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# Confining Beta Function For Strongly Coupled QED

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We give a field theoretic derivation of the beta function for a recently proposed model of strongly coupled QED. The beta function is non-trivial in the absence of virtual fermions due to the non-perturbative vacuum fluctuation of the gauge field. The running coupling approaches an ultra-violet stable fixed point $\alpha_c = 1$ at short distances and increases at large distances indicating confinment. This behaviour depends crucially on the Minkowskian signature and does not occur in the Euclidean signature.

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A new model of QED exhibiting two distinct phases was recently proposed [1]. The model arises through a non-perturbative gauge invariant regularization of the Wilson loop. It is characterized by a curvature correction to the perimeter law in the strong coupling phase which is absent in the weak coupling phase. In this letter we shall study the beta function in the strong coupling phase of the model. The running coupling constant $\alpha$ decreases at short distances to the fixed point $\alpha_c = 1$ while at large distances the coupling increases indicating confinement.

The new model is phrased in terms of the Feynman spacetime picture of QED. The matter is described by a qunatum mechanical point particle $x^\mu$ , $\mu = 1, 2, ...D$, and the radiation by a quantum field $A_\mu$. The effective action obtained by integrating out $A_\mu$ is

$$S_{eff} = \mu_0 \oint_C ds + \alpha_0 \oint_C ds\kappa(s) + 2\alpha_0 \oint_{C_{e^+}} \oint_{C_{e^-}} dx^\mu dy^\mu \delta\left(\left|(x-y)^2\right|\right) \qquad (1)$$

where $\mu$ is the mass scale, $\alpha$ the fine structure constant, $ds$ the arclength, the curvature $\kappa$ is the length of the proper acceleration , $C_{e^+}$ the positron trajectory, $C_{e^-}$ the electron trajectory, and $C = C_{e^+} U C_{e^-}$. In eq.(1) we have ignored a complex piece which is irrelavent for the analysis in this paper. The last term of (1) represents the relativistic version of the coulomb interaction[2]. The curvature term arises from a non-perturbative gauge invariant regularization of the expectation value of the Wilson loop [1]. This new term characerizes our model. An essential feature of the effective action is the adoption of new boundary conditions in the higher derivative variational principle. The velocities and positions of $e^+$ and $e^-$ at the initial and final points are fixed. There are two principle

results of [1]. First the model of QED defined by the quantum theory of (1) has two distinct phases. One is conventional QED and the other is a new strongly coupled phase. Second the bound states of $e^+e^-$ exhibits a Majorana spectrum depending inversely on the square of the spin. It resembles an inverted hydrogen spectrum and distinguishes the strongly coupled phase. This is in sharp constrast with linear potential models motivated by lattice QED which have Regge type spectrum increasing with spin.

It has been long conjectured [3] that the mysterious narrow $e^+e^-$ peaks observed in heavy ion collisions [4] should be explained by a strong coupling confining phase of QED. Indeed, the spectrum predicted by our new model is in remarkable agreement with the observed peaks. The coupling $\alpha$ determined phenomenologically from the mass formula is of order 2.5 which implies that the theory is certainly strongly coupled. Furthermore the mass formula predicts four new resonances as well as the spin assignment. The validity of the strong phase explanation of the GSI experiments requires that the $e^+e^-$ state produced is confined. It is the purpose of this paper to show that our new model of strongly coupled QED has a running coupling $\alpha$ with a confining character which approaches $\alpha_c = 1$ at high energies.

The two distinct phases can be exposed most simply by considering $S_{eff}$ in absence of Coulomb interactions. More detailed features of the two phases can later be seen when we include the long range Coulomb interactions. The phases of the quantum theory of (1) can be seen in an Euclidean lattice regularization [1], [5]. The dramatic effects of continuuing to Minkowski space-time will be discussed subsequently. We discretize the domain space of $x^\mu(s)$ by setting $x_i^\mu = x^\mu(ia)$



where a is the lattice spacing. The action then reads

$$S = \mu_{0L}|x_i - x_{i-1}| + 2\alpha_{0L}|sin\frac{\theta_i}{2}| \tag{2a}$$

where $\theta_i$ is the angle between the two links $l_i = x_i - x_{i-1}$ and $l_{i+1}$, and $\alpha_{0L}, \mu_{0L}$ are the values of the bare coupling and mass scale respectively on the lattice. To proceed further in quantizing $S$ we replace the Euclidean spacetime $R^d$ by a lattice $Z^d$. The path in $Z^d$ is now a random walk $z = x_0, x_1, ..., x_n$ where $x_i$ and $x_{i-1}$ differ by the unit vector $l_i$ on the lattice. We can view $z$ as a set of links $l_1, ..., l_n$. The action can be cast as an anisotropic random walk

$$S(z) = \sum_{i=1}^{n} E(l_i \cdot l_{i+1}) \tag{2b}$$

where $E = \delta_1$ if $l_i$ is parallel to $l_{i+1}$, $\delta_2$ if $l_i$ is perpendicular, and $\delta_3$ if $l_i$ is antiparallel. Agreement with action (2a) implies

$$\delta_1 = \mu_{0L} \qquad \delta_2 = \mu_{0L} + \sqrt{2}\alpha_{0L} \qquad \delta_3 = \mu_{0L} + 2\alpha_{0L}\ . \tag{2c}$$

The propagator of the discretized theory is

$$G_\delta(X_0, Y_0) = \sum_{z:X_0 \to Y_0} e^{-S(z)} \tag{3}$$

where $X_0 = (x_0, \dot{x}_0)$ and $Y_0 = (y_0, \dot{y}_0)$.

The two distinct phases correspond to the two distinct ways of taking the continuum limit. This tantamount to having two fixed points on the critical surface of the theory,

$$\xi = e^{-\mu_{0L}} + e^{-\mu_{0L} - 2\alpha_{0L}} + 2(D-1)e^{-\mu_{0L} - \sqrt{2}\alpha_{0L}} = 1\ . \tag{4}$$

Eq.(4) is obtained by setting the mass scale $M(\mu_{0L}, \alpha_{0L})$ associated with the propagator (3) to zero. The distinguishing feature of the curvature term, which

represents the repulsive interactions between the links, is the rigidity that it gives to the paths. This rigidity is quantitatively measured in terms of the tangent to the chain of the Wilson loop of length L by the normalized tangent-tangent correlation function

$$\psi(s-s') =< t(s)t(s') >= e^{-\nu_{0L}\frac{(s-s')}{a}} = e^{-\nu_{0L}(n-n')} \tag{5a}$$

where the average is weighted with respect to the Boltzman factor $e^{-S}$, $s$ and $s'$ specify points along the chain by their arc-length distance, and $\nu_{0L}$ is the inverse correlation length of the tangents,

$$\nu_{0L}(D,\alpha_{0L}) = ln(\frac{1+2(D-1)e^{-\sqrt{2}\alpha_{0L}}+e^{-2\alpha_{0L}}}{1-e^{-2\alpha_{0L}}}) \tag{5b}$$

The strongly coupled phase arises by demanding (3) have a non-zero continuum limit,i.e. $\nu_{0L} = 0$. The fixed point satisfying $\nu_{0L} = 0$ and equation (2) is $(\mu_{0L},\alpha_{0L}) = (0,\infty)$ for fixed $D$. The other fixed point corresponds to the absence of rigidity and this occurs for $\nu_{0L} = +\infty$. This is the infared stable point $(3ln2, 0)$ corresponding to the weakly coupled phase of QED. Thus the curvature term is irrelevent and $\psi$ goes to zero in the continuum limit.

Our new strongly coupled phase is characterized by three physical mass scales $M$, $\mu$, and $\nu$ two of which are independent. They are defined by $M = \Lambda_L M_0(\mu,\alpha_{0(}\Lambda_L))$, $\mu = \Lambda_L\mu_{0L}$, and $2(D-1)\nu = \Lambda_L\nu_{0L}(\alpha_{0L})$ where the scale $\Lambda_L = \frac{1}{a} \to \infty$ is the lattice cutoff. The new mass scale $\nu$ is analogous to $\Lambda_{QCD}$ (not to be confused with the above $\Lambda_L$) and determines the bare coupling at a particular large scale $\Lambda_L$ thru equation (5b). One obtains $\alpha_{0(}\Lambda_L) = \frac{1}{\sqrt{2}}ln\frac{\Lambda_L}{\nu}$. This is equivalent to the mass gap relation $\nu = \Lambda_L e^{-\frac{\sqrt{2}}{t_{0L}}}$ which gives a non-trivial



beta-funtion in the coupling $t_{0L} = \alpha_{0L}{}^{-1}$ (c.f.eq.(10a)). However, the fixed point $\nu_{0L}$ must also exist in the limit $D \to \infty$ i.e $\nu$ must have a non-trivial value except in $D = 1$ where it is zero. Therefore $t_{0L}$ must be proportional to $D-1$. Hence we set $t_{0L}$ to be $c^{-1}\sqrt{2}(D-1)t_0^*$ where $c$ is yet an arbitrary normalization constant. The mass gap relation then reads

$$\nu = \Lambda_L e^{-\frac{c}{(D-1)t_0^*}} . \tag{6a}$$

To determine $c$ we note that since the mass scale $\nu$ is being fixed, its physical value must be the same in the continuum and on the lattice. We will calculate the mass gap relation in the continuum by considering the large $D$ limit of the curvature effective action in the arc-length gauge:

$$S_{eff} = \alpha_0 \int_C ds(\sqrt{\ddot{x}^2} + \frac{\nu}{2}(\dot{x}^2-1) + \mu_0) \tag{1'}$$

where $\nu$ is the lagrange multiplier enforcing the arc-length gauge condition. In the large D limit the saddle point approximation applies. In this case the stationary loop $C$ with two fixed points consists of two classical trajectories of opposite directions travelling togther. Consequently a multiplicative factor of two will appear in front $S_{eff}$. Our analysis gives the following result:

$$\nu = \Lambda_{mom} e^{-\frac{2\pi}{D}(\frac{1}{t_0^*}+k)} \tag{6b}$$

where $t_0^*$ is the continuum bare coupling and k is a finite constant that is independent from the coupling but depends on finite renormalization effects of the coupling. Because of closed paths trajectories, eq.(6b) differs by a factor of two in the exponent from a similar result obtained in [10] for open paths. In computing (6b) we have neglected finite size effects.

From (6a,b) we obtain in the large $D$ limit

$$\frac{\Lambda_L}{\Lambda_{mom}} = e^{\frac{2\pi - c}{Dt_0^*} + k} \ . \tag{7a}$$

To fix $c$ we recall that the lattice and momentum cut-offs for (non)-abelian gauge theories are just related by an overall constant [12]. This determines $c$ to be $2\pi$. This result should not be suprising because we know that the coefficent of $t_0^*$ in (6) which is the coefficent of the beta function (c.f.(10a)) is a universal constant that is independent of the regularization scheme at least in the weak coupling $t$ (strong coupling $\alpha$). The normalized bare running coupling is therefore (we will drop the superscript $*$ in the rest of the paper unless stated otherwise)

$$\frac{1}{t_0(\Lambda)} = \alpha_0(\Lambda_L) = \frac{D-1}{2\pi} ln \frac{\Lambda_L}{\nu} \ . \tag{8}$$

Though the free parameters of the weak phase are the electron mass $M = m_e$ and the fine structure constant $\alpha$, in the strong phase, $\alpha$ at a particular scale is determined by dimensional transmutation and the free parameters are now $M$ and $\nu$.

The discrete order parameter of the two phases is obtained by computing the mean square distance

$$< R^2 >= \int_0^L ds \int_0^L ds' \psi(s - s').$$

We find

$$< R^2 >= L^2 \qquad strongphase \tag{9a}$$

$$< R^2 >= \frac{2La}{\nu_0} \qquad weakphase \tag{9b}$$

(In the continuum limit of the weak phase $a \to 0$ , $\nu_0 \to \infty$ and therefore



$< R^2 > \to 0$.) This is equivalent to the statement that the Hausdorf dimension $d_h$=1 for the strong phase and $d_h$=2 for the weak phase. Equivalently we can regard the presence or absence of the curvature in the continuum thoery as the order parameter which distinguishes the weak phase from the strong phase.

The beta function computed from (8) is

$$\beta(t_0(\Lambda_L)) = \frac{-(D-1)}{2\pi} {t_0}^2(\Lambda_L) \tag{10a}$$

where ${\alpha_0}^{-1} = t_0$. This implies that $t_0 = 0$ ($\alpha_0 = \infty$) is an ultraviolet stable fixed point. This beta function is non-perturbative in the sense of QED (power series expansion in $\alpha_0$) although perturbative in $t_0 = \frac{1}{\alpha_0}$.

The beta function (10a), for very small $t_0$, is exactly the expression for the beta function of the $O(D+1)$ non-linear $\sigma$-model in two dimensions with coupling $t_0$. It is know that the inclusion of long range interactions shifts the critical point of the $\sigma$-model in one or two dimensions to a finite value $t_{0_c} \neq 0$ and the $\sigma$-model beta function then reads

$$\beta(t_0(\Lambda_L)) = \frac{-(D-1)}{2\pi} (t_0(\Lambda_L) - t_{0_c})^2 \qquad t_0 > t_{0_c} \tag{10b}$$

Therefore we expect that the effect of the previously ignored long-range Coulomb interactions in our model is also described by equation (10b). We shall give an argument for the value of $t_{0_c}$ shortly.

Our discussion so far has been limited to the Euclidean signature and we now discuss the effect of continuuing to the Minkowski signature. The internal symmetry of our model can be easly seen from the effective action (1′). The velocity $\dot{x}^\mu$ is analogous to the spin vector $S^\mu$ of the O(3,1) $\sigma$-model albeit with

a square root-like interaction. In fact from the $\beta$-function comparison for very small t, the model (1′) is equivalent to the usual O(4,1) non-linear $\sigma$-model in two dimensions. Evidently the effect of the square root in (1′) is to increase the dimensionality of the domain and the range of the spin vector by one. The non-compact nature of the symmetry group, as is well known, effectively changes the sign of the of the coupling constant $t$ in the compact models [6]. In particular, the sign of the $\beta$-function in equation (10) changes sign. The beta function of our model of strongly coupled QED in the Minkowski signature is therefore:

$$\beta(\alpha_0(\Lambda_L)) = \frac{-(D-1)}{2\pi}(\frac{\alpha_0(\Lambda_L)}{\alpha_{0_c}} - 1)^2 \qquad \alpha_0 > \alpha_{0_c} \,. \tag{11}$$

Thus the dramatic effect of continuuing to Minkowski signature is that $\alpha_0$ approaches $\alpha_{0_c}$ at short distances. We now obtain the running coupling from (11). The solution is

$$\alpha_0(\Lambda_L) = \alpha_{0_c}(1 + \frac{\alpha_{0_c}}{\Omega + \frac{\alpha_{0_c}^2}{\alpha_0{}^* - \alpha_{0_c}}}) \tag{12a}$$

where

$$\Omega = \frac{3}{4\pi} ln \frac{\Lambda_L{}^2}{\nu^2} \tag{12b}$$

and $\alpha_0{}^*$ is the coupling at the scale $\Lambda_L = \nu$. For large $\Lambda_L$ the coupling becomes

$$\alpha_0(\Lambda_L) = \alpha_{0_c}(1 + \frac{\alpha_{0_c}}{\Omega}) \tag{13}$$

which is independent of $\alpha_0{}^*$. That the coupling is determined by the fixed point $\alpha_{0_c}$ and $\nu$ is the so called dimensional transmutation. A similar phenomenon occurs in QCD except there the fixed point is zero. The coupling $\alpha_0(\Lambda_L)$ approaches $\alpha_{0_c}$ from above at high energies i.e $\alpha_{0_c}$ is an U.V stable fixed point (cf.



Fig.1). at low energies $\alpha_0(\Lambda_L)$ increases until $\alpha_0 = \infty$ at $\Lambda_L = \nu_{QED}$:

$$ln\nu_{QED}^2 = ln\nu^2 - \frac{4\pi}{3}\frac{\alpha_{0_c}^2}{\alpha_0{}^*(\nu^2) - \alpha_{0_c}} \,. \tag{14}$$

This behaviour of $\alpha$ suggests that $e^+e^-$ states in our model of strongly coupled QED are confined. Unlike QCD, the beta function (11) is reliable at large distances since the perturbative expression in $\frac{1}{\alpha_0}$ is a very good approximation to the exact result at strong coupling. In further contrast to QCD the confining nature of the model arises from treating matter dynamically (summing over loops) and would not arise by considering the Wilson loop in the static limit.

We now turn to the question of the value of $\alpha_{0_c}$. The analogy with statistical mechanics previously discussed suggest that the effect of the long range Coulomb interactions is to lift the critical temprature from zero to a finite value $t_{0_c} = \frac{1}{\alpha_{0_c}}$. To determine this value, we examine the seemingly unrelated problem of the $e^+e^-$ bound states. In the weak phase there is no difficulty in solving the bound state problem for $\alpha$ less than one. For $\alpha$ greater than one , the problem is not well defined unless the Coulomb potential is cut-off at some distance $a = \frac{1}{\Lambda}$. To remove the cut-off and obtain a finite ground state energy, the coupling $\alpha$ must approach one as $\Lambda \rightarrow \infty$ [7], [9] . However from the running coupling (13) we know that this is nothing but the value $\alpha_{0_c} = 1$. This result is not affected by the Majorana nature of the bound states of $e^+e^-$ [1], [8].

We close this letter with some remarks on related works[9]. A proposal for the beta functions of strongly coupled QED with an ultraviolet fixed point was given by Miransky. This beta function was obtained by simply postulating a dependence of the coupling on the cut-off $\Lambda$ and demanding the ground state energy be fixed as $\Lambda \rightarrow \infty$. In contrast to Miransky, our coupling $\alpha_0(\Lambda_L)$ has

a non-trivial dependence on the scale in absence of virtual fermions due to the nontrivial one dimensional quantum field theory associated with our curvature model(1). Our beta function, modulo the precise value of $\alpha_{0_c}$, was determined by standard field theoretical techniques and differs from that of Miransky.

The confining Minkowskian beta function discussed here should also have implications for the string analog of the arc-length plus curvature model discussed by Polyakov [11].

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