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A New Model Of Strongly Coupled QED

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Abstract***

We present a new model of QED which exhibits two distinct phases. One is conventional QED and the other is a new strongly coupled phase characterized by a curvature law. In this new phase we give a field theoretic derivation of the beta-function and show that it is non-trivial in the absence of virtual fermions due to the new curvature term. At short distances the running coupling approaches an ultra-violet stable fixed point $\alpha_c = 1$ rather than zero and increases at large distances indicating confinment. This behaviour depends crucially on the Minkowskian signature and does not occur in the Euclidean signature. We argue that the positronium spectrum in the strong phase is approximtely described by a Majorana mass formula which varies inversely with the spin. We examine this spectrum in connection with the GSI experiments. The spectrum not only is in excellent agreement with the three observed narrow e^+e^- peaks, but also predicts four new resonances with masses 1486 KeV, 1341 KeV and 1208 KeV and 1091 KeV. The spins of all the resonances are also predicted.

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ON

QED is historically our principal reason for believing quantum field theory is usefull and at the same time its existence is uncertain. Arguments due to Landau [1] suggest that QED breaks down at some short distance scale. The screening of charge by virtual fermions implies the coupling at shorter and shorter distance becomes arbitrarily large to maintain $\alpha = \frac{1}{137}$ at large distances. The question of the existence of QED can be avoided by embedding it in a larger asymptotically free theory where other forces are not artificially turned off. Antiscreening of charges implies the coupling aprroaches zero at short distances and the theory is entirely well behaved. An alternative response is that QED without other forces may exhibit a phase transition to a well behaved theory. Instead of the coupling growing indefinitely it may approach a critical value α_c of order one at short distances. Such theories may represent a new class of well defined quantum field theories. If QED is to exhibit a phase transition there presumably must be another source of coupling constant renormalization opposing screening to avoid Landau's argument.

In this paper we propose a model of QED [2] which indeed possesses a new source of coupling constant renormalization. The model has a weak phase corresponding to ordinary QED and a new strongly coupled phase. In the strongly coupled phase we argue that a Majorana type mass formula, i.e. where the mass varies inversely with spin, approximately describes the bound states of e^+e^-. We compare the spectrum with the GSI peaks. The model is phrased in terms of Feynman's spacetime picture of QED where matter is described by a quantum mechanical point particle. The origin of the new source of coupling constant renormalization is the contribution to the action of the curvature of the world line associated with the point particle. The curvature implies a non-trivial 1-dimensional quantum field theory and thus source of coupling constant renormalization is at the first quantized level.

The curvature term arises through a new regularization method applied to the relativistic point particle Maxwell system. Integrating out the vector potential produces an effective action for the relativistic point particle. The effective action is divergent due to the singularity of the photon propogator at short distances. Ordinarily this divergence is regulated by introducing a cuttoff ϵ at short distances and amounts to mass renormalization. A closer examination of the form of the divergence reveals an alternative way to regularize the theory. Because the relativistic point particle current has support on a curve the new term in the effective action resembles the self inductance of an ideal current carrying wire corresponding to the world line C of the point particle. The divergence can be regulated by considering the mutual inductance of two disjoint wires C and C_ϵ where C_ϵ is obtained by displacing C a distance ϵ along a unit normal n to the world line. So as to avoid dependence on a special choice of normal a crucial step is to regard n as a constrained dynamical variable. This is a key departure from

ordinary QED and forms the new element in the definition of our model. The new regularization method yields not only mass renormalization but also a new finite term that depends on n. Eliminating n by its equation of motion implies that the new term is precisely the curvature of the world line times $\frac{e^2}{4\pi} = \alpha h$.

The curvature term characterizes our new model of QED and a physical picture of its origin can be formulated. Although the calculation outlined was in terms of funtional integrals, the curvature arises at the classical level and can be understood as a generalization of classical mass renormalization [3]. The simple idea of a charged point particle is not well defined because the interaction of the charged particle with its own field gives a divergent contribution to the total mass. This infinity can be regulated by modifying the force law on distances of order ϵ (e.g. Pauli-Villars regularization). Here we regulate the infinity in a new way by arranging the charged particle with arbitrary trajectory C not to interact with its own field but with the field of a charged particle with trajectory C_ϵ obtained by displacing C a distance ϵ along a dynamical unit normal n. The proceedure clearly regulates the infinity in the Coulomb field as there is now a finite separation ϵ between the charged particles. However, there are now mutual inductive effects in addition to the change in inertia since there are two infintesimally nearby charged particles. The presence of the other particle's electromagnetic field modifies the usual straight line motion into helical motion characteristic of magnetic fields. The inductive effects are encoded by the addition of the curvature term in the action whose extrema are helicies. It is interesting to note that the relativistic point particle scalar field system does not exhibit a curvature term with our new regularization method. The classical solutions of this system do not include helical motion characteristic of magnetic fields and consequently no curvature term should be present.

The addition of $\alpha = \frac{e^2}{4\pi}$ times the curvature to the arc-length action implies a non-trivial one dimensional quantum field theory with running coupling α and two phases. In arc-length gauge the theory is analogous to a σ-model model where the spin σ constrained to lie on a unit sphere corresponds to the velocity $\dot{x}$ of the particle. The curvature term then corresponds to the square root of the usual σ-model kinetic term. The square root effectively increases the dimension by one so that our model actually resembles a two dimensional non-linear σ-model. It is well known that the beta function of the temperature T is negative in compact non-linear σ-models. Since T corresponds to $\frac{1}{\alpha}$ we might expect the beta function in α to be positive and not oppose screening. The internal symmetry of our model, however, is the Lorentz group rather than the rotation group of the σ-model. The non-compactness reverses the sign so that the beta function of our model is negative. Thus the curvature term is a new source of coupling constant renormalization which opposes screening.

The two phases of the arc-length plus curvature theory can be seen in a lattice regularization. The theory, being one dimensional, is an exactly solvable

quantum field theory in the sense of Wilson. For any finite value of the bare α, the continuum limit of the theory is equivalent to the pure arc-length theory. This is the weakly coupled phase we identify with ordinary QED. However, for infinite bare α, the curvature survives in the continuum limit and defines the strongly coupled phase. Although there is a general rule that one dimensional systems cannot exhibit a phase transition, this rule has exceptions. At $T = 0$ a phase transition can occur and a phase transition at $T \neq 0$ can occur if sufficiently long range forces are included. Since α corresponds to $\frac{1}{T}$, the point $\alpha = \infty$ is indeed such an exception. Furthermore, we shall be interested in virtual e^+e^- pairs (summing over loops instead of open curves) where the action includes the long range Coulomb field. In this more realistic system, the phase transition will occur at a finite α. It is not clear whether these result will survive the introduction of virtual fermions in full QED. However, the running of the coupling constant due to the curvature does so on all distance scales as in the σ-model in contrast to the running due to virtual fermions which does so only out to a Compton wavelength. At sufficiently large distances the curvature effects may therefore dominate over screening effects. Furthermore, the negative sign of the beta function would then be a qualitative indication of confinement.

The order parameter of the two phases is the continuum coupling constant of the curvature term. This is similar to QCD without fermions where the order parameter is the coefficient of the area law in the Wilson loop. In contrast to QCD, the strong coupling phase of our model corresponds to a low temperature statistical mechanical system and the weak phase to a high temperature one. Similar statistical mechanical models arise in polymer physics. Polymers at low tempature posses a rigidity that is modeled by a curvature squared term in the energy. The mean square end to end distance R^2 goes as L^2 at low temperatures [4]. At high temperatures the rigidity is overcome by thermal fluctuations and R^2 goes as L. Thus the strong coupling phase of our model is analogous to rigid polymers and the weak phase to non-rigid polymers. An alternative order parameter, motivated by polymer physics, is to take the power of L in R^2 in the large L limit. In polymer physics it is known that the addition of long range interactions does not change this exponent but gives subleading corrections in lnL. Thus the long range Coulomb interaction, in contrast to the curvature, does not change the phase.

The presence of the curvature term in the strong phase dramatically alters the particle spectrum. It transforms a single particle of mass m described by the arc-length theory into an infinite number of particles with mass varying inversely with spin [5]. Such a mass spectrum is called a Majorana mass spectrum. The spin degrees of freedom arise from the extra degrees of freedom associated with the higher derivative nature of the curvature term. The solutions of the arc length plus curvature theory are helicies about a time like direction and the different spin states correpond to helicies with different radii and angular fre-

quency. Although the arc length theory has 3 configuration degrees of freedom. the curvature term increases the degrees of freedom to 5. Our starting point was the ordinary relativistic point particle Maxwell system and one may ask where the 2 extra degrees of freedom associated with the higher derivatives originated. Essenially, the 2 degrees of freedom correspond to those in the constrained dynamical unit normal n which when eliminated by its equations of motion gave the higher derivative curvature term. Another example of a particle transformed by its interaction with its own field occurs in the point particle Chern-Simons system studied [6] in connection with high temperature superconductivity. There a single bosonic particle of mass m is transformed into a fermion of mass m. The extra degrees of freedom here are discrete owing to the topological character of the Chern-Simons action.

A central question for comparison with obsevation is the spectrum of bound states of e^+e^- in the strongly coupled phase. This problem possesses two new features not present in the weak phase. First there are an infinite number of Majorana states available for each particle. Second the strong phase is in the supercritical region $\alpha > 1$ where the Coulomb attraction overcomes the centrifugal barrier. Although we have not solved the supercritical Majorana bound state problem, we believe the principal features of the bound state spectrum are determined by which mass state the particles are in rather than which orbital state. This belief is based on the observation that the bound state spectrum in the supercritical case with ordinary particles (not Majorana) has excited levels with exponentially small separations[7]. Thus we expect the spectrum of bound Majorana particles in the supercritical case also to have a Majorana form. This answer bears an interesting analytic relation to the ordinary positronium spectrum in that the Majorana spectrum can essentially be obtained by inverting both the quantum number and the coupling in the ordinary positronium spectrum.

The observation of a Majorana behavior for the bound state spectrum can serve as an experimental signiture of our new model of the strongly coupled phase of QED. Attention has recently been focused on the strong phase of QED in an attempt to explain data from heavy ion collisions at GSI. An unexpected feature of this data is the observation of e^+e^- resonances with narrow peak energies in the range 1.4 to 1.8 MeV. The observation of 3 to 4 such peaks suggests a composite structure made out of e^+e^-. However, such a system cannot be the usual positronium spectrum of conventional QED as the levels lie bellow the $2m_e$=1.022MeV threshold. A more plausible explanation is to consider this system as new quasi-bound states of e^+e^- in the strongly coupled phase of QED. The strong electromagnetic background field of the colliding heavy ions is conjectured to trigger the new phase. Although these peaks are not well understood experimentally, it is nonetheless interesting to compare them with the Majorana spectrum of our model. The spectrum not only agrees well with the observed peaks but predicts the spin and four new resonances. The coupling constant

α determined phenomenologically from the mass formula is of order 2.5. This value exceeds the critical value $\alpha_c = 1$ where the transition to the strong phase is expected to occur and the theory is indeed strongly coupled.

I A NEW MODEL OF QED

The starting point of our new model of QED with two phases is phrased in terms of Feynman's spacetime picture of QED. The dynamical variables are the point in space-time x and the photon field A_μ. The action is

$$S(x, A) = m_0 \int ds + e_0 \int A \cdot dx - \frac{1}{4} \int F^2 \tag{1}$$

Integrating out the vector potential produces an effective action for the point particle defined by

$$e^{iS_{eff}} = \int DA e^{iS(x,A)} = e^{i(m_0 \int_C ds - \frac{e_0^2}{4\pi^2} \int_C \int_C dx^\mu dy^\mu \frac{1}{|x-y|^2+i\epsilon})} \tag{2}$$

where we have introduced C for the trajectory of the point particle x. If C is open and moving forward in time, S_{eff} describes a single electron and its interaction with its own field. If C is a closed loop, S_{eff} describes an e^+e^- pair and their mutual and self interactions. The real part of the double line integral represents the relativistic Coulomb interaction. Note that (2) differs from the Feynmen-Wheeler[8] action at a distance theory as they include only the real piece and not the imaginary piece of the double line intergral which represents radiation reaction. S_{eff} is simply a representation of ordinary QED with the usual Feynman boundary conditions and not the half-advanced half-retarded boundary conditions of Feynman and Wheeler.

The double line integral in S_{eff} is divergent when $x = y$. Our new model of QED arises by a fundamentally new method of regulating this divergence. Instead of modifying the propogator (e.g. Pauli-Villars regularization) which breaks gauge invariance for non-zero ϵ and amounts only to mass renormalization, we shall modify one of the trajectories C in the double line integral. This procedure preserves gauge invariance because it does not break current conservation.

The divergences of the double line integral are similar to those encountered in the self-inductance of a current-carrying wire C. These divergences can be regulated by considering the mutual inductance of two wires C and C_ϵ . The wire C_ϵ is determined by displacing C a distance ϵ along a unit normal n^μ to C. We define the self inductance given by the double line integral as the limit of the mutal inductance as $\epsilon \to 0$. Geometrically this can be viewed as replacing C by a tiny ribbon with edges C and C_ϵ . This geometrical regularization introduces ϵ and n to regularize the theory. We treat the normal n as a dynamical variable

and integrate over all normal configurations in order to avoid dependence on a specific choice of normal. This is in contrast to ordinary regularization methods which essentially introduce only a scalar parameter ϵ. Thus part of our definition of regularized QED involves the dynamical normal n. The result of the regularization is a new term $I_0 = -\alpha \int ds \frac{d^2x(s)}{ds^2} \cdot n$ in addition to the usual mass renormalization. Since n is subject to the constraints that it be unit and normal implies the addition of the two terms $I_1(n) = \int ds[\eta_0(\frac{dx(s)}{ds} \cdot n) + \eta_1(n^2 - 1)]$ to I_0. Extremizing n in $I(n) = (I_0 + I_1)(n)$ we find that n must be $\pm$ the principal normal (i.e., the unit vector of the proper acceleration). However a unit normal n can in principle be subject to a further constraint $\frac{d^2x(s)}{ds^2} \cdot n = 0$. Hence one can have the choice of adding a third term $I_2(n) = \int ds[\eta_2(\frac{d^2x(s)}{ds^2} \cdot n)]$ to the action I(n). Extremizing n in $\tilde{I}(n) = (I + I_2)(n)$ one finds the solution for n^μ are the remaining two normals in D=4. They can be expressed in terms of $v_i^\mu = \epsilon^{\mu\nu\rho\sigma} \frac{dx^\nu(s)}{ds} \frac{d^2x^\rho(s)}{ds^2} w_i^\sigma$ as $\frac{v_i^\mu}{|v_i|}$ where w_i^σ are two arbitrary vectors that lie in a plane. However these two normals are in fact degenerate for class of curves whose equations are $\dot{x}^\mu = w_i^\mu$ or $\ddot{x}^\mu = w_i^\mu$. Therefore they cannot seperate (regularize) such curves in the double line integral (2). If we insist that the regularization should apply for any arbitrary curve C other than the straight line then $\tilde{I}$ is ruled out in favour of I. Thus the action I(n) is the unique minimal choice for determining n dynamically. From the $\pm$ principal normal solutions of I(n) we will choose the $-$ sign which gives helical motion for x rather than the $+$ sign which gives only exponential runaway motion. Quantum mechanically we integrate over n, η_0, η_1 in the path integral. The resulting effective action contains a previously undiscovered curvature correction to the arc-length action

$$S_{reg}(C) = \mu_0 \oint_C ds + \alpha_0 \oint_C ds k(s) + 2\alpha_0 \oint_{C_{e-}} \oint_{C_{e+}} dx^\mu dy^\mu \delta\left(\left|(x-y)^2\right|\right) + \text{I.P} \quad (3)$$

where $\mu_0 = m_0 + \delta m_0$ is the renormalized mass, k(s) = $|\frac{d^2x(s)}{ds^2}|$ is the curvature of C, the union of electron trajectory C_{e-} and the positron trajectory C_{e+} and I.P stands for the imaginary part. To obtain S_{eff} as it stands it was crucial to check that quantum fluctuations due to η_1 are negligable. Furthermore a particular choice of sign of the curvature term is again made so as the path integral is bounded. This choice of sign is part of the definition of our theory. In the language of Wilson loops, we have a previously undiscovered curvature correction to the perimeter law. Remarkably the bare coupling constant α_0 is the "fine structure" constant $\frac{e_0^2}{4\pi}$. The third term of (3) represents the relativistic version of the Coulomb interaction. The above result can also be obtained by employing a different regularization scheme due to Polyakov [6]. A novel feature

of this regularization is the independence of the coefficient of the curvature term from the cut-off ϵ. In all other dimensions d, the coefficient depends on the cut-off as ϵ^{4-d}. The curvature term in (3) for open paths has been studied in the completely different context of rigid paths [9] as toy models of rigid strings [10].

The higher derivative nature of this action implies that the number of degrees of freedom has apparently increased over that of the point particle we started with. The extra degrees of freedom, as discussed in the introduction, arise from the dynamical unit normal n. In the ordinary derivative variational principle for the action, x and n were held fixed at the initial and final points. To obtain equivalent equations of motion for the higher derivative action resulting after eliminating n, both x and $\dot{x}$ must be fixed at the initial and final points. In the path integral quantization of the theory we therefore sum over all trajectories $C_{(X_0,Y_0)}$ with $X_0 = (x_0, \dot{x}_0)$ and $Y_0 = (y_0, \dot{y}_0)$ held fixed. It is the spin of the Majorana particle that is encoded in the new quantum numbers associated with $\dot{x}$.

II THE NEW STRONG PHASE

The two distinct phases can be exposed most simply by considering S_{eff} in absence of Coulomb interactions. More detailed features of the two phases can later be seen when we include the long range Coulomb interactions. The phases of the quantum theory of (3) can be seen in an Euclidean lattice regularization in which the loop C is replaced by a closed chain of links. The dramatic effects of continuuing to Minkowski space-time will be discussed subsequently. On the lattice, the curvature theory (3) describes a self-avoiding random walk (SAW) in which the curvature term represents the repulsive interactions between the links of the walk. These interactions cause the walk to be "rigid" and have analogs in polymer physics. In fact rigid polymer chains are described by a classical action similar to (3) with squared curvature term. The phases of a rigid polymer are distinguished by the behaviour of the mean square distance $< R^2 >$ between the two ends of the molecule. In fact one obtains two phases: A low temperature T phase with $< R^2 >= L^2$, and a high temperature phase with $< R^2 >= \frac{2\gamma}{T} L$ with γ being the curvature coupling. At very high temperature the polymer loses its rigidity and the chain becomes the usual gaussian random (ideal) chain. These remarkable properties of polymers are shared by our curvature model of QED. The strong phase of QED is analogus to the low temperature phase of polymers while conventional QED is analogus to the high temperature phase of polymers. The justification of this statement is due to the remarkable scale invariant property of the curvature term in (3). If we rescale x by αx in (3) the effective action S_{eff} rescales by an overall factor α. This implies that α corresponds to the inverse temperature $\frac{1}{T}$ in contrast to gauge theories where the gauge coupling constant is the temperature. The fact that our coupling is the inverse temperature has an important consequence that follows from properties of one dimensional statistical systems. In the absence of long range interactions,

these systems exhibit a phase transition only at $T = 0$. Therefore one can intuatively understand that our model will exhibit a phase transition at $\alpha = \infty$ in the absence of the long range Coulomb interactions. The inclusion of these interactions ultimately shift the critical coupling to a finite value α_c. Our model on the lattice will therefore have a non-trivial fixed point at $\alpha = \infty$ whose continuum limit corresponds to a strongly coupled theory with finite rigidity ($\alpha_{continuum} \neq 0$). The model also exhibits a trivial fixed point at $\alpha = 0$ whose continuum limit is the usual arc-length theory (gaussian random walk) which when coupled to Coulomb interactions gives rise to the usual perturbative weak QED.

We proceed by discretizing the domain space of $x^\mu(s)$ by setting $x_i^\mu = x^\mu(ia)$ where a is the lattice spacing. The action then reads

$$S = \mu_{0L}|x_i - x_{i-1}| + 2\alpha_{0L}|sin\frac{\theta_i}{2}| \tag{4a}$$

where θ_i is the angle between the two links $l_i = x_i - x_{i-1}$ and l_{i+1}, and α_{0L}, μ_{0L} are the values of the bare coupling and mass scale respectively on the lattice. The action for the self-avoiding random walk (SAW), S_{SAW}, that follows from (4a) is obtained by replacing the Euclidean spacetime R^d by a lattice Z^d. The path consists of the sequence $z = x_0, x_1, ..., x_n$ where x_i and x_{i-1} differ by the unit vector l_i on the lattice. We can view z as a set of links $l_1, ..., l_n$. The action can be cast as an anisotropic random walk

$$S_{SAW}(z) = \sum_{i=1}^{n} E(l_i \cdot l_{i+1}) \tag{4b}$$

where $E = \delta_1$ if l_i is parallel to l_{i+1}, δ_2 if l_i is perpendicular, and δ_3 if l_i is antiparallel. Agreement with action (4a) implies

$$\delta_1 = \mu_{0L} \qquad \delta_2 = \mu_{0L} + \sqrt{2}\alpha_{0L} \qquad \delta_3 = \mu_{0L} + 2\alpha_{0L} \ . \tag{4c}$$

The propagator of the discretized theory is

$$G_\delta(X_0, Y_0) = \sum_{z: X_0 \to Y_0} e^{-S(z)} \tag{5a}$$

Define

$$p_1 = e^{\delta_1} \qquad p_2 = e^{\delta_2} \qquad p_3 = e^{\delta_3} \tag{5b}$$

then explicitly we obtain the following expression [illegible] the Green's function:

$$G(x, p_1, p_2, p_3) = \int_{-\pi}^{\pi} \frac{d^D k}{(2\pi)^D} e^{ik.x} G(k) \tag{5c}$$

$$G(k) = \frac{\xi}{D}[\sum_{\mu=1}^{D} \frac{(1 - \frac{2Dp_3 cosk_\mu}{\omega})}{1 - \lambda_\mu} + \frac{2Dp_3}{\omega} \sum_{\mu=1}^{D} \frac{cosk_\mu}{(1 - \chi_\mu)}]$$

where $\chi_\mu = \lambda_\mu + \omega$ with

$$\lambda_\mu = (p_1 + p_2 - 2p_3)cosk_\mu \qquad \omega = 2p_3 \sum_{\mu=1}^{D} cosk_\mu \ . \tag{5d}$$

In the absence of the curvature term the above result reduces to that of the usual gaussian random walk. The above answer is obtained after imposing a convergence condition which is the requirement that

$$\xi = e^{-\mu_{0L}} + e^{-\mu_{0L} - 2\alpha_{0L}} + 2(D-1)e^{-\mu_{0L} - \sqrt{2}\alpha_{0L}} \ . \tag{6}$$

is less than one. The critical surface of the theory is defined to be the point $k_\mu = 0$ i.e the vanishing of the bare mass. Alternatively it is the line for which the susceptibility $\phi = \sum_x G(x, \xi)$ diverges. From (5c) one simply obtains

$$\phi = \frac{\xi}{1 - \xi}$$

thus the critical surface corresponds to the condition

$$\xi = 1 \ . \tag{7}$$

The poles of G can be easly determined for $\xi << 1$ to be

$$k_\mu = i\phi^{-1/2} + O(1 - \xi) \tag{8}$$

hence the mass M_0 has a critical exponent equals to half.

Our objective is to look for non-trivial fixed points on the critical line. This tantamount to having non-trivial continuum theories whose scaling limits correspond to the action (3).

The distinguishing feature of the curvature term, which represents the repulsive interactions between the links, is the rigidity that it gives to the paths. This rigidity $< R^2 >$ is quantitatively measured in terms of the tangent to the chain of the Wilson loop of length L by the normalized tangent-tangent correlation function

$$< R^2 >= \int_0^L ds \int_0^L ds' \psi(s - s')$$

where

$$\psi(s - s') =< t(s)t(s') >= e^{-\nu_{0L}\frac{(s-s')}{a}} = e^{-\nu_{0L}(n-n')} \tag{9a}$$

with the average being weighted with respect to the Boltzman factor e^{-S}. Here s and s' specify points along the chain by their arc-length distance, a is the length of the link of the chain and ν_{0L} is the inverse correlation length of the tangents. Its exact value is,

$$\nu_{0L}(D, \alpha_{0L}) = ln(\frac{1 + 2(D-1)e^{-\sqrt{2}\alpha_{0L}} + e^{-2\alpha_{0L}}}{1 - e^{-2\alpha_{0L}}}) . \tag{9b}$$

If we require the continuum limit to have a finite rigidity then we demand the scaling limit $a \to 0$, $\nu_{0L} \to 0$ to be such that their ratio $\nu_{continuum}$ is kept fixed. The only fixed point on the critical surface which satisfies this scaling limit is $(\mu_{0c}, \alpha_{0c}) = (0, \infty)$ for fixed D. All other points with finite $\alpha_0 < \alpha_{0c}$ give a zero rigidity or equivalently a zero continuum α. The critical line flows towards a trivial fixed point at $\nu_{0L} = +\infty$. In D=4 this is the infared stable point $(3ln2, 0)$ whose continuum limit corresponds to the ordinary gaussain random walk. In the presence of Coulomb interactions this is the weakly coupled phase of QED where the curvature term becomes irrelevent and ψ goes to zero in the continuum limit. The mean square distance $< R^2 >$ in the weakly coupled limit is $\frac{2La}{\nu_0}$ which goes to zero in the continuum limit indicating the absence of rigidity.

The continuum theory that corresponds to the fixed point $(0, \infty)$ is a theory that lives, in the absence of Coulomb interactions, in one phase which is strongly coupled. This is our new proclaimed strong phase of QED. It is characterized by three physical mass scales M, the poles of the Green's function, μ, and ν two of which are independent. They are defined by $M = \Lambda_L M_0(\mu, \alpha_0(\Lambda_L))$, $\mu = \Lambda_L \mu_{0L}$, and $2(D-1)\nu = \Lambda_L \nu_{0L}(\alpha_{0L})$ where the scale $\Lambda_L = \frac{1}{a} \to \infty$ is the lattice cutoff. The new mass scale ν is analogous to Λ_{QCD} (not to be confused

with the above Λ_L) and determines the bare coupling at a particular large scale Λ_L thru equation (9b). One obtains $\alpha_0(\Lambda_L) = \frac{1}{\sqrt{2}} ln \frac{\Lambda_L}{\nu}$. This is equivalent to the mass gap relation $\nu = \Lambda_L e^{-\frac{\sqrt{2}}{t_{0L}}}$ which gives a non-trivial beta-funtion in the temperature $t_{0L} = \alpha_{0L}{}^{-1}$ (c.f.eq.(15a)). The inverse corrolation length ν_{0L} must also exist in the large D limit ($D \to \infty$) except in $D = 1$ where it is infinite (absence of rigidity) because the curvature term is identically zero. Therefore t_{0L} must be proportional to $D - 1$. Hence we set t_{0L} to be $c^{-1}\sqrt{2}(D-1)t_0^*$ where c is yet an arbitrary normalization constant. The mass gap relation then reads

$$\nu = \Lambda_L e^{-\frac{c}{(D-1)t_0^*}} . \tag{10a}$$

To determine c we note that since the mass scale ν is being fixed, its physical value must be the same in the continuum and on the lattice. We will therefore calculate the mass gap relation in the continuum by considering the large D limit of the curvature effective action in the arc-length gauge:

$$S_{eff} = \alpha_0 \int_C ds(\sqrt{\dot{x}^2} + \frac{\nu}{2}(\dot{x}^2 - 1) + \mu_0) \tag{1'}$$

where ν is the lagrange multiplier enforcing the arc-length gauge condition. In the large D limit the saddle point approximation applies. In this case the stationary loop C with two fixed points consists of two classical trajectories of opposite directions travelling togther. Consequently a multiplicative factor of two will appear in front S_{eff}. Our analysis gives the following result:

$$\nu = \Lambda_{mom} e^{-\frac{2\pi}{D}(\frac{1}{t_0^*} + k)} \tag{10b}$$

where t_0^* is the continuum bare coupling and k is a finite constant that is independent from the coupling but depends on finite renormalization effects of the coupling. Because of closed paths trajectories, eq.(10b) differs by a factor of two in the exponent from a similar result obtained in [Pisarski,9] for open paths. In computing (10b) we have neglected finite size effects.

From (10a,b) we obtain in the large D limit

$$\frac{\Lambda_L}{\Lambda_{mom}} = e^{\frac{2\pi - c}{Dt_0^*} + k} . \tag{11}$$

To fix c we recall that the lattice and momentum cut-offs for (non)-abelian gauge theories are just related by an overall constant [11]. This determines c to be 2π.

This result should not be suprising because we know that the coefficent of t_0^* in (10) which is the coefficent of the beta function (c.f.(15a)) is a universal constant that is independent of the regularization scheme at least in the weak coupling t (strong coupling α). The normalized bare running coupling is therefore (we will drop the superscript $*$ in the rest of the paper unless stated otherwise)

$$\frac{1}{t_0(\Lambda)} = \alpha_0(\Lambda_L) = \frac{D-1}{2\pi} ln \frac{\Lambda_L}{\nu} \ . \tag{12}$$

Though the free parameters of the weak phase are the electron mass $M = m_e$ and the fine structure constant α, in the strong phase, α at a particular scale is determined by dimensional transmutation and the free parameters are now M and ν.

The discrete order parameter of the two phases is obtained by computing the mean square distance $< R^2 >$ We find

$$< R^2 >= L^2 \qquad strongphase \tag{13a}$$

$$< R^2 >= \frac{2La}{\nu_0} \qquad weakphase \tag{13b}$$

This implies that the Hausdorf dimension d_h=1 for the strong phase and d_h=2 for the weak phase. In the presence of Coulomb interactions the above two phases would be part of the same continuum theory. Therefore it makes sense to talk about the order paremeter which distinguishes these two phases. We define the analogus of the magnetization in statistical models Θ to be $\frac{R^2}{L^2}$ then in the limit of long random walks we have:

$$< \Theta >= 1 \qquad strongphase \tag{14a}$$

$$< \Theta >= 0 \qquad weakphase \tag{14b}$$

The result (13) states that conventional QED with the scale $\frac{m}{a}$ identified as $\frac{T}{\gamma}$ is analogus to high temperature polymers while our new stongly coupled QED is analogus to low temperature polymers.

The beta function computed from (12) is

$$\beta(t_0(\Lambda_L)) = \frac{-(D-1)}{2\pi} {t_0}^2(\Lambda_L) \tag{15a}$$

where ${\alpha_0}^{-1} = t_0$. This implies that $t_0 = 0$ ($\alpha_0 = \infty$) is an ultraviolet stable fixed point. This beta function is non-perturbative in the sense of QED (power series expansion in α_0) although perturbative in $t_0 = \frac{1}{\alpha_0}$.

The beta function (15a), for very small t_0, is exactly the expression for the beta function of the $O(D+1)$ non-linear σ-model in two dimensions with coupling t_0. In fact in [12] we show that our curvature model in Euclidean (resp.Minkowski) is indeed equivalent to O(D+1) (resp. O(D,1)) non-linear σ-model in two dimensions where the σ field is nothing but $\dot{x}$. It is known that the inclusion of long range interactions shifts the critical point of the σ-model in one or two dimensions to a finite value $t_{0_c} \neq 0$ and the σ-model beta function then reads

$$\beta(t_0(\Lambda_L)) = \frac{-(D-1)}{2\pi}(t_0(\Lambda_L) - t_{0_c})^2 \qquad t_0 > t_{0_c} \tag{15b}$$

Therefore we expect that the effect of the previously ignored long-range Coulomb interactions in our model is also described by equation (15b). The detailed analysis is complicated and outside the scope of this article. However we can intutively understand the physics of these Coulomb interactions and their subtlities by drawing once more the analogy with polymers. It is important to remark that the long Coulomb interactions in Minkowski signature become short range in Euclidean signature with a power law $\frac{e^2}{x^{D-3}}$ behaviour. It is natural to ask under what circumstances do such forces shift the critical coupling from zero or infinity to a finite value α_c. In the context of polymers it is known [5] that the inclusion of short range repulsive forces between two different chains can in principle shift the critical coupling to a finite value but the critical exponent η of the mean square distance $< R^2 >= L^{2\eta}$ is only affected by the self repulsive interactions between the links of the one chain. The curvature term is one such example of these self-interactions where η becomes one in the strong (rigid) phase. Therefore the presence of Coulomb interactions will not alter the rigidity of the paths of the point particle and one expects two distinct phases as in (13) seperated by a finite fixed point α_{0_c}. For any finite bare coupling $\alpha_0 < \alpha_{0_c}$ the continuum (renormalized) coupling constant $\alpha_{cont.}$ associated with the curvature is zero because the tangent-tangent correlation function vanishes in the continuum (absence of rigidity). This does not mean that the continuum coupling constant $\tilde{\alpha}_{cont.}$ associated with Coulomb interactions is zero. This is because the bare coupling in the Coulomb sector should renormalize differently. For $\alpha_0 > \alpha_{0_c}$ the coupling constant runs as in (c.f (16)) due to the non-triviality of the one dimensional curvature theory and the strong phase is characterized by a non-vanishing $\alpha_{cont.}$ as well as $\tilde{\alpha}_{cont.}$

Our discussion so far has been limited to the Euclidean signature and we now discuss the effect of continuuing to the Minkowski signature. The internal symmetry of our model can be easly seen from the effective action (1'). The velocity $\dot{x}^\mu$ is analogous to the spin vector σ^μ of the O(3,1) σ-model albeit with a square root-like interaction. In fact from the β-function comparison for very small t, the model (1') is equivalent [12] to the usual O(4,1) non-linear σ-model

in two dimensions. Evidently the effect of the square root in (1′) is to increase the dimensionality of the domain and the range of the spin vector by one. The non-compact nature of the symmetry group, as is well known, effectively changes the sign of the of the coupling constant t in the compact models [13]. In particular, the sign of the β-function in equation (15) changes sign. The beta function of our model of strongly coupled QED in the Minkowski signature is therefore:

$$\begin{aligned} \beta(\alpha_0(\Lambda_L)) &= \frac{-(D-1)}{2\pi}\left(\frac{\alpha_0(\Lambda_L)}{\alpha_{0_c}} - 1\right)^2 \qquad \alpha_0 > \alpha_{0_c} \\ &= 0 \qquad\qquad\qquad\qquad\qquad\qquad \alpha_0 < \alpha_{0_c} \end{aligned} \quad . \tag{16}$$

Thus the dramatic effect of continuuing to Minkowski signature is that α_0 approaches α_{0_c} at short distances. An important question is whether the above beta function preserves its sign after the inclusion of virtual fermions. We know that in weak QED the running of the coupling constant is due to screening effects (vacuum polarization). The quantum fluctuations of the fermion field which creates the virtual e^+e^- pairs are confined within a wavelength ω less than the Compton wavelength m_e^{-1}. Because of the dielectric nature of the medium, the effective charge e_{eff} of an electron is less than its bare charge e_0 and becomes smaller and smaller as we approach the Compton radius m_e^{-1} away from the origin $r = 0$. As $r >> m_e^{-1}$ the effective charge reaches an asymptotic fixed value e_{as} that is independent of the distance r. In the language of the renormalization group this means that the coupling constant stops running for distance scales which are much larger than m_e^{-1} . The coupling constant of our model on the otherhand, runs at all scales due to the nonperturbative fluctuations of virtual photons. Therefore we expect that the inclusion of fermion loops will not change the sign of the beta function beyond the Compton wavelength. In particular at large distance scales where confinement becomes important. We now return to (16) which we can solve by

$$\alpha_0(\Lambda_L) = \alpha_{0_c}\left(1 + \frac{\alpha_{0_c}}{\Omega + \frac{\alpha_{0_c}^2}{\alpha_0{}^* - \alpha_{0_c}}}\right) \tag{17a}$$

where

$$\Omega = \frac{3}{4\pi} ln \frac{\Lambda_L{}^2}{\nu^2} \tag{17b}$$

and $\alpha_0{}^*$ is the coupling at the scale $\Lambda_L = \nu$. For large Λ_L the coupling becomes

$$\alpha_0(\Lambda_L) = \alpha_{0_c}\left(1 + \frac{\alpha_{0_c}}{\Omega}\right) \tag{18}$$

which is independent of $\alpha_0{}^*$. That the coupling is determined by the fixed point α_{0_c} and ν is the so called dimensional transmutation. A similar phenomenon

occurs in QCD except there the fixed point is zero. The coupling $\alpha_0(\Lambda_L)$ approaches α_{0_c} from above at high energies i.e α_{0_c} is an U.V stable fixed point. At low energies $\alpha_0(\Lambda_L)$ increases until $\alpha_0 = \infty$ at $\Lambda_L = \nu_{QED}$:

$$ln\nu_{QED}^2 = ln\nu^2 - \frac{4\pi}{3}\frac{\alpha_{0_c}^2}{\alpha_0{}^*(\nu^2) - \alpha_{0_c}} \quad . \tag{19}$$

This behaviour of α suggests that e^+e^- states in our model of strongly coupled QED are confined. Unlike QCD, the beta function (16) is reliable at large distances since the perturbative expression in $\frac{1}{\alpha_0}$ is a very good approximation to the exact result at strong coupling. In further contrast to QCD the confining nature of the model arises from treating matter dynamically (summing over loops) and would not arise by considering the Wilson loop in the static limit.

We now turn to the question of the value of α_{0_c}. The analogy with statistical mechanics previously discussed suggest that the effect of the long range Coulomb interactions is to lift the critical temprature from zero to a finite value $t_{0_c} = \frac{1}{\alpha_{0_c}}$. To determine this value, we examine the seemingly unrelated problem of the e^+e^- bound states. In the weak phase there is no difficulty in solving the bound state problem for α less than one. For α greater than one . the problem is not well defined unless the Coulomb potential is cut-off at some distance $a = \frac{1}{\Lambda}$. To remove the cut-off and obtain a finite ground state energy, the coupling α must approach one as $\Lambda \to \infty$ [14], [15] . However from the running coupling (17) we know that this is nothing but the value $\alpha_{0_c} = 1$. This result is not affected by the Majorana nature of the bound states of e^+e^- [2].

We close this section with some remarks on related works [15]. A proposal for the beta functions of strongly coupled QED with an ultraviolet fixed point was given by Miransky. This beta function was obtained by simply postulating a dependence of the coupling on the cut-off Λ and demanding the ground state energy be fixed as $\Lambda \to \infty$. In contrast to Miransky, our coupling $\alpha_0(\Lambda_L)$ has a non-trivial dependence on the scale in absence of virtual fermions due to the nontrivial one dimensional quantum field theory associated with our curvature model (3). Our beta function, modulo the precise value of α_{0_c}, was determined by standard field theoretical techniques and differs from that of Miransky.

The confining Minkowskian beta function discussed here should also have implications for the string analog of the arc-length plus curvature model discussed by Polyakov [10].

III MASS FORMULA

The mass spectrum of our new curvature model of strong coupled QED can be calculated from the poles in the amplitude for e^+e^- annihilation. In the ordinary phase of QED (no curvature), the singularities of the amplitude consist of the spectrum of the usual positronium poles accumulating at the threshold 2μ.

In the strong phase of QED, there should actually be a spectrum of thresholds as well as poles accuumulating at each of these thresholds. The reason is that a single particle described by arc length plus curvature action is transmuted into an infinite number of particles with masses varying inversely with spin (Majorana particle). This transmuation phenomena is also encountered in D=3 [6] where a bosonic particle described by arc-length plus torsion action is transmuted into a fermionic particle. Here we shall assume without proof that the Coulomb interactions induce a fine structure on the spectrum of thresholds so that the poles are effectively given by the threshold spectrum. In any event, the Majorana spectrum should be contained in the exact spectrum. We shall calculate the thresholds (branch cuts) of the amplitude A in the strongly coupled phase through the semi-classical approximation. The stationary loops with two fixed points consist of two classical trajectories of opposite directions travelling togther. A factor of two in μ and α then arise from the stationary phase approximation. Given this double propagator picture of the stationary loop, the singularities can be calculated using the classical solutions of S_{eff} with μ and α doubled. The solutions in the arc-length gauge $\dot{x}^2 = 1$ are helicies about the time axis:

$$x^0 = y^0 + \sqrt{(1+(R\omega)^2)}t, \quad x^1 = y^1, \quad x^2 = y^2 + R sin\omega t, \quad x^3 = y^3 + R cos\omega t \quad (20)$$

where y^μ is a constant vector and μ, R, ω and α are related by $\mu R = \alpha(1+(R\omega)^2)$. The action calculated in terms of the classical solutions is

$$S_{eff} = MX + 2\alpha\dot{X} = \frac{2\mu}{\sqrt{1+(R\omega)^2}}X + 2\alpha\dot{X} \quad (21)$$

where X (resp. $\dot{X}$) is the distance between the intial and final points (resp. velocities). The amplitude in the stationary phase approximation is then

$$A = (\frac{1}{2M}(\frac{M}{2\pi X})^{3/2} e^{-\frac{M}{2}X - \alpha\dot{X}})^2 \quad (22)$$

where the over all power two arises from the two classical trajectories comprising the closed stationary loop. The singularities M_J^2 are obtained from the Fourier transform of (22). The subscript J is the intrinsic spin of the closed path (total spin) which is equal to $2\alpha R\omega$. In terms of the fine structure constant α, the scale μ, and the spin J the singularities are:

$$M_J^2 = \frac{(2\mu)^2}{1+(2\alpha_{QED})^{-2}J(J+1)}, \qquad J = 0, 1, 2, ... \quad (23)$$

where the classical J^2 is replaced by its quantum mechanical value $J(J+1)$. A spectrum such as (22) where mass varies inversely as spin is called a Majorana spectrum. This spectrum resembles an inverted hydrogen spectrum and distinguishes the strongly coupled phase.

The observation of the mass spectrum (23) in nature can serve as an experimental signature of our model of the strongly coupled phase of QED.

IV CONNECTION WITH GSI EXPERIMENTS

Recent observations of multiple narrow e^+e^- peaks resulting from heavy ion collisions [16] have been the center of many theoretical studies [17]. The most promising so far is the conjecture that such peaks are due to the decay of the bound e^+e^- system formed in a new strongly coupled phase of QED induced by the strong electromagnetic field of the colliding heavy nuclei. Although these peaks are not well understood experimentally, it is nonetheless intriguing to compare them with (23). In the context of our model, the observed peaks can be used to fit the free parameters 2μ and α. We find*

$$M_0 = 2\mu = 1833 \pm 8 KeV \quad (24a)$$

and indeed a strong "fine structure" constant

$$\alpha = 2.4 \pm 0.08 \, . \quad (24b)$$

The resulting mass spectrum obtained from (23) shown in table 1 is in excellent agreement with the observed peaks. It fits the data very well with $\chi^2 = 1.6$ for one degree of freedom. Although strictly speaking the peaks are in energy rather than invariant mass, qualitative arguments based on Doppler broadening [16] indicate that the energy coincides with the invariant mass to a good accuracy. The error incurred in this assumption is of order 0.1% and we shall neglect it since it is below the experimental errors of the observed peaks (cf.table 1).

The mass formula also predicts new phenomena which could serve as a test of our model. Though the spectrum implies an infinite number of e^+e^- bound states, their observation in terms of the decay into separate e^+ and e^- states in the ordinary phase of QED is limited due to the descending nature of the Majorana spectrum. A bound state with mass less than $2m_e$ could not decay into e^+ and e^- by conservation of energy. Since the bound state masses are infered through their decay into e^+e^-, there are only four new mass points to be observed. Taking errors into account, a fifth new mass point $M_7 = 989$ KeV may exist. The spin of each state is also predicted and is a signature of our model of strongly coupled QED distinguishing it from linear potential models [18] motivated by lattice QED.We summarize our predictions in table 1.

* It is important to note that the parameter 2μ is essentially determined by the heaviest observed mass. The heaviest mass reported by EPOS (1.83 MeV) differs from that reported by Orange (1.92MeV). Here we follow EPOS.

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References

[1] L.D. Landau, in Niels Bohr and the Development of Physics, ed. by W. Pauli (McGraw Hill, New York 1955).

[2] M. Awada and D. Zoller, To appear in Phys.LettB. and Cincinnati preprint 02-92 (1992)

[3] P.A.M Dirac, Proc. Roy. Soc., Ser.A 167, 148 (1938).

[4] M. deGennes Concepts in Polymer Physics.

[5] M.S. Plyushchay Mod.Phys.Lett.A Vol.3, 1299 (1988).

[6] A.M. Polyakov, Mod. Phys. Lett., A3, 325 (1988).

[7] K. M. Case Phys.Rev. 80 ,797 (1950).

[8] R. Feynman and J. Wheeler Rev. Mod. Phys.,17, (1945), 157 : R. Feynman, Phys. Rev.,74, (1948),939.

[9] R. Pisarski, Phys. Rev. D34, 670J, Ambjorn et al., Euro. Phys. 3 1059 (1987) (1986).

[10] A. M. Polyakov, Nucl. Phys. B268(1986)406.

[11] M. Creutz, Quarks, Gluons, and Lattices, Cambridge, CUP (1983)

[12] M.Awada, M.Ma, and D.Zoller Cincinnati preprint August 04-1992

[13] Zinn-Justin, Quantum Field Theory and Critical Phenomena, Oxford University press 1989.

[14] Ya.B. Zeldovich and V.S.Popov, Usp.Fiz.Nauk 105 (1971) 405

[15] V.A. Miransky et al., Il Nuovo Cim. 90A, 149 (1985); V.A. Miransky and P.I. Fromin, Sov. J. Part. Nucl. 16, 203 (1985).

[16] T. Cowen and J. Greenberg "Physics of Strong Fields"NATO ASI series ed. W. Greiner (1987), J. Schweppe et al., Phys. Rev. Lett. 51, 2261 (1983); M. Clemente et al., Phys. Lett. 137B , 41 (1984); T. Cowen et al., Phys. Rev. Lett. 56, 444 (1986); H. Tsertos et al., Phys. Lett. 162B, 273 (1985); H. Tsertos et al., Z. Phys. A 326, 235 (1987); W. Koenig et al. Phys. Lett 218B, 12 (1989).

[17] A. Chodos, Comments Nucl. Part. Phys. 17, 211 (1987). D.G. Calidi and A. Chodos, Phys. Rev. D36, 707 (1987); L.S. Celenza et al., Phys. Rev. Lett. 57, 55 (1986); Y.J. Ng and Y. Kikuchi, Phys. Rev. D36 2880 (1987).

[18] Paola Cea, Phys. Rev. D39, 340 (1989).

$\alpha = 2.4 \pm 0.08$		$\chi^2 = 1.6$	
Mass (KeV)	Experiment	Theory	Spin
M_0	1831 ± 8	1833	0
M_1	1782 ± 20	1758	1
M_2	1630 ± 8	1632	2
M_3		1486	3
M_4		1341	4
M_5		1208	5
M_6		1091	6

Table I

It is interesting to point out that the fourth point we predict based on the EPOS data agrees with a fourth observed peak at 1498 ± 20 KeV reported by the ORANGE group.