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GRAVITATIONAL RADIATION

AND

MODIFICATION IN EINSTEIN EQUATION

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Abstract

Einstein's radiation formula has some weaknesses related to its derivation. It is shown that much of the criticisms based on the derivation are not valid to the formula itself because none of the weakness is essential to the derivation of a radiation formula. A modified derivation results in essentially the same formula for the binary stars. In this connection, it is crucial to realize that the "gauge condition" is not generally applicable. Moreover, this analysis shows that Einstein has implicitly made some important modifications of his equation. The resulting equation is a more versatile theory which includes radiation. Concurrently, it is also shown that linearized gravity would be valid only if the linearized conservation law is exact.

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I. Introduction.

General relativity suggests the existence of gravitational waves. However, gravity waves have never been directly observed although there is indirect evidence which supports energy loss by gravitational radiation [1,2]. Theoretically, the existence of gravity waves had been assumed a certainty to the first approximation. However, Einstein himself had serious doubts about their existence because he discovered in 1936 that linearized gravity is not reliable [3]. Although during 1950s theorists reached a consensus that gravity waves did indeed exist [4], the controversy concerning the famed "quadrupole radiation" formula remains [2,5,6]. In fact, Einstein's radiation formula is intrinsically inconsistent with the linearized "gauge condition" (see §III). This condition necessarily implies that the linearized conservation law is exact (see §II). This in turn implies that there is no radiation [2,6].

It appears that the radiation formula has some weaknesses in its derivation [2,5]. However, criticisms based on the derivation may not be valid to the formula itself. In this paper, it will be shown none of the weakness is essential to the derivation of a radiation formula. A modified derivation results in the same formula with an additional term which is expected to be small for the binary stars. In this analysis, it is crucial to realize that the "gauge condition" is not generally applicable [7].

II. The Initial Derivation of Radiation Formula.

Much of the criticism of Einstein's radiation formula is based on its derivation. Here, to identify the problems, major steps of the derivation are presented. Much of the presentation is based on Wald's [2] and Yu's analysis [6].

First, the non-linear Einstein's field equation reads

$$G_{ab} \equiv R_{ab} - \frac{1}{2} R g_{ab} = - K T_{ab} , \tag{1a}$$

where its source $K T_{ab}$ generally depends on the space-time metric g_{ab}. The harmonic coordinate condition [2,8] is

$$\frac{\partial}{\partial x^a}(|g|^{\frac{1}{2}} g^{ab}) = 0, \tag{1b}$$

where g is the determinant of the metric. For weak gravity, it is convenient to consider equations, which is expressed in terms of deviations γ_{ab} ($\equiv g_{ab} - \eta_{ab}$) from the flat metric. Then, eqs. (1a) and (1b) are respectively linearized to [2,8]

$$G_{ab}{}^{(1)} = - K T_{ab} , \tag{2a}$$

where

$$G_{ab}{}^{(1)} \equiv \frac{1}{2}\partial^c\partial_c\bar{\gamma}_{ab} - \partial^c\partial_{(b}\bar{\gamma}_{a)c} + \frac{1}{2}\eta_{ab}\partial^c\partial^d\bar{\gamma}_{cd} ;$$

and

$$\partial^a\bar{\gamma}_{ab} = 0 , \tag{2b}$$

where

$$\bar{\gamma}_{ab} \equiv \gamma_{ab} - \frac{1}{2}\eta_{ab}\gamma \ , \text{ and } \quad \gamma \equiv \eta^{ab}\gamma_{ab} \ .$$

The "transverse gauge" (2b) sufficiently reduces (2a) to

$$\frac{1}{2}\partial^c\partial_c\bar{\gamma}_{ab} = - K\ T_{ab}. \tag{2c}$$

It follows eq. (2b) that the linearized conservation law,

$$\partial^a\ T_{ab} = 0 \ , \tag{2d}$$

is necessarily <u>exact</u>. The effective stress-energy tensor of the gravitational field, valid to second order, is assumed to be [2]

$$t_{ab} = G_{ab}{}^{(2)}/K \tag{3a}$$

where

$$G_{ab}{}^{(2)} = G_{ab} - G_{ab}{}^{(1)}.$$

Then the total energy associated with γ_{ab} is

$$E = \int t_{00}d^3x \tag{3b}$$

and the rate of energy loss due to radiation is

$$\frac{dE}{dt} = -\int t_{a0}dS^a = \frac{-1}{K}\int G_{a0}{}^{(2)}dS^a \tag{3c}$$

To evaluate formula (3c), one must solve eq. (2c). The solution is

$$\bar{\gamma}_{ab}\ (x^i,\ t) = -\frac{K}{2\pi}\int\frac{1}{R}T_{ab}[x'^i,(t - R)]d^3x' \tag{4a}$$

where

$$R^2 = \sum(x^i - x'^i)^2$$

In the far field from the source, eq.(4a) can be approximated by

$$\bar{\gamma}_{ab} = -\frac{K}{2\pi}\frac{1}{r}\int T_{ab}[x'^i,(t - R)]d^3x' \tag{4b}$$

To establish relationship between different components of T_{ab}, let us consider the Fourier transform

$$\hat{T}_{ab}(\omega,\ x^i) = -\frac{1}{\sqrt{2\pi}}\int T_{ab}(t,\ x^i)e^{i\omega t}\ dt \tag{5a}$$

Then the linearized conservation law eq.(2d) implies

$$-i\omega\hat{T}_{0b}(\omega,\ x^i) = \sum_{a=1}^{3}\frac{\partial\hat{T}_{ab}}{\partial x^a} \tag{5b}$$

It follows (5b) that

$$\int\hat{T}^{jk}\ d^3x = -\frac{\omega^2}{2}\int\hat{T}^{00}\ x^k x^j\ d^3x \tag{5c}$$

Consequently,

$$\int T^{jk}\ d^3x = \frac{1}{2}\frac{\partial^2}{\partial t^2}\int T^{00}\ x^k x^j\ d^3x\ . \tag{6a}$$

Substituting (6a) to formula (4b), one obtains,

$$\bar{\gamma}^{jk} = -\frac{K}{2\pi r}\frac{\partial^2}{\partial t^2}\int T^{00}\, x^k x^j\, d^3x \tag{6b}$$

Then the rate of energy loss formula (3c) becomes

$$\frac{dE}{dt} = -\frac{G}{45}\,(\ddot{q}'_{kj}\,\ddot{q}'^{kj}) \tag{7}$$

where q_{jk} is the quadrupole moment tensor of the material system. Eq. (7) is the famed "quadrupole radiation" formula.

III. Theoretical Issues and Modifications.

Einstein once remarked that the left-hand side of his equation was granite, but the right-hand side (the source) was sand. It will be shown that, in the problem of radiation loss, he has actually modified the source with the concept of anti-gravity implemented. This modification would have been agreed upon by Pauli [9] who pointed out that general relativity does not provide a physical interpretation for the sign of the gravitational coupling constant. At vacuum, eq.(2a) and eq.(3a) imply that

$$G_{ab} = G_{ab}^{(1)} + G_{ab}^{(2)} = Kt_{ab}\ . \tag{8a}$$

Thus, t_{ab} is actually a <u>tensor</u> although it appeared to be a pseudotensor in eq. (3a). Eq.(8a) means that the <u>factual</u> assumption at vacuum is

$$T_{ab} = -\,t_{ab}\ . \tag{8b}$$

Eq. (8) means that the tensor t_{ab} has anti-gravity coupling. This is also

necessary. The Einstein tensor of a gravitational plane wave has, on the average, a different sign from that of the massive matter [7,10].

It will be shown that the current justification for eq. (3a) is not valid. Let us consider the second order approximation that

$$G_{ab}(\gamma_{cd}) \simeq G_{ab}^{(1)}(\gamma_{cd}^{(1)}) + G_{ab}^{(1)}(\gamma_{cd}^{(2)}) + G_{ab}^{(2)}(\gamma_{cd}^{(1)}),$$

where

$$\gamma_{cd} = \gamma_{cd}^{(1)} + \gamma_{cd}^{(2)} , \tag{9a}$$

and $\gamma_{cd}^{(1)}$ and $\gamma_{cd}^{(2)}$ are respectively the first and the second order approximations of γ_{cd}. Eq. (9a) seems to imply

$$G_{ab}^{(1)}(\gamma_{cd}^{(2)}) \simeq - G_{ab}^{(2)}(\gamma_{cd}^{(1)}) , \tag{9b}$$

and eq. (9b) was used to justify (3a) [2]. However, eq. (9b) is not generally valid since $\partial^a G_{ab}^{(1)} \equiv 0$, but eq. (1a) implies

$$\partial^a G_{ab}^{(2)}(\gamma_{cd}^{(1)}) \simeq - K\partial^a T_{ab} . \tag{9c}$$

Thus, eq. (9b) is not valid unless the term $K\partial^a T_{ab}$ is of third order deviations.

Eq. (8) and eq. (9) reveal that, in practice, Einstein has extended his formula to the following form,

$$R_{ab} - \frac{1}{2} Rg_{ab} = - K[T_{ab}(m) - t_{ab}(g)] = - KT_{ab} \tag{10a}$$

where $T_{ab}(m)$ is the stress tensor for massive matter and $t_{ab}(g)$ is for the field energy which includes radiation energy. Eq. (10a) implies

$$\nabla^a T_{ab}(m) = \nabla^a t_{ab}(g) = 0. \tag{10b}$$

Energy-momentum conservation requires they are conserved separately. This seems strange at the first sight. But, this is necessary. The gravitational force, as shown in the geodesics, has no separate term as the other force such as the Lorentz force. Also, from eq. (10a), if the stress tensor T_{ab} is zero at vacuum, then it is not possible to have gravitational radiation since $t_{ab} = 0$ (see also [3,11]).

To justify the validity of the following Maxwell equation,

$$\partial^c \partial_c \bar{\gamma}_{ab} = -2KT_{ab}(m) \ , \tag{10c}$$

for weak gravity, linearized gravity and the gauge condition were used. However, in linearized gravity, eq. (2a) requires that the linearized conservation law eq. (2d) is exact because $\partial^a G_{ab}{}^{(1)} \equiv 0$. For eq. (10c), the gauge condition eq. (2b) would creat a similar problem. Fortunately, it has been shown that, neither eq (2a) nor eq. (2b) is a necessary condition for eq. (10c) [7,10]. Thus, eq. (10c) can be independently justified from a physical ground. Eq. (10c) produces Newton's law. In vacuum, eq. (10c) implies that a gravity wave propagates with the speed of light. Moreover, the inclusion of radiation energy in eq. (10c) would lead to an unphysical equation. For example, if an electromagnetic plane wave is the source, then eq. (10c) is exact [7], but $G_{ab}{}^{(1)} = - K\ T_{ab}(m)$ is not valid.

The fact that Einstein's derivation is not self-consistent [2,6] is due to the "gauge" condition (2b). As pointed out by Wald [2] that eq. (2d) implies that "two stars would not orbit each other but would move on geodesics of the flat metric." This means $\ddot{q}'_{jk}$ is zero and therefore no gravitational radiation. Stephani [8] also recognizes that eq. (2d) is not consistent with self-gravitation. However, it seems, they have overlooked the fact that it is eq. (2b) that requires eq. (2d) to be <u>exact</u>. If eq. (2b) were valid, it should not lead to an unphysical consequence unless eq. (10c) is not valid. Thus, it is necessarily to replace (2b) with a condition of less restrictive. Fortunately, the "gauge condition" has been proven to be not generally valid [7,10].

In the above derivation, eq. (2d) is used only to obtain eq. (5b). Due to weak gravity, eq. (5b) can be derived with $\nabla^a T_{ab} = 0$. Note that the second order accuracy of $G_{ab}{}^{(2)}$ remains the same. Thus, the "gauge condition" is not needed if eq. (10c) can be justified independently.

However, when eq. (10c) is used directly without the gauge condition, the tensor t_{ab} becomes

$$t_{ab} = [G_{ab}{}^{(2)} + H_{ab}{}^{(1)}]/K \tag{11a}$$

where

$$H_{ab}{}^{(1)} = \frac{1}{2}(\eta_{ab}\partial^c L_c - \partial_b L_a - \partial_a L_b), \text{ and } \quad L_a = \partial^c \bar{\gamma}_{ca} .$$

And

$$-2\partial^a H_{ab}{}^{(1)} = \partial^c \partial_c L_b = \partial^c \partial_c \partial^a \bar{\gamma}_{ab} = -2K\ \partial^a T(m)_{ab} . \tag{11b}$$

Eq. (11b) implies that the inhomogeneous solution of L_a is of second order

deviations. If the motion is periodic, the metric and $T(m)_{ab}$ would also be periodic. Then, on the time average, L_a and and therefore $H_{ab}^{(1)}$ would be zero. Now, due to weak gravity, eq. (10b) implies

$$\partial^a t_{ab} = \partial^a G_{ab}^{(2)}/K + \partial^a T(m)_{ab} \simeq \nabla^a t_{ab} = 0. \qquad (12a)$$

Thus, the field energy is from matter. On the average, the tensor t_{ab} is essentially $G_{ab}^{(2)}/K$ as assumed [2]. It follows eq. (12a) that

$$\frac{dE}{dt} = - \int t_{a0} dS^a = - \frac{1}{K}\int G_{a0}^{(2)} dS^a + \frac{1}{2K}\int (\partial_0 L_a + \partial_a L_0) dS^a \qquad (12b)$$

Eq. (11) implies that L_b is linear to $K\partial^a T(m)_{ab}$. For the case of a near periodic motion, the time average of $H_{a0}^{(1)}$ would be negligible. Then, eq. (12b) would in effect be the same as eq. (3c).

IV. Conclusion and Discussion.

In conclusion, linearized gravity would be valid only if the linearized conservation law is exact. The radiation formula requires that the gauge condition should not be valid. To handle radiation due to self-gravitation, Einstein has implicitly extended his equation to a more general form. Such an extension would be allowed by the three crucial tests (see also [11]). The Einstein tensor requires the coupling of t_{ab} must be anti-gravity. Physically, this is expected since the field tensor t_{ab} is induced by $T(m)_{ab}$. Moreover, t_{ab} only appears to be a pseudotensor which is a focus of severe criticisms [5]. Now, it seems, much of the criticism of general relativity has become meaningless. They are as follows:

i) Einstein's quadrupole formula for gravitational radiation is not a corollary of general relativity.

ii) When a gravitational field and matter are taken in conjunction, the general theory of relativity has not, and cannot have, energy momentum conservation laws.

iii) It does not follow from general relativity, in principle, that a double star losses its energy by gravitational radiation.

iv) General relativity does not have the classical Newtonian limit and, hence, it does not satisfy a fundamental physical principle, that is the correspondence principle.

Some colleagues have linked the validity of "gauge" and linearized gravity with Einstein's radiation formula. The fact is that both linearized gravity and the "gauge condition" is inconsistent with the radiation formula. Either of them requires the approximate linearized conservation law to become exact (althoug such a conservation law does not assure the gauge condition [7,10]). Fortunately, the validity of the "linearized" equation (10c) does not depend on either of them. From the viewpoint of physics, radiation should be independent of the mathematical method used. However, this would leave the justification of the "linearized" equation (10c) open. In this paper, its justification is based on physical considerations. It seems, once the "linearized" equation is accepted, in this new derivation, the weakness related to linearized conservation law no longer exists.

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