 were given for the particular case of a Gaussian correlator and a symmetric configuration in momentum space,

$$q_{12}^2 = q_{13}^2 = q_{23}^2 = \ldots = q_{(n-1)n}^2 = q^2 = Q_{nx}^2 \frac{2}{n(n-1)} \tag{12}$$

[2]Of course, Bose-Einstein correlation functions involve only diagonal elements (in the particle number representation) of the density matrix. Therefore, another characteristic feature of coherence, namely, the presence of non-vanishing off-diagonal matrix elements, cannot be tested in BEC measurements.

[3]This is not to be confused with the case discussed in ref.[7] where it is assumed that the higher order intensity correlations can be reduced to the second order *intensity* correlations. The data of ref.[8] prove explicitly that such a "non-standard" prescription does not hold for hadron-hadron reactions, a conclusion reached also by the authors of ref.[7] from the analysis of multiplicity distribution data.

where $Q_{nx}^2 = \sum_{i<j}^n q_{ij}^2$. These expressions are:

$$C_2 = 1 + 2p(1-p)e^{-R^2Q_{2x}^2} + p^2 e^{-2R^2Q_{2x}^2} \tag{13}$$

$$C_3 = 1 + 6p(1-p)e^{-R^2Q_{3x}^2/3} + 3p^2(3-2p)e^{-2R^2Q_{3x}^2/3} + 2p^3 e^{-R^2Q_{3x}^2} \tag{14}$$

$$\begin{aligned} C_4 = {} & 1 + 12p(1-p)e^{-R^2Q_{4x}^2/6} + 6p^2(7-8p+2p^2)e^{-R^2Q_{4x}^2/3} + \\ & 4p^3(11-9p)e^{-R^2Q_{4x}^2/2} + 9p^4 e^{-2R^2Q_{4x}^2/3} \end{aligned} \tag{15}$$

$$\begin{aligned} C_5 = {} & 1 + 20p(1-p)e^{-R^2Q_{5x}^2/10} + 10p^2(13-18p+6p^2) \times \\ & e^{-R^2Q_{5x}^2/5} + 20p^3(16-21p+6p^2)e^{-3R^2Q_{5x}^2/10} + \\ & 5p^4(53-44p)e^{-2R^2Q_{5x}^2/5} + 44p^5 e^{-R^2Q_{5x}^2/2} \end{aligned} \tag{16}$$

where $C_2(1...n)$ are defined in analogy to the second-order case:

$$C_2(1,...,n) = \frac{\frac{1}{\sigma} d^n\sigma / d^3k_1 ... d^3k_n}{\prod_{i=1}^n \frac{1}{\sigma} d\sigma / d^3k_i} \tag{17}$$

As can be readily seen the formulae for C_3, C_4, C_5 emerge from the two correlation form of C_2 and thus reflect the relationship (10), (11) characteristic for coherent fields. The presence of coherence ($p < 1$) manifests itself e.g. in C_3 by the existence of 3 correlations terms rather than 2, as would be the case for purely chaotic fields. Furthermore the coefficients of these correlation terms and the "radii" are again in well defined relations, analogous to eqs.(10) , (11). Herefrom follows that a strong test of (a) the standard form of the density matrix and (b) the presence of coherence is the simultaneous fit of second and higher order correlation data by the QS formulae with the same parameters p and R. A recent experiment by the UA1 minimum bias Collaboration [8] has tested this last QS prediction for higher order BE correlations and found that the data can be better described by an exponential ansatz for d than a Gaussian one. However the authors of [8] also found that when comparing the data with an ad-hoc exponential version of the correlation

functions obtained by simply replacing Gaussians by exponentials in the formulae (13)-(16), for this "exponential" case the radii extracted from higher order correlations are not the same as those found from the second order correlation. As a matter of fact the authors of [8] obtained a "purely phenomenological" relation for this apparent dependence of the radii on the order m of the correlation,

$$r_m = R\sqrt{m(m-1)/2} \tag{18}$$

where R is independent of the order of the correlation.

In this letter we show that this "phenomenological" relation (18) is nothing else but a straighforward consequence of the QS approach provided that the form of the higher order correlation functions is derived from the initial correlator according to the rules of quantum statistics and not simply by "replacing" C_G by C_{exp}. Once this is realized it follows that the "true" radius is independent of the order of the correlation function and equal to the parameter R present in the correlator. The fact that relation (18) was obtained empirically without any theoretical prejudice is in our view strong evidence for the presence of coherence as well as for the standard form of the density matrix which we write below:

$$\begin{aligned}\rho &= N\int D\alpha^* D\alpha \exp\{-\int d\omega_1 \int d\omega_2(\alpha^*(k_1)-\gamma^*(k_1))\times \\ &\quad M(k_1,k_2)(\alpha(k_2)-\gamma(k_2))\}\ |\alpha><\alpha|\end{aligned} \tag{19}$$

where $|\alpha>$ is a coherent state ($\hat{a}(\vec{k})|\alpha>=\alpha(\vec{k})|\alpha>$) and the definition $d\omega = d^3\vec{k}/\{(2\pi)^3 2E_{\vec{k}}\}$ is used. Here γ stands for the coherent part of the field. In eq.(19) M is the inverse operator of the chaotic field correlator F (see eq.(1))

$$\int d\omega_2 M(k_1,k_2)F(k_2,k_3)=\delta(k_1-k_3).$$

Here, $D\alpha$ denotes the functional differential and N is a normalization constant.

Note that ρ is Gaussian in α and α^*. This is not to be confused with the gaussian parametrization of the correlator mentioned above. Expression (19) for the density matrix is standard in quantum optics and together with the linear superposition of coherent and chaotic fields has been introduced in particle physics in ref. [10] and used subsequently in the QS analysis of multiplicity data, (cf. ref. [11]) for a recent review). Its first test in correlation data was performed in ref. [8].

Given the fact that the density matrix is the most general operator which characterizes completely any quantum system, the relevance of this finding is obvious. In some sense, however, the evidence for coherence is even a more important achievement. Indeed, QS coherence is the manifestation of a global collective property of the system, present in all modern quantum field theoretical (QFT) approaches to particle physics and in particular in all QFT treatments of confinement. Nevertheless the experimental evidence for this fundamental property in particular in Bose-Einstein correlations has been controversial. This is so because : (i) the intercept $C_2(0)$, which was inititally proposed as a test of coherence [10] is not easily accessible to experiment and is also sensitive to other effects difficult to control (for a review cf. e.g. [1]); (ii) the distinction between one and two correlation terms in C_2 demands, as mentioned above, higher statistics than available at present.

In the remainder of this paper we shall give the formulae for higher order correlations up to and including fifth order for an arbitrary correlator d, show how eqs. (13) - (16) emerge from them as a particular case, write the corresponding relations for the exponential parametrization of d and in this way derive eq.(18) from QS. One obtains:

$$\begin{aligned}C_2(1,2) &= 1 + 2p(1-p)Re\theta_1^* d_{12}\theta_2 + p^2 d_{12}d_{21} \\ C_3(1,2,3) &= 1+\sum_{\substack{perm\\1,2,3}}\{\frac{p^2}{2}d_{12}d_{21}+p(1-p)\theta_1^* d_{12}\theta_2\end{aligned} \tag{20}$$

$$+ \frac{p^3}{3} d_{12}d_{23}d_{31} + p^2(1-p)\theta_1^* d_{12}d_{23}\theta_3\} \tag{21}$$

$$\begin{aligned} C_4(1,..,4) &= 1 + \sum_{\substack{perm \\ 1,2,3,4}} \{\frac{p^2}{4} d_{12}d_{21} + \frac{p}{2}(1-p)\theta_1^* d_{12}\theta_2 + \frac{p^3}{3} d_{12}d_{23}d_{31} \\ &+ p^2(1-p)\theta_1^* d_{12}d_{23}\theta_3 + \frac{p^4}{8} d_{12}d_{21}d_{34}d_{43} \\ &+ \frac{p^2}{2}(1-p)^2\theta_1^* d_{12}\theta_2\theta_3^* d_{34}\theta_4 + \frac{p^3}{2}(1-p)d_{12}d_{21}\theta_3^* d_{34}\theta_4 \\ &+ \frac{p^4}{4} d_{12}d_{23}d_{34}d_{41} + p^3(1-p)\theta_1^* d_{12}d_{23}d_{34}\theta_4\} \end{aligned} \tag{22}$$

$$\begin{aligned} C_5(1,..,5) &= 1 + \sum_{\substack{perm \\ 1,2,3,4,5}} \{\frac{p^2}{12} d_{12}d_{21} + \frac{p}{6}(1-p)\theta_1^* d_{12}\theta_2 + \frac{p^3}{6} d_{12}d_{23}d_{31} \\ &+ \frac{p^2}{2}(1-p)\theta_1^* d_{12}d_{23}\theta_3 + \frac{p^4}{8} d_{12}d_{21}d_{34}d_{43} + \frac{p^2}{2}(1-p)^2\theta_1^* d_{12}\theta_2\theta_3^* d_{34}\theta_4 \\ &+ \frac{p^3}{2}(1-p)d_{12}d_{21}\theta_3^* d_{34}\theta_4 + \frac{p^4}{4} d_{12}d_{23}d_{34}d_{41} + p^3(1-p)\theta_1^* d_{12}d_{23}d_{34}\theta_4 \\ &+ \frac{p^5}{6} d_{12}d_{21}d_{34}d_{45}d_{53} + \frac{p^4}{3}(1-p)\theta_1^* d_{12}\theta_2 d_{34}d_{45}d_{53} \\ &+ \frac{p^4}{2}(1-p)d_{12}d_{21}\theta_3^* d_{34}d_{45}\theta_5 + p^3(1-p)^2\theta_1^* d_{12}\theta_2\theta_3^* d_{34}d_{45}\theta_5 \\ &+ \frac{p^5}{5} d_{12}d_{23}d_{34}d_{45}d_{51} + p^4(1-p)\theta_1^* d_{12}d_{23}d_{34}d_{45}\theta_5\} \end{aligned} \tag{23}$$

where we have introduced $\theta(k) = \gamma(k)/|\gamma(k)|$ and assumed as in ref.[2] that the chaoticity parameter p is independent of momenta (for a more general form cf. [12]).

For the symmetrical momentum configuration (12) and for $\theta = const$, we recover for the Gaussian form of the field correlator the results (13) - (16) obtained in ref.[9].

For the exponential parametrization however one gets

$$C_2 = 1 + 2p(1-p)e^{-RQ_{2\pi}} + p^2 e^{-2RQ_{2\pi}} \tag{24}$$

$$C_3 = 1 + 6p(1-p)e^{-RQ_{3\pi}/\sqrt{3}} + 3p^2(3-2p)e^{-\frac{2}{\sqrt{3}}RQ_{3\pi}} + 2p^3 e^{-\sqrt{3}RQ_{3\pi}} \tag{25}$$

$$\begin{aligned} C_4 &= 1 + 12p(1-p)e^{-RQ_{4\pi}/\sqrt{6}} + 6p^2(7-8p+2p^2)e^{-2RQ_{4\pi}/\sqrt{6}} + \\ &\quad 4p^3(11-9p)e^{-3RQ_{4\pi}/\sqrt{6}} + 9p^4 e^{-4RQ_{4\pi}/\sqrt{6}} \end{aligned} \tag{26}$$

$$\begin{aligned} C_5 &= 1 + 20p(1-p)e^{-RQ_{5\pi}/\sqrt{10}} + 10p^2(13-18p+6p^2) \times \\ &\quad e^{-2RQ_{5\pi}/\sqrt{10}} + 20p^3(16-21p+6p^2)e^{-3RQ_{5\pi}/\sqrt{10}} + \\ &\quad 5p^4(53-44p)e^{-4RQ_{5\pi}/\sqrt{10}} + 44p^5 e^{-5RQ_{5\pi}/\sqrt{10}} \end{aligned} \tag{27}$$

and these are different from those derived empirically and used in [8]. Nevertheless they can be made to coincide with those used in [8] after the substitution (18). This proves that the eq.(18) found empirically in [8] is nothing else but a consequence of quantum statistics and in particular of the presence of partial coherence.

In conclusion, we have shown that the UA1-data on the higher order Bose-Einstein correlations can be considered as evidence for the standard form (19) of the density matrix and for the presence of coherence. The fact that the QS relations between the coefficients of the correlations and the "radii" are not only satisfied by the second order correlation function but simultaneously also by the third, fourth and fifth, and that this discovery was made by "trial and error", suggests that the QS laws are robust enough to resist attempts of "falsification". To paraphrase Voltaire: if quantum statistical coherence would not have existed, one would have to invent it. To have made possible this conclusion is a great merit of the experiment of ref. [8]. Furthermore this strengthens very much the confidence in the experimental values for R and p obtained in [8], i.e. $R \cong 1.4\ fm$ and $p \cong 0.45$. As a matter of fact the value $p \sim 0.45$ is consistent with the value obtained for the chaoticity from the analysis of higher moments of the multiplicity distribution [13], especially if one takes into account that only charged data were available for that analysis[4].

[4]Even after the result has been corrected according to eq.(18), a small dependence of the radius R on the order of the correlation remains. There may be several reasons for this. Firstly the parametrization of the function d is not general enough, see ref.[12]. In particular as emphasized in ref.[2] the assumption of "stationarity" in Q space, on which (24) - (27) are based, may not hold exactly. Moreover, in general the

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chaoticity parameter p is momentum dependent [12, 14]. Secondly the symmetry assumption (12) may be too strong. This particularization can be avoided (cf. e.g. [6]). Thirdly the corrections introduced in ref.[8] in the higher order correlations for the Coulomb interaction and for the limited detector acceptance were obtained by "straightforward" extension of the corresponding correction factors from the second order correlation function. That such a procedure may introduce biases has been proven above in the case of correlators.

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