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IS THERE ANY UPPER LIMIT ON THE HIGGS MASS ?

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ABSTRACT

A simple argument to recover in perturbation theory the recently discovered asymptotic freedom of massless $\lambda\phi^4$ theories is presented. It is pointed out that no upper limit on the Higgs mass can be obtained from the vanishing of the renormalized scalar self-coupling in the broken phase.

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The problem of the "triviality" of $\lambda\phi^4$ theories is one of the most challenging (and controversial) in the area of today quantum field theories. In fact, by accepting the triviality hypothesis, the self- interacting scalar sector of the Weinberg-Salam[1] theory has to be replaced by some more fundamental structure at some, unknown, energy scale whose magnitude increases by decreasing the Higgs boson mass.

Despite of the general belief, however, rigorous arguments[2] do not exclude, in four space-time dimensions, a meaningful continuum limit for $\lambda\phi^4$ theories which corresponds to asymptotic freedom. Therefore, it is remarkable that such a result has been obtained in refs.[3,4,5] by analyzing the effective potential of massless $\lambda\phi^4$ theories which exhibit spontaneous symmetry breaking (SSB)[6].

The peculiarity of massless $\lambda\phi^4$ theories[7] can be understood by recalling some properties of the exact effective potential[8] $V(\phi)$. It may happen that in some approximation (one loop, variational method,..) say $V^{appr.}(\phi)$, the effective potential is not convex downward in some range of ϕ. This result implies that $V^{appr.}(\phi)$ cannot be trusted in that region but, still, its absolute minima may have an important physical meaning (i.e. signaling the occurrence of SSB). For instance, as discussed in ref.[9], it is possible to recover the convexity property by improving on the gaussian approximation, e.g. going beyond the gaussian subspace of the whole functional space, but, in the end, in an infinite 3-dimensional volume, only the absolute minima of the non convex approximation have a non vanishing probability content. Therefore, the situation is similar to a quantum mechanical double well potential in the limit $\hbar \to 0$ where there is no tunnelling and the ground state wave function is non vanishing only at the minima of the classical potential. This result, however, does not imply that SSB in quantum field theory is a classical phenomenon. In fact the convexity property implies that the exact effective potential has to be completely flat

[10,11] in the region enclosed by the gaussian minima $\phi = \pm\overline{\phi}$ as any positive curvature at $\phi = 0$ would never allow for SSB. As a consequence, in the presence of SSB, the effective potential cannot be an analytic function[8] since it cannot be reconstructed by a Taylor expansion starting in the interval $-\overline{\phi} < \phi < +\overline{\phi}$ where all its derivatives vanish identically. Therefore, the only mass renormalization condition which is compatible with the exact properties of $V(\phi)$ of being the Legendre transform of the generating functional for connected Green functions and, at the same time, can produce SSB is the massless case as defined from the condition[7]

$$\frac{d^2V(\phi)}{d\phi^2}\Big|_{\phi=0} = 0 \tag{1}$$

In other words, even starting with a non convex classical potential whose negative mass squared is redefined order by order in perturbation theory, one will, unavoidably, end up with a non negative curvature at the origin and the only possibility to obtain SSB is associated with the above condition thus discovering, in the end, that the theory does not possess any intrinsic scale, in its symmetric phase, and SSB is the effect of dimensional transmutation i.e. of the asymptotic freedom of the massless theory.

As shown in refs.[4,5], asymptotic freedom is discovered in connection with a non trivial rescaling of the bare vacuum field $\phi =< \Phi >$, whose anomalous dimension starts to $O(\lambda)$, rather than $O(\lambda^2)$, as first suggested in ref.[12]. In fact, by defining the renormalized field $\Phi_R(x) = \phi Z_\phi^{-1/2} + h(x)Z_h^{-1/2}$, one finds the different behaviour of the two renormalization constants $Z_\phi \sim 1/\lambda$ and $Z_h = 1+O(\lambda)+O(\lambda^2 ln\Lambda)$ so that $Z_\phi^{-1/2} \to 0$ and $Z_h^{-1/2} \to 1$ in the infinite cutoff limit $\Lambda \to \infty$ where $\lambda \to 0$ and $\lambda ln\frac{\Lambda}{m_h} = fixed$, m_h being the mass of the shifted field $h(x)$. The behaviour $Z_\phi^{-1/2} \to 0$, typical of a composite field, expresses the non trivial dynamical content of the long range components of the underlying massless theory which, as discussed in ref.[13], needs a singular field renormalization

constant to satisfy the stringent constraints associated with its conformal invariance and, at the same time, be interacting (the non triviality of conformally invariant, massless $\lambda\phi^4$ theory, has been recently rediscovered, with a completely different technique, also in ref.[14]). The shifted field, on the other hand, is trivially free in the infinite cutoff limit due to $Z_h = 1$ and all interaction effects are reabsorbed into its mass.

The fact that the same theory, namely massless $\lambda\phi^4$, can exhibit both asymptotic freedom in its symmetric phase and triviality in its broken phase, could have been deduced directly from the perturbative expansion for the massive theory without no need of investigating the effective potential. Indeed, let us consider the zero momentum coupling as defined in terms of the running coupling constant $\lambda = \lambda(\mu)$ in the $\overline{MS}$ scheme by taking into account the basic one loop bubble of particles with mass m

$$\lambda_R = \lambda(\mu) - \frac{3\lambda^2(\mu)}{32\pi^2} ln \frac{\mu^2}{m^2} \tag{2}$$

and introduce $t = \frac{1}{2} ln \frac{\mu^2}{m^2}$. Our problem is to keep fixed (i.e. μ- independent) λ_R and, at the same time, to be able to take the limit $t \to \infty$ to recover the properties of the continuum theory. From the equation

$$\frac{d\lambda_R}{dt} = 0 \tag{3}$$

we obtain (we assume $\frac{dm^2}{dt} = 0$, and set $\lambda(\mu) = \lambda(t)$)

$$\frac{d\lambda(t)}{dt} - \frac{3\lambda^2(t)}{16\pi^2} - \frac{3t\lambda(t)}{8\pi^2}\frac{d\lambda(t)}{dt} = 0 \tag{4}$$

In the usual perturbative analysis, one neglects the third term on the left hand side of the above equation since, for small $\lambda(t)$, it contributes a term $O(\lambda^3(t))$ and solves the equation

$$\frac{d\lambda(t)}{dt} = \frac{3\lambda^2(t)}{16\pi^2}$$

Then, by introducing the arbitrary boundary condition at $t = 0$, one finds

$$\lambda(t) = \frac{\lambda(0)}{1 - \frac{3\lambda(0)t}{16\pi^2}} \tag{5}$$

and is *forced* to identify $\lambda_R = \lambda(0)$ for consistency with the basic one loop relation (2) which is seen as the the first term of the infinite expansion

$$\lambda_R = \lambda(0) = \frac{\lambda(t)}{1 + \frac{3\lambda(t)t}{16\pi^2}} \tag{6}$$

This procedure, where one ignores the possibility that for small t $\lambda(t) \sim 1/t$, leads to the well known problem of the Landau pole, since no limit $t \to \infty$ is possible and it is clear that the way in which one handles with the one loop expression will affect the whole renormalization procedure. On the other hand, by keeping the third term in eq.(4) we obtain the equation

$$\frac{d\lambda(t)}{dt} = \frac{-3\lambda^3(t)}{16\pi^2} \frac{1}{\lambda(t) - 2\lambda_R} \tag{7}$$

By assuming that $\lambda(t)$ and λ_R are both non negative we find that eq.(7) has to be studied separately for $\lambda(t) - 2\lambda_R > 0$ and for $\lambda(t) - 2\lambda_R < 0$ to preserve the uniqueness of the solution. In both cases, however, no limit $t \to \infty$ is possible if $\lambda_R > 0$. The only possibility is associated with the case $\lambda_R = 0$, i.e. ($b_o = \frac{3}{16\pi^2}$)

$$\lambda(t) = \frac{1}{b_o t} \tag{8}$$

$$\frac{d\lambda(t)}{dt} = -b_o \lambda^2(t) \tag{9}$$

which corresponds to assume that the particle mass is related to μ and $\lambda = \lambda(\mu)$ through the relation

$$m^2 = \mu^2 exp - \frac{32\pi^2}{3\lambda} \tag{10}$$

just the condition which corresponds to minimize the one loop potential for the massless theory[7] where $m^2 = \frac{\lambda\phi^2}{2}$. In this case, in fact, the limit $t \to \infty$ is possible and one

recovers that the asymptotic freedom of the massless theory implies the triviality of the shifted theory.

Let us now consider the higher loop contributions and introduce the variable

$$z = b_o t \lambda(t)$$

so that the basic one loop correction (2) has the form

$$\lambda_R^{(o)} = \lambda(t)(1-z)$$

In the region $z = 1-\epsilon$ ($\epsilon > 0$) and we are interested in the limit $z \to 1$ (i.e. in establishing the definite relation in eq.(10)) it is convenient to rearrange the terms of the perturbative expansion in $\lambda_R^{(o)}$ rather than in $\lambda(t)$. This procedure amounts to include, for any leading logarithmic term, the corresponding leading logarithmic correction, so that the effective expansion parameter is $z(1-z) < z$ rather than z itself. By adopting this type of expansion, which works also in the case $z << 1$ where, by the way, does not introduce any significant modification with respect to the usual analysis, we obtain the structure

$$\lambda_R^{(o)} = \lambda(t)(1-z)$$

$$\lambda_R^{(1)} = \lambda(t)(1-z)(1+z^2)$$

$$\lambda_R^{(2)} = \lambda(t)(1-z)(1+z^2+z^4)$$

.....

$$\lambda_R^{(N)} = \lambda(t)(1-z)(1+z^2+z^4+...z^{2N}) = \lambda(t)\frac{1-(z^2)^{N+1}}{1+z} \tag{11}$$

which, for any N, gives

$$\lambda_R^{(N)}|_{z=1} = 0$$

Notice that the limits $z \to 1$ and $N \to \infty$ do not commute. Indeed, for any $z = 1 - \epsilon < 1$ we obtain

$$\lambda_R = lim_{N\to\infty}\, \lambda_R^{(N)}(z) = \frac{\lambda(t)}{1+z} \tag{12}$$

as in eq.(6), whereas

$$\lambda_R = lim_{N\to\infty}\, lim_{z\to 1}\, \lambda_R^{(N)}(z) = 0 \tag{13}$$

implies again eqs.(8,9).

The analysis can be easily extended to include the next-to leading, next-to-next-to-leading...terms, by realizing that the perturbative relations have a validity only for $z < 1$. For instance, let us consider the recursive relations ($\lambda_R^{(N)}$ is defined in eq.(11), $b = \frac{b_1}{b_o}$)

$$\lambda_R^{(N,1)} = \frac{\lambda_R^{(N)}}{1 - b\lambda_R^{(N)} ln \frac{\lambda_R^{(N)}(1+b\lambda(t))}{\lambda(t)(1+b\lambda_R^{(N)})}}$$

$$\lambda_R^{(N,2)} = \frac{\lambda_R^{(N)}}{1 - b\lambda_R^{(N)} ln \frac{\lambda_R^{(N,1)}(1+b\lambda(t))}{\lambda(t)(1+b\lambda_R^{(N,1)})}}$$

.....

$$\lambda_R^{(N,M+1)} = \frac{\lambda_R^{(N)}}{1 - b\lambda_R^{(N)} ln \frac{\lambda_R^{(N,M)}(1+b\lambda(t))}{\lambda(t)(1+b\lambda_R^{(N,M)})}} \tag{14}$$

Eq.(14) contains, in the limit $N \to \infty$, $M \to \infty$, all the leading and next-to leading corrections to the zero momentum coupling. For any $z < 1$, by defining

$$\lambda_R = lim_{N\to\infty} lim_{M\to\infty} \lambda_R^{(N,M)}(z) \tag{15}$$

the condition

$$\frac{d\lambda_R}{dt} = 0$$

implies the two loop relation

$$\frac{d\lambda(t)}{dt} = +b_o(\lambda(t))^2 + b_1(\lambda(t))^3$$

However, in the limit $z \to 1$, we obtain identically

$$\lambda_R = lim_{N\to\infty} lim_{M\to\infty} lim_{z\to 1} \lambda_R^{(N,M)}(z) = 0 \tag{16}$$

and the associated relations (8,9).

The same procedure, when used in the effective potential for the massless theory, shows that the one loop minimum of ref.[7] is not modified by the higher order terms.

In conclusion, our results, which are in agreement with ref.[2] where it is shown that asymptotic freedom is the only possibility to generate a meaningful continuum limit for $\lambda\phi^4$ theories in four dimensions, indicate that the usual belief concerning the triviality of $\lambda\phi^4$ theories has to be reconsidered (see also ref.[14]). Indeed, the vanishing of the renormalized coupling in the broken phase for infinite cutoff is just the *consequence* of SSB and of an underlying dinamics which has no intrinsic ultraviolet cutoff. Therefore, a new approach to derive theoretical upper limits on the Higgs mass is required.

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